Anomalous spin-flip scattering rate near the magnetic-ordering temperature

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The spin-flip scattering rate of an electron, in a material which does exhibit long-ranged magnetic ordering, is calculated for temperatures close to the magnetic-ordering temperature T_M. The spin-flip scattering rate $1/\tau_s$ does have a critical temperature dependence and may be determined experimentally by its effects on superconducting materials.

I. INTRODUCTION

We consider the scattering of the conduction electrons by the thermal fluctuations of the localized magnetic moments, in a material which does exhibit long-ranged, magnetic ordering. As the temperature is reduced towards the magnetic-ordering temperature, the correlation function for the localized magnetic moments shows a critical temperature dependence. The magnetic correlation function determines the rate at which the conduction electrons undergo spin-flip transitions and provides the spin-flip transition rate with an anomalous temperature dependence.

Previous authors, Ora Entin-Wohlman *et al.*,[1] considered the spin-flip transition rate for ferromagnets, and we shall generalize the calculation to cover all the cases of long-ranged magnetic ordering, such as antiferromagnetic or helical ordering. The model used (that of P. G. de Gennes and J. Friedel[2]) consists of an *s-f* or *s-d* exchange Hamiltonian, to model the interactions between the conduction electrons and the localized magnetic moments, together with a Heisenberg exchange interaction between the localized magnetic moments. The calculation of the spin-flip transition rate of the conduction electrons is based on the Fermi golden-rule expression. This involves the correlation function for the localized magnetic moments. Near the magnetic-ordering temperature the magnetic-moment correlation function is sharply peaked for scattering wave vectors near the magnetic-reciprocal-lattice wave vectors $\vec{Q}$. However, the correlation function is weighted, not only by the usual phase-space density, but also by a factor which favors scattering through small wave vectors. The competition between the weighting and the correlation function for the localized magnetic moments produces a temperature dependence of the spin-flip transition rate that is sensitive to the type of magnetic ordering. This is to be contrasted with the anomalous resistivity at magnetic critical points, which always has the critical temperature dependence associated with the internal energy per spin.[2,3,5,10]

II. THE MODEL

The model is essentially the same as that of Ref. 2. The Hamiltonian is comprised of three terms.

$$\hat{H} = \hat{H}_s + \hat{H}_d + \hat{H}_{sd} , \tag{1}$$

where the components are as follows:

(i) The Hamiltonian for the conduction electrons $\hat{H}_s$, where

$$\hat{H}_s = \sum_{\vec{q},\sigma} E_{\vec{q}\sigma} a^{\dagger}_{\vec{q}\sigma} a_{\vec{q}\sigma} , \tag{2}$$

and $a^{\dagger}_{\vec{q}\sigma}$, $a_{\vec{q}\sigma}$, respectively create and destroy a conduction electron in the conduction band state that is characterized by a Bloch wave vector $\vec{q}$ and spin σ.

(ii) The Hamiltonian for the localized magnetic moments $\hat{H}_d$, where

$$\hat{H}_d = \frac{1}{N} \sum_{\vec{k}} J_{\vec{k}} \vec{S}_{\vec{k}} \cdot \vec{S}_{-\vec{k}} \tag{3}$$

and $\vec{S}_{\vec{k}}$ is the Fourier transform of the localized magnetic moments, and $J_{\vec{k}}$ is the Heisenberg exchange-interaction parameter.

(iii) The Hamiltonian which determines the interaction between the conduction electrons and the localized magnetic moments of $\hat{H}_{sd}$, where

$$\hat{H}_{sd} = -\frac{1}{N} \sum_{\vec{q},\vec{q}'} I_{\vec{q}-\vec{q}'} [(a^{\dagger}_{\vec{q}'\uparrow} a_{\vec{q}\uparrow} - a^{\dagger}_{\vec{q}'\downarrow} a_{\vec{q}\downarrow}) \times S^{z}_{\vec{q}-\vec{q}'} + a^{\dagger}_{\vec{q}'\downarrow} a_{\vec{q}\uparrow} S^{+}_{\vec{q}-\vec{q}'} + a^{\dagger}_{\vec{q}'\uparrow} a_{\vec{q}\downarrow} S^{-}_{\vec{q}-\vec{q}'}] , \tag{4}$$

and $I_{\vec{q}-\vec{q}'}$ is the de Gennes–Friedel interaction

strength, which is a smooth function of the scattering wave vector.

III. SPIN-FLIP TRANSITION RATE

The transition probability per unit time for a conduction-electron spin to flip is calculated from the Fermi golden-rule expression

$$\frac{1}{\tau_s} = \frac{4}{hN^2} \sum_{\vec{k},\vec{q}} |I_{\vec{k}}|^2 f(E_{\vec{q}})[1 - f(E_{\vec{k}+\vec{q}})] \times \int_0^\infty dt \exp[i/h(E_{\vec{q}} - E_{\vec{k}+\vec{q}})t] g_{\vec{k}}(t) , \tag{5}$$

where $f(x)$ is the Fermi distribution function and $g_{\vec{k}}(t)$ is the time-dependent correlation function for the localized magnetic moments

$$g_{\vec{k}}(t) = \langle \vec{S}_{\vec{k}}(t) \cdot \vec{S}_{-\vec{k}} \rangle - \langle \vec{S}_{\vec{k}} \rangle \cdot \langle \vec{S}_{-\vec{k}} \rangle . \tag{6}$$

Below the magnetic-ordering temperature, we do not believe that this formula will be accurate, but in Sec. IV we shall compare the results with that obtained by a calculation which is of higher order in $|I|^2$. The functional form of the temperature dependence is shown to be unaltered in this approximation. Assuming a free electron band for the conduction electrons, and that the derivative of the Fermi function is large in a width of $k_B T_M$ around the Fermi-level μ and zero elsewhere, we may rewrite the spin-flip transition rate as the power series

$$\frac{1}{\tau_s} = \frac{v_F}{\pi} \frac{m^2}{\hbar^5} \frac{1}{N} \sum_{\vec{k}} |I_{\vec{k}}|^2 \frac{1}{|\vec{k}|} \int \frac{d\omega}{2\pi} g_{\vec{k}}(\omega)[k_B T_M - \hbar\omega f(\mu) + \cdots] , \tag{7}$$

where $g_{\vec{k}}(\omega)$ is the Fourier transform of $g_{\vec{k}}(t)$.

The first term gives the contribution of the quasielastic scattering processes to the spin-flip transition rate, while the other terms represent the contributions from the inelastic scattering processes. In Sec. V, we shall argue that this series expansion is rapidly convergent so that, to a good approximation, the spin-flip scattering rate is given by the first term. In this case, when the limits on the ω integration are taken to be $+\infty, -\infty$, then the spin-flip transition rate is given by the weighted instantaneous correlation function for the localized magnetic moments, i.e.,

$$\frac{1}{\tau_s} = \frac{v_F}{\pi} \frac{m^2}{\hbar^5} \frac{k_B T_M}{N} \sum_{\vec{k}=0}^{2k_F} |I_{\vec{k}}|^2 \frac{1}{|\vec{k}|} g_{\vec{k}}(t=0) . \tag{8}$$

IV. TEMPERATURE DEPENDENCE OF $1/\tau_s$ ABOVE THE MAGNETIC-ORDERING TEMPERATURE

As the temperature T is reduced towards the magnetic-ordering temperature T_M, the correlation function for the localized magnetic moments will start to diverge at wave vectors associated with the magnetic-reciprocal-lattice wave vectors $\vec{Q}_i$. For the antiferromagnetic, the $\vec{Q}_i$ will lie on the Brillouin-zone boundary, while for the ferromagnetic we have $\vec{Q} = 0$. If there are z points $\vec{Q}_i$, we shall divide the Brillouin zone into a set of z nonoverlapping regions, in each of which there is only one magnetic-reciprocal-lattice wave vector $\vec{Q}_i$. The magnetic-moment correlation function in the vicinity of $\vec{Q}_i$ is expressed in terms of the scaling function of M. E. Fisher and A. Aharony.[4] In this manner we have associated the fluctuations at each point in the Brillouin zone with the nearest $\vec{Q}_i$. The dominant, quasielastic part of the temperature derivative of the spin-flip transition rate is given by

$$\frac{d}{dt}\left(\frac{1}{\tau_s}\right) \propto \sum_i \sum_{\vec{k}_i} \frac{1}{|\vec{k}_i|} \frac{d}{dT} \times \left[\Delta T^{-\gamma} D\left(\frac{|\vec{k}_i - \vec{Q}_i|\xi_0}{\Delta T^\nu}\right)\right] , \tag{9}$$

where $\xi = \xi_0 \Delta T^{-\nu}$ is the coherence length, and in the vicinity of $\vec{Q}_i$ the scaling function $D(x)$ is given by

$$D^{-1}(x) = C^{-1}(1 + x^2 - \Sigma_4 x^4 - \cdots)$$

for values of $x << 1$, and

$$D^{-1}(x) = C^{-1} x^{2-\eta}(0.962 - 2x^{(1-\alpha)/\nu} - \cdots) , \tag{10}$$

for values of $x >> 1$. The critical exponents γ, ν, η, and α are defined as usual.[5] It can be seen that the temperature derivative of the magnetic correlation function $dg_{\vec{k}}(t=0)/dt$ becomes negative in the regions with wave vectors $|\vec{k} - \vec{Q}_i| < \xi_0^{-1} \Delta T^\nu$, the inverse correlation length. For wave vectors $|\vec{k} - \vec{Q}_i| < \xi_0^{-1} \Delta T^\nu$ the correlation function becomes positive and goes through a maximum (Fig. 1). Above T_M, the correlation function satisfies the sum rule[6]

$$\sum_{\vec{k}} \frac{d}{dt}[g_{\vec{k}}(t=0)] = 0 , \quad T > T_M , \tag{11}$$

where the summation is over the entire Brillouin zone. Most of the cancellation of the negative region by the positive region occurs in a volume, with a radius of order $\xi_0^{-1} \Delta T^\nu$, around $\vec{Q}_i$. The temperature derivative of the spin-flip transition rate gives a larger weighting to the fluctuations of small wave vectors by

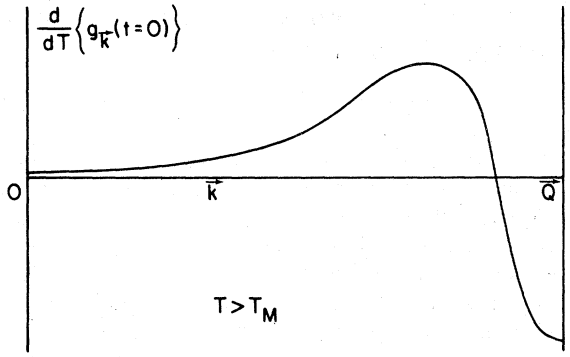

FIG. 1. Temperature derivative of the zero-time magnetic-moment correlation, for scattering wave vectors along $\vec{Q}_i$, the magnetic-reciprocal-lattice wave vector, the temperature being greater than the magnetic ordering temperature T_M.

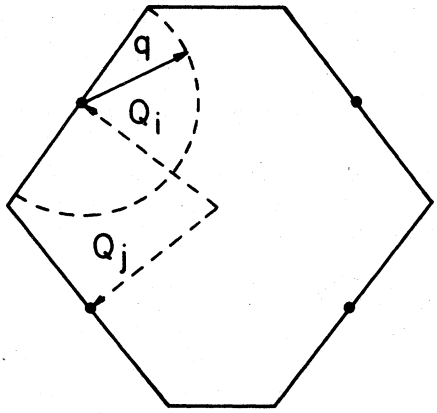

FIG. 2. Section of the Brillouin zone of an antiferromagnet in which Q_i and Q_j are magnetic-reciprocal-lattice wave vectors. After Ref. 6.

a factor of $|\vec{k}|^{-1}$, as compared to the above sum rule,

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \propto \sum_{\vec{k}} \frac{1}{|\vec{k}|} \frac{d}{dT} g_{\vec{k}}(t=0) \ . \tag{12}$$

Thus the temperature variation of $1/\tau_s$ is dependent on the positions of the magnetic-reciprocal-lattice wave vectors $\vec{Q}$ and on the Fermi cutoff on the scattering wave vector $2k_F$. In each of the z regions, defined by the magnetic-reciprocal-lattice wave vectors $\vec{Q}_i$, we shall change the variable from $\vec{k}$ to $\vec{q}=\vec{k}-\vec{Q}_i$. Then for large enough Q, we may express the temperature derivative of the spin-flip transition rate as a power series

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \propto \Delta T^{-\gamma-1} \sum_i 2\pi \int q^2\,dq \int d\cos\theta \sum_n P_n(\cos\theta)(q/Q)^n \frac{1}{Q}\left[\gamma D(q) - \Delta T \frac{d}{dT} D(q)\right], \tag{13}$$

where $P_n(x)$ are the Legendre polynomials. The range of integration is restricted by the conditions

$$|\vec{q}+\vec{Q}_i| < 2k_F \ , \quad |\vec{q}+\vec{Q}_i-\vec{Q}_j| > |\vec{q}| \ ,$$

together with the constraint that $\vec{q}+\vec{Q}_i$ should lie within the first Brillouin zone (Fig. 2).

We shall first consider the case of the proper antiferromagnet, in which $\vec{Q}_i$ lies on the Brillouin-zone boundary. The angular integration can be performed over the range $1>\cos\theta>0$, giving

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \propto -\Delta T^{-\gamma-1} \sum_i \frac{2\pi}{Q} \int q^2\,dq \sum_{n=0}^{\infty} P_n^{-1}(0)(q/Q)^n \left[\gamma D(q) - \Delta T \frac{d}{dT} D(q)\right], \tag{14}$$

where $P_n^m(x)$ are the associated Legendre polynomials. We may rewrite this expression

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \propto -\Delta T^{-\gamma-1} \frac{2\pi}{Q} \int q^2\,dq \left[1 + \sum_{n=1}^{\infty} \frac{\Gamma(\frac{1}{2})\Gamma[\frac{1}{2}(2+n)]}{\frac{1}{2}[n(n+1)]\Gamma[\frac{1}{2}(1+n)]} \frac{\sin\frac{1}{2}(\pi n)}{\pi} \left[\frac{q}{Q}\right]^n\right] \left[\gamma D(q) - \Delta T \frac{d}{dT} D(q)\right], \tag{15}$$

where $\Gamma(x)$ is the Euler γ function. Above the Néel temperature T_N, the first term is simply the average of the temperature derivative of the moment correlation function. The higher-order terms give a larger weighting to the region in which the magnetic-moment correlation function has a positive derivative than the region in which the temperature derivative is negative. For the metallic antiferromagnet, which has $2k_F > Q$, the first term is zero, due to the sum rule. The higher-order terms when integrated

over q, give the temperature dependence of the spin-flip transition rate to be an increasing function of temperature. The exponent being that associated with the large q behavior of the magnetic correlation function[7]

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \propto +\Delta T^{-\alpha}, \quad T > T_N .$$

In the antiferromagnet with a small Fermi wave vector $2k_F < Q$ the semiconducting antiferromagnet, the range of integration is that of only large q. Again, $d(1/\tau_s)/dT$ is dominated by the asymptotic behavior of the large q correlation function, and again we have

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \propto +\Delta T^{-\alpha}, \quad T > T_N .$$

For the cases in which Q is still large, but lies within the Brillouin-zone boundary, such as a helical magnet, we have a different behavior for the spin-flip transition rate. The metallic helical magnet $2k_F > Q$ has the critical region well inside the limits of integration. In this case, there is an approximate spherical symmetry around $\vec{Q}_i$. The angular integration can be performed over the whole range of $\theta, 1 > \cos\theta > -1$. This results in the temperature derivative of $1/\tau_s$ to be given by

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \propto -\Delta T^{-\gamma-1} \sum_i 2\pi \int q^2\, dq\, \frac{1}{Q}\left[\gamma D(q) - \Delta T \frac{d}{dT} D(q)\right] . \tag{16}$$

From the sum rule on the magnetic correlation function we see that for the metallic helical magnet, with a Fermi cutoff larger than the reciprocal-lattice vector, gives

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \approx 0, \quad T > T_M .$$

However, as the geometry is not perfectly spherically symmetric, the leading will exhibit a $\Delta T^{-\alpha}$ dependence. The semiconducting helical magnet is similar to the case of the semiconducting antiferromagnet. The behavior of the spin-flip transition rate is determined by the large q part of the magnetic-moment correlation function, and gives

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \propto +\Delta T^{-\alpha} .$$

For materials which have small magnetic-reciprocal-lattice wave vectors $\vec{Q}$, one is no longer justified in using the expansion

$$\frac{1}{|\vec{Q}+\vec{q}|} = \sum_n \frac{P_n \cos\theta}{Q} (q/Q)^n, \quad q/Q < 1 , \tag{17a}$$

but for large q one should use the complementary expansion

$$\frac{1}{|\vec{Q}+\vec{q}|} = \sum_n \frac{P_n(\cos\theta)}{q} (Q/q)^n, \quad Q/q < 1 . \tag{17b}$$

For $\vec{Q}$ near the center of the Brillouin zone with $2k_F > \vec{Q}$, then the integration is spherically symmetric. As $\vec{Q} \to 0$, the expression for $d(1/\tau_s)/dT$ becomes that of the ferromagnet as calculated in Ref. 1. In this case, the scattering wave vector $\vec{k}$ and $\vec{q}$ coincide. The factor $1/q$ favors spin-flip transitions that occur at small wave vectors, as compared to the phase-space average of the magnetic-moment correlation function. The temperature derivative of the spin-flip scattering rate is thus dominated by the negative peak in the temperature derivative of the magnetic correlation function. The radial integrative gives, in agreement with Ref. 1,

$$\frac{d}{dT}\left(\frac{1}{\tau_s}\right) \propto -\Delta T^{\nu\eta-1}, \quad T > T_c ,$$

where we have used the scaling law $\nu\eta = 2\nu - \gamma$. Thus for a ferromagnet $d(1/\tau_s)/dT$ is an increasing function of T.

V. INELASTIC SCATTERING

We shall argue that the inelastic component of the spin-flip transition rate is negligible compared to the quasielastic component of the spin-flip transition rate. The leading inelastic spin-flip scattering term is proportional to the integral

$$\int \omega g_{\vec{k}}(\omega) \frac{d\omega}{2\pi} . \tag{18}$$

The phase space which is available for the elastic scattering processes limits the energy transfer ω to be less than $k_B T_M$. The dynamical scaling hypothesis[8] asserts that the magnetic correlation function has the form

$$g_{\vec{k}}(\omega) = \frac{2}{\omega_c(\vec{k})} g_{\vec{k}}(t=0) F_{\vec{k}}\left(\frac{\omega}{\omega_c(\vec{k})}\right), \tag{19}$$

where $F_{\vec{k}}(\omega/\omega_c(\vec{k}))$ is the shape function which is normalized, with respect to ω, to have unit area and $\omega_c(\vec{k})$ is the characteristic frequency which has the form

$$\omega_c(\vec{k}) = |\vec{k}-\vec{Q}|^z \Omega\left(\frac{|\vec{k}-\vec{Q}|\xi_0}{\Delta T^{\nu}}\right) . \tag{20}$$

Near the critical temperature, $F_{\vec{k}}(x)$ is dominated by a central peak,[9] and so the integral in the leading inelastic-spin-flip scattering term may be carried out with a Lorentzian shape function

$$\int \omega g_{\vec{k}}(\omega)\frac{d\omega}{2\pi}=\frac{-i\omega_c(\vec{k})}{\pi}\tan^{-1}\times\left[\frac{k_BT_M}{\omega_c(\vec{k})}\right]g_{\vec{k}}(t=0)\ . \tag{21}$$

For positive z, this term tends to zero as $\vec{k}\to\vec{Q}$. The decrease of $\omega_c(k)$ occurs just at the scattering wave vectors for which $g_{\vec{k}}(t=0)$ is maximum. This is the phenomenon of critical slowing down. This critical slowing down may be understood, physically, in terms of the ease in which fluctuations, with $\vec{k}\simeq\vec{Q}$, occur and their consequent slow relaxation. In this regime, the inelastic scattering is smaller than the quasielastic scattering by a factor of $\omega_c(\vec{k})/k_BT_M$. The above type of argument might lead one to expect that for large values of $|\vec{k}-\vec{Q}|$, the inelastic scattering should be comparable to the elastic scattering. However, for the large values of $|\vec{k}-\vec{Q}|$, the magnetic correlation function is small and has the same type of leading temperature dependence as the energy, that is $\Delta T^{1-\alpha}$. The set of wave vectors, for which the elastic scattering processes and the inelastic scattering processes are equal, provides only a small contribution to the total spin-flip scattering rate. Physically, one can understand the dominance of the quasielastic terms as the fast conduction electrons will see the slow moment fluctuations as if they were frozen. In this case, $\omega_c(\vec{k})/k_BT_M$, is the ratio of the relevant time scales which is estimated to have a value of 10^{-2}. Also for the large values of $|\vec{k}-\vec{Q}|$, the validity of the dynamical scaling expressions for the characteristic frequency and the shape function is questionable.

Following the same procedure as for the quasielastic scattering, we see that the temperature derivative of the inelastic scattering rate may be written

$$\frac{d}{dt}\left[\frac{1}{\tau_s}\right]\propto-\Delta T^{-\gamma-1}\sum_i\int_0 d^3q\,\frac{\omega_c(q)}{|\vec{q}+\vec{Q}_i|}\tan^{-1}\times\frac{k_BT_M}{\omega_c(\vec{k})}\left[\gamma D(q)-\Delta T\frac{d}{dt}D(q)\right]. \tag{22}$$

The Van Hove theory of critical dynamics predicts that, for small q, we have $z=2$. For sufficiently small q, the critical slow-down argument applies, and the average energy transferred between the critical fluctuations and the conduction electrons is small compared to k_BT_M. The temperature dependence comes from the large q part of the correlation function and is of the form $\Delta T^{-\alpha}$. The coefficient of the inelastic scattering processes is small compared to the quasielastic scattering processes.

The dominance of the quasielastic scattering is similar in the related phenomena of critical resistivities. Geldart[10] has shown that the inelastic scattering contribution to the resistivity is small compared with the quasielastic terms, and that the temperature dependence is similar to that of the quasielastic part.

VI. SPIN-FLIP TRANSITION RATE BELOW THE MAGNETIC-ORDERING TEMPERATURE

The behavior of the spin-flip scattering rate below T_M is far more complicated. This is due to the existence of a nonzero order parameter, and due to the opening up of gaps in the electronic band structure, which is associated with the new magnetic periodicity of the system.[10] In particular, the magnetic-moment correlation function is altered. At the magnetic-ordering temperature, the positive peak, and the negative region in $dg_{\vec{k}}(t=0)/dT$ contract at $\vec{Q}$ and cancel. The sum rule on the temperature derivative of the magnetic correlation function is modified due to the existence of the temperature-dependent order parameter. This allows $dg_{\vec{k}}(t=0)/dT$ to be positive for all the scattering wave vectors $\vec{k}$ (Fig. 3). The sum rule for the correlation function becomes

$$\frac{1}{N^2}\sum_{\vec{k}}g_{\vec{k}}(t=0)=S(S+1)-M^2\ , \tag{23}$$

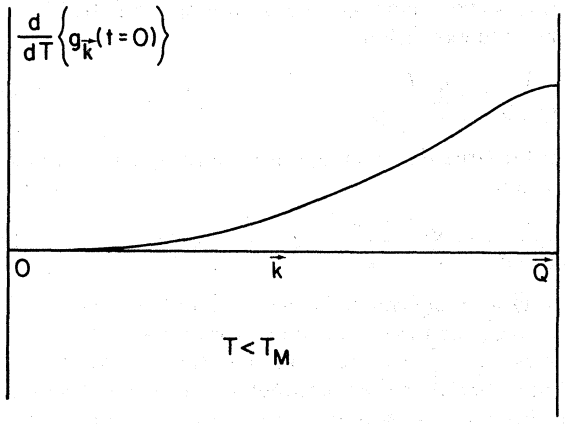

FIG. 3. Temperature derivative of the magnetic-moment correlation function, for wave vectors along $\vec{Q}_i$. The temperature is assumed to be below the magnetic-ordering temperature T_M.

where M is the order parameter $\langle S_Q^z \rangle$. Thus the temperature derivative of the Fermi golden-rule approximation to the spin-flip transition rate, for both antiferromagnetic and ferromagnetic materials, increases as T is lowered below the ordering temperature, the critical exponents being the same as those above the ordering temperature. The resulting behavior of the approximate spin-flip transition rate is plotted in Fig. 4.

In the above we have neglected the changes in the electronic band structure that are expected to occur as a result of the new periodicity, which is associated with the magnetic ordering. The repeated scattering of the conduction electrons off of the ordering magnetic moments will produce a new periodic average potential that will determine the band structure. At the ordering temperature, gaps will appear in the band structure at wave vectors associated with the magnetic Brillouin-zone boundaries. These types of effects do not occur as a renormalization of the band

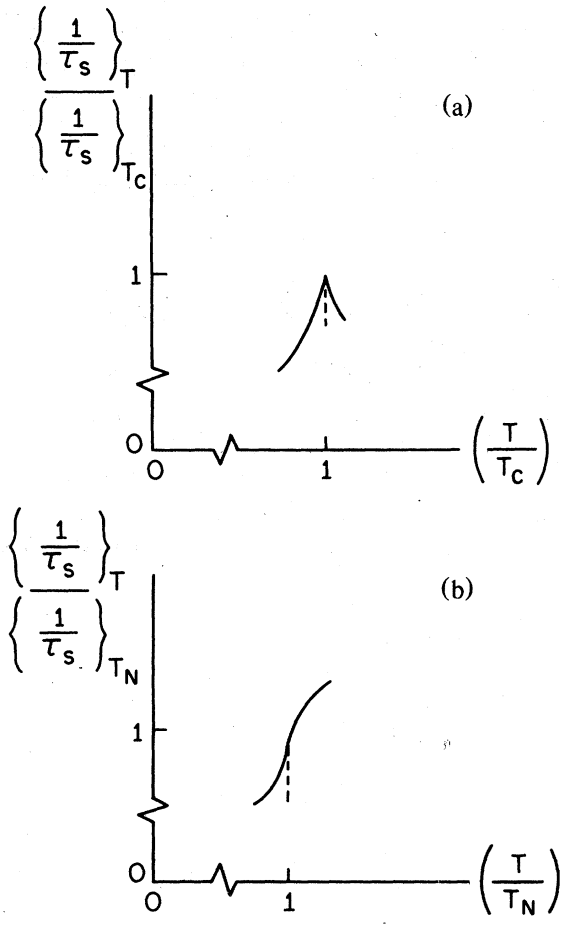

FIG. 4. (a) Schematic plot of the spin-flip transition rate for a ferromagnet. The critical exponent is $\nu\eta$. (b) Spin-flip transition rate for an antiferromagnetic metal. The critical exponent is $1-\alpha$.

structure, but occur, naturally, through the higher-order terms of the perturbation series. We have evaluated the next order contributions to the spin-flip transition rate, in order to ascertain the effects of the differing electronic spectra below the magnetic-ordering temperature. Thus formula (5) is modified. The next contributions to the spin-flip transition rate are proportional to the fourth power of the de Gennes-Friedel interaction strength and involve four moment correlation functions. The four moment correlation functions were decoupled into two moment correlation functions, and terms involving the diagonal z component of the magnetization. This decoupling procedure is unjustified, but it is done in the expectation that the irreducible terms are negligible. We Fourier-transform the magnetic-moment correlation functions and then perform the time integrations. We only retain the terms which represent the quasielastic spin-flip scattering of the conduction electrons. We find that the quasielalstic scattering contribution is proportional to

$$\sum_{\vec{k}} \int \frac{g_{\vec{k}}(\omega)}{|\vec{k}|} \frac{d\omega}{2\pi} \left[\frac{1}{Q} \langle S_Q^z \rangle^2 + \sum_{\vec{q}-\vec{q}'} \int \frac{g_{\vec{q}-\vec{q}'}(\omega')}{|\vec{q}-\vec{q}'|} \frac{d\omega'}{2\pi} \right] . \tag{24}$$

In the vicinity of the magnetic-reciprocal-lattice wave vector, $\vec{q}-\vec{q}' \simeq \vec{Q}$, the last term in the parantheses may be approximated by

$$\sum_{\vec{q}-\vec{q}'} \frac{g_{\vec{q}-\vec{q}'}(\omega')}{Q} \frac{d\omega'}{2\pi} .$$

Indeed, if the region of summation is approximately shperically symmetric, this will be a good approximation. On making this approximation we note that there is a cancellation in the temperature dependence of the two terms in the parantheses due to the modified sum rule

$$\frac{1}{N^2} \sum_{\vec{k}} g_{\vec{k}}(t=0) = S(S+1) - M^2 , \quad T < T_M . \tag{25}$$

This gives the fourth order contribution to have the same type of critical temperature dependence as the second order contribution.

VII. CONCLUSION

We have calculated the spin-flip transition rate $1/\tau_s$ for conduction electrons in systems which exhibit long-ranged magnetic ordering. We have shown that the interplay of the phase space available for electrons to scatter into and the fluctuations of the magnetic moments lead to differing behaviors of in the various cases of magnetic ordering. In the antiferromagnet, the spin-flip scattering is dominated by

the long-wavelength behavior of the moment correlation function. This gives $1/\tau_s$ an energy-like temperature dependence, i.e., it decreases like $\Delta T^{1-\alpha}$ as T decreases through T_N. For the ferromagnet, the spin-flip transition rate is dominated by the peak in the moment correlation function. Thus, in the ferromagnet $1/\tau_s$ behaves like $\Delta T^{+\eta\nu}$ giving a positive cusp at T_C. In a helical magnet, $1/\tau_s$ is not expected to have any significant critical type of temperature dependence.

The spin-flip transition rate $1/\tau_s$ behaves differently to the resistivity which is proportional to

$$\sum_{\vec{k}} |\vec{k}| g_{\vec{k}}(t=0) \ .$$

The weighting factor $|\vec{k}|$ occurs in the resistivity, as, in the resistivity the important fact is not that the electron is scattering, but the amount that the wave vector along the electric field is changed in the process. However, the temperature dependence of the spin-flip scattering rate can be measured by its effects on superconductors. In the superconductors, the spin-flip scattering strongly suppresses the superconducting transition temperature. It has already been seen on thin Pd_xNi_{1-x}:Sn proximity effect sandwiches.[11] In these alloys the Curie temperature may be varied by changing the concentration x. The superconducting transition temperature can be made greater than the critical temperature of the alloy. For a small range of concentrations x, the superconducting transition occurs at a lower temperature. This occurs at the point where the critical temperature becomes of the order of the superconducting transition temperature. A much more sensitive measurement would be that of tunneling experiments on proximity effect sandwiches. In the proximity effect tunneling experiments, the zero bias conductivity is a direct measure of the superconducting density of states $\rho_s(\omega)$.

$$\sigma_s(0) \propto \int_{-\infty}^{\infty} \rho_s(\omega) \frac{\partial f(\omega)}{\partial \omega} d\omega \ .$$

The spin-flip scattering rate exhibits itself through the superconducting density of states.[12]

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