Remanent magnetization in spin-glasses

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A spin-glass model consisting of a kinetic Ising model with random nearest-neighbor interactions is studied by Monte Carlo methods. As in real experiments the system is cooled, and a magnetic field is applied and then switched off. Below a freezing temperature T_f both an irreversible and a reversible magnetic susceptibility are observed. A remanent magnetization M occurs which decays very slowly with time t with a power law $M \sim t^{-a}$ to the equilibrium value $M=0$. For different cooling procedures different remanent magnetizations are discussed as a function of temperature and previously applied field. A characteristic difference between field cooled (TRM) and isothermal (IRM) remanent magnetization is observed in the field dependence of the exponent a. Many of the predictions resemble experimental results. In the second part an exactly solvable spin-glass model incorporating a symmetric distribution of random interactions and frustration is introduced. Since the range of the interactions is infinite there exist no local clusters in this model. A phase transition with a cusp in the susceptibility, a remanent magnetization, and a ferromagnet–spin-glass transition are found.

I. INTRODUCTION

Spin-glasses are random magnetic systems with a puzzling magnetic ordering at low temperatures.[1-3] Experimentally the susceptibility shows a sharply defined temperature T_f indicating a sudden freezing of the spins.[4] On the other hand the specific heat does not indicate any phase transition at T_f,[5] and neutron scattering shows no long-range periodic order.[6] However, all the experimental results depend on the time scale of the measurements. For instance, neutron scattering[7] and Mössbauer experiments[8] with measurement times of about 10^{-11} and 10^{-8} sec give evidence for the gradual formation of correlated regions or clusters of spins with decreasing temperature. For times of about 10^{-2} sec the cusp of the ac susceptibility depends logarithmically on the frequency in some but not in all of the spin-glasses.[9-11] On the time scale of minutes, hours or even longer spin-glasses show remarkable properties such as an irreversible susceptibility, a remanent magnetization, or a time-dependent specific heat.[12-15] Such detailed studies of time-dependent properties are necessary to understand the nature of magnetic ordering in spin-glasses.

In many spin-glasses the magnetic moments interact via the oscillating RKKY [Ruderman-Kittel-Kasuya-Yosida] interaction.[1-3] For the ideal RKKY coupling the nearest-neighbor interaction, say in *Au*Fe, is about 100 times larger than the freezing temperature $k_B T_f$. Thus it is evident that at $T = T_f$ many spins are coupled to clusters of strongly correlated but presumably randomly orientated spins. The coupling between such clusters is of the order $k_B T_f$, and because of the random orientation of the spins it has a short range. At temperatures much larger than T_f it may be a useful approximation to describe spin-glasses by a spectrum of isolated clusters with effective moments S_i and relaxation times τ_i due to an anisotropy caused by the internal dipolar interaction.[13,15-17] But for temperatures lower than or equal to T_f the collective behavior due to the interaction between the clusters should be considered, and an appropriate model for this situation is a system of localized spins on a regular lattice with an interaction with random sign and random absolute value.[2]

This model, first proposed by Edwards and Anderson,[18] has been investigated very extensively in the last years.[2,3] Using the so-called "replica trick", which is still under discussion[19-21] the mean-field solution of this model gives a cusp in the susceptibility at a temperature T_f but also a cusp in the specific heat.[18,22,23] More powerful methods have been used to study the freezing process of this model, for instance, computer simulations, high-temperature expansion, and renormalization-group methods, but the existence of a phase transition is still unclear.[24-26]

One characteristic property of spin-glasses is the existence of conflicting bonds (or "frustration"[27]). In a simple mean-field approach it has been shown that the nonzero low-temperature susceptibility $\chi(T=0)$, the shape of the cusp of $\chi(T)$ at T_f, the dependence of T_f on the impurity concentration and on the finite mean free path of the conduction electrons depend

strongly on the degree of frustration.[28,29] As a consequence of the random sign of the interaction all extended modes contribute to the dynamic and static homogeneous susceptibility leading to unusual critical behavior.[30]

Although many approximations have been proposed to study the Edwards-Anderson model[2,3] up to now the analytic methods have not given any conclusive answers. Therefore computer simulations are very useful in deciding whether this model can describe real spin-glasses.[31] In fact the Monte Carlo simulations show a rather sharp peak in the susceptibility, a broad specific-heat maximum, remanent magnetization, and nonexponential relaxation processes in qualitative agreement with experiments.[32,33] Although simulations of a finite system of about 10^4 spins cannot prove the existence of a phase transition,[25,26] fluctuations of a suitable order parameter, which cannot be observed in real experiments but seem to diverge at T_f in computer experiments, and the observation of critical exponents seem to indicate a phase transition.[34]

In this paper we study irreversible processes of spin-glasses. In Sec. II using Monte Carlo simulations of the Edwards-Anderson model we calculate different remanent magnetizations as a function of field and temperature and we investigate the time dependence of relaxation in detail. All the simulations are closely related to experimental methods.[35] In Sec. III we introduce an exactly solvable model of spin-glasses which shows an irreversible susceptibility, a remanent magnetization and a ferromagnet–spin-glass transition. Section IV contains some conclusions.

II. COMPUTER SIMULATIONS OF A TWO-DIMENSIONAL ISING SPIN-GLASS MODEL

A. The model

We consider the Ising model on a square lattice with Hamiltonian

$$H = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j - B \sum_i S_i \ , \tag{1}$$

where $S_i \in \{+1, -1\}$ and the nearest-neighbor interaction J_{ij} is randomly distributed according to a symmetric Gaussian

$$P(J_{ij}) = \exp\left[-\frac{J_{ij}^2}{2(\Delta J)^2}\right] \ , \tag{2}$$

with width ΔJ.

The dynamics are simulated by a master equation for the probability of a given spin configuration at a given time t. The Monte Carlo simulations closely follow the earlier work of Binder *et al.*[26,32–34] For a detailed description of the Monte Carlo method see Ref. 36. We simulate pure relaxational processes without any conserved quantities. Thus our model cannot account for propagating collective excitations which become important far below T_f.[37] The time scale of the kinetic Ising model is the microscopic time τ which corresponds to the relaxation time of a single spin in an external heat bath. In the simulations we measure the time in units of Monte Carlo steps per spin (MCS/spin). All energies are given in units of the width ΔJ of the random interactions. From earlier work we know that ΔJ has the same order of magnitude as the freezing temperature $k_B T_f$ (in the square lattice[32]). We have treated a 50×50 system with periodic boundary conditions using computer runs of about 4000 MCS/spin.

B. Reversible and irreversible susceptibility

Figure 1 shows a typical computer run. We start from a random spin configuration, that is with infinite temperature. The system is suddenly cooled to $T = 3\Delta J/k_B$ and then *slowly* cooled to the final temperature, here $T = \Delta J/2k_B$. If a magnetic field is applied during the cooling [TRM (thermo remanent magnetization) curves] a magnetization develops until the field is suddenly switched off. Then the magnetization drops within a few MCS/spin to a remanence value which decays very slowly to zero.

The points in Fig. 2(a) show the field cooled magnetization for $B = 0.2\Delta J$ as a function of temperature. The B dependence as $B \to 0$ determines the irreversible susceptibility χ_{irr}, which is nearly constant below $T_f = \Delta J/k_B$ and follows a Curie law above T_f. (Note that due to the nonzero field there are deviations from the Curie law above T_f). A measure for the fluctuations of the system is the jump ΔM between the magnetization in an external field and the remanent magnetization shortly after the field is switched off. This jump ΔM is shown as a function of temperature in Fig. 2(a) by the crosses. There is a steep increase with temperature up to T_f. Then the points approach the values of the irreversible magnetization indicating the absence of remanent magnetization. The behavior as $B \to 0$ determines the reversible susceptibility of the metastable state.

The results for $\Delta M(T)$ agree with the susceptibility calculated from the fluctuations of the magnetization in earlier computer simulations. The $M(B)$ values agree with the slope of the magnetization as a function of field obtained by slowly lowering the field from infinity to zero[32] (again note the rounding due to a nonzero field). Although all these susceptibilities were calculated in different ways, there is a unique temperature $T_f \simeq \Delta J/k_B$ indicating the onset of some collective behavior.

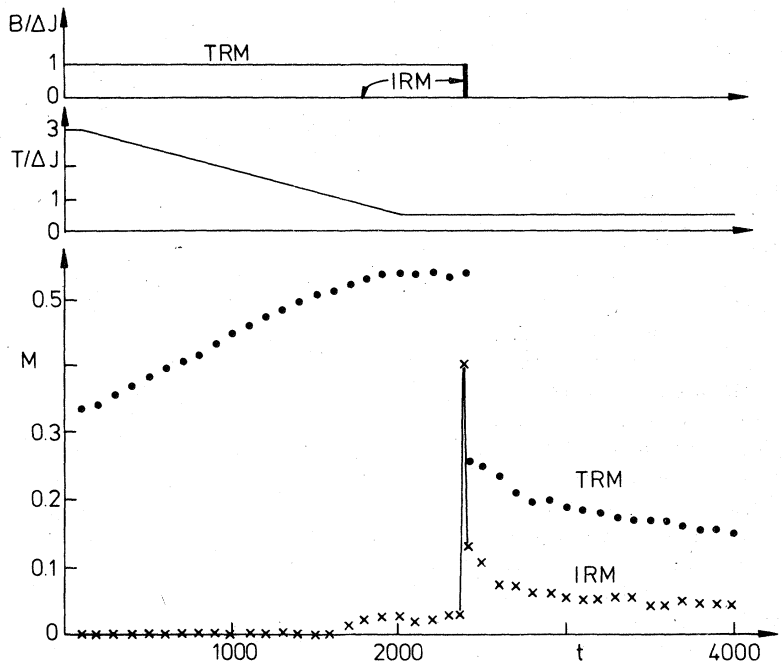

FIG. 1. Magnetization M as a function of time t for cooling in an external field B (TRM) or for cooling in zero field and short application of the field (IRM). Temperature T and field B are shown in units of the width $\Delta J = \langle J_{ij}^2 \rangle_{\text{av}}^{1/2}$ of the interactions, time t is shown in units of Monte Carlo steps per spin.

To compare our results with experiments we consider the reversible susceptibility (short applications of a dc field) and irreversible susceptibility (field-cooled magnetization) of *Au*Fe taken from Ref. 13. These experimental data shown in Fig. 2(b) are in remarkable agreement with the Monte Carlo data of Fig. 2(a) and Ref. 32.

C. Field-cooled and isothermal remanent magnetization

Figure 1 shows that the remanent magnetization has been produced by different ways: (a) TRM (thermo remanent magnetization): The system is slowly cooled *in* an external field and then the field is switched off. (b) IRM (Isothermal remanent magnetization): The system is slowly cooled in *zero* external field. Then the field is applied for a short time (20 MCS/spin) and switched off. Afterwards a slowly decaying remanent magnetization appears. It was measured by taking its average over 1000 MCS/spin.

Because of the slow cooling (slow means that we could not see any differences on enlarging the cooling time internal by a factor 10) we expect to start from thermal equilibrium in an external field in the TRM case and from the equilibrium in zero field in the IRM case. We have also investigated a nonequilibrium start position, IRM (fc) (fc: fast cooled). First the system was cooled very rapidly within 20 MCS/spin. Then the field was applied as in the usual IRM for 20 MCS/spin and then switched off.

The remanent magnetization depends on the final temperature and the field which was applied initially. Figure 3(a) shows the remanence as a function of field. Since in the TRM case the system starts from a high magnetization in a stable state we expect the TRM values to be higher than the IRM ones. In the IRM (fc) case the system can relax for a while in the applied field. Therefore we expect the IRM (fc) values to be larger than IRM ones. For the IRM in Fig. 3(a) we cannot exclude the existence of a critical field below which no IRM occurs, but the TRM grows linearly with the field. The TRM has a maximum for field energies of the order of the interaction energy ($\simeq k_B T_f$).

The IRM and TRM results are in qualitative agreement with experiments on *Au*Fe,[12] CuMn,[38] and EuSrS.[8] The experimental data of *Au*Fe taken from Ref. 12 are shown in Fig. 3(b). The ratio of the remanence at its maximum to the remanence for very large fields seems to be somewhat smaller for the metallic spin-glasses *Au*Fe and CuMn than the ratio calculated here, whereas the nonmetallic spin glass EuSrS has a much larger ratio. However, it should be noted that this ratio depends on the measuring time (see Sec. I D).

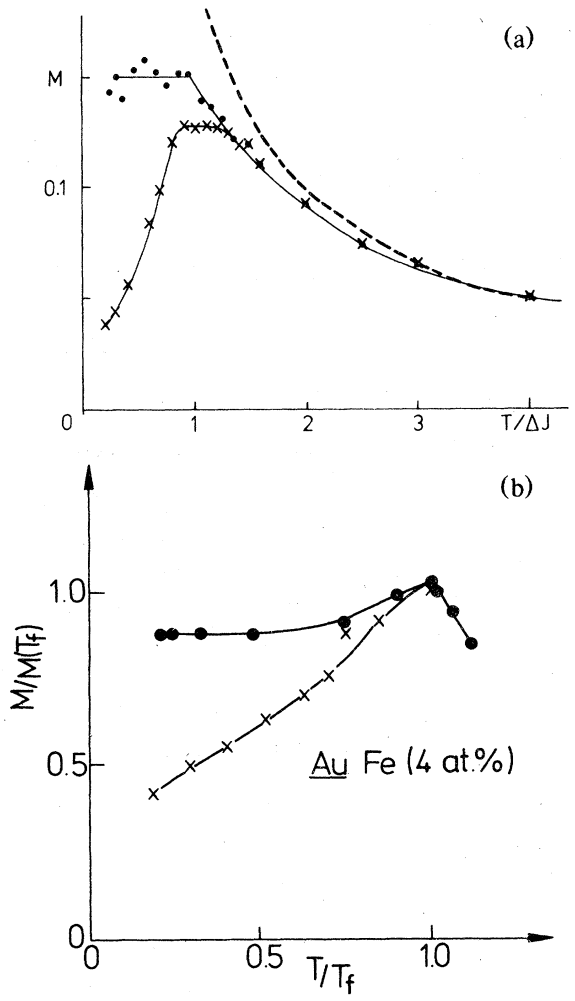

FIG. 2. (a) Magnetization M (dots) obtained by cooling in an external field $B=0.2\Delta J$. It corresponds to the irreversible susceptibility. The crosses show M minus the TRM, which corresponds to the reversible or short-time susceptibility. The dashed line follows the Curie law $M=B/k_BT$. (b) For AuFe (4 at.%) the magnetization obtained by cooling in a field of 20 Oe (dots) and by applying a field of 20 Oe for a short time (crosses) is shown as a function of temperature. The data are taken from Ref. 13.

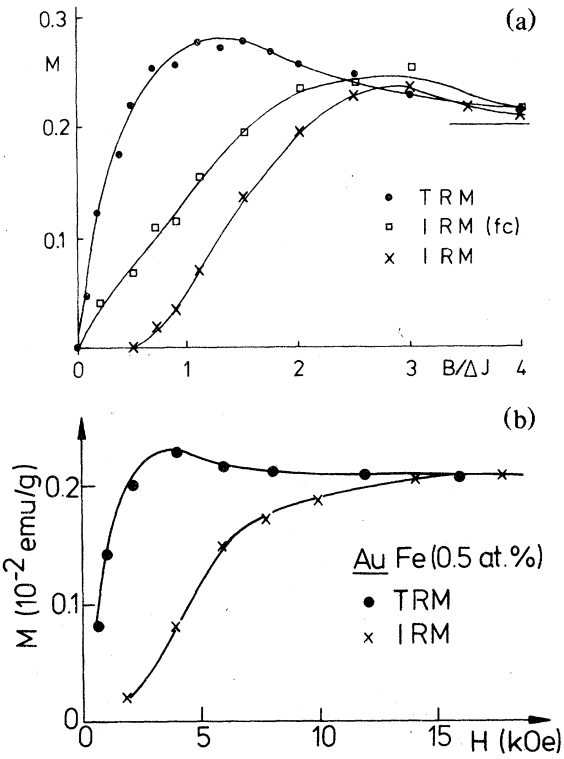

FIG. 3. (a) Remanent magnetization M obtained by cooling in a field (TRM) or shortly applying a field at constant temperature (IRM) is shown as a function of the initially applied field. IRM (fc) is obtained by some mixed cooling procedure (see text). M was measured at the temperature $T=\Delta J/4k_B$. (b) The same kinds of remanent magnetizations as in Fig. 3(a) but for AuFe (0.5 at.%). The data are taken from Ref. 12.

Each metastable state may be characterized by its energy and magnetization. Figure 4 shows the states of Fig. 3(a) in the energy-magnetization plane. With increasing time all points approach the equilibrium value $M=0$ and $E=-1.29\Delta J$. Thus all points within the region surrounded by the outer states are metastable states. Of course we cannot rule out other metastable states produced by different cooling procedures than those considered here.

Figures 3(a) and 4 were measured at temperature $T=\frac{1}{4}T_f$. We now discuss the temperature dependence. Figure 5 shows the field-cooled remanent magnetization (TRM) as a function of temperature. As for infinite fields[32] the remanence decays nearly linearly and only occurs for temperatures below T_f. The same holds for the isothermal remanent magnetization (IRM). However, in comparing the IRM with experiments, we have to take the temperature dependence of the microscopic time τ into account. As noted above, our time scale is the single spin relaxation time τ. If we interpret each Ising spin as the effective spin of a rigid cluster,[2] τ should have the temperature dependence

$$\tau \propto e^{E/k_BT}, \tag{3}$$

where E is some mean anisotropy energy due to the internal dipolar energy of a cluster.[15,16]

The IRM depends on the length of the time interval $\Delta t/\tau$ over which the magnetic field was applied. This is shown in Fig. 6. Of course, for $\Delta t=0$ we have no remanence whereas for $\Delta t=\infty$ we expect the IRM to approach the TRM marked by an arrow. If we apply the field for a constant time Δt, the ratio

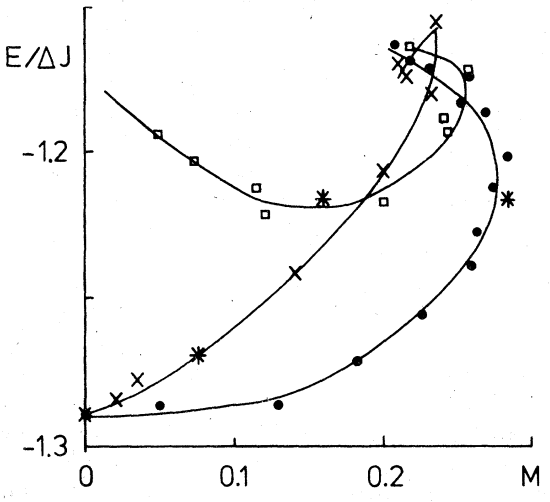

FIG. 4. Energy-magnetization plane for the metastable states of Fig. 3(a). The stars show the results for an initially applied field $B = 1.5\Delta J$.

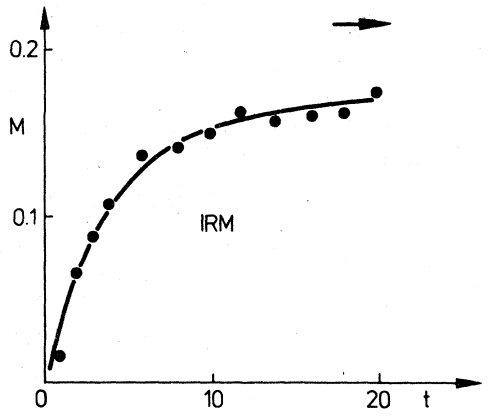

FIG. 6. Isothermal remanent magnetization (IRM) as a function of the time interval t over which the field B has been applied ($T = 0.25\Delta J$, $B = 2\Delta J$). The arrow marks the field-cooled remanent magnetization which should correspond to IRM ($t = \infty$).

$\Delta t/\tau$ increases with temperature according to Eq. (3). This effect combines with the linear temperature dependence to yield the results of Fig. 7. The IRM has a maximum as a function of temperature, the location of which seems to depend only on the anisotropy E. We stress that this maximum is not due to the interaction between the clusters but due to the variation of the single cluster relaxation time with temperature.

The temperature dependence of IRM and TRM has also been investigated in experiments on *Au*Fe and EuSrS. In *Au*Fe with weak fields a linear decay in TRM vs T below T_f and a maximum in IRM vs T was found,[13] in agreement with our computer simulations. For larger fields a nearly exponential decrease of the TRM with temperature was observed.[8,12,15]

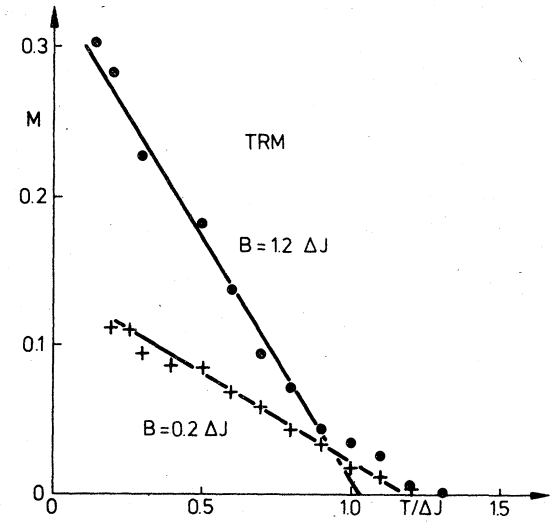

FIG. 5. Thermoremanent magnetization (TRM) as a function of temperature for two values of the initially applied field B.

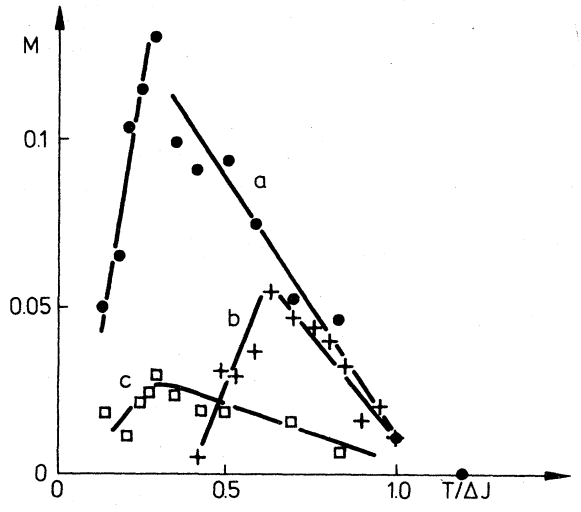

FIG. 7. Isothermal remanent magnetization (IRM) as a function of temperature. The single-spin relaxation time τ changes with temperature as $\tau \sim e^{E/k_B T}$. The parameters for the anisotropy energy E and initially applied field B are $(E/\Delta J, B/\Delta J) = a$: (0.5,2); b: (2.08,2); c: (0.5,1).

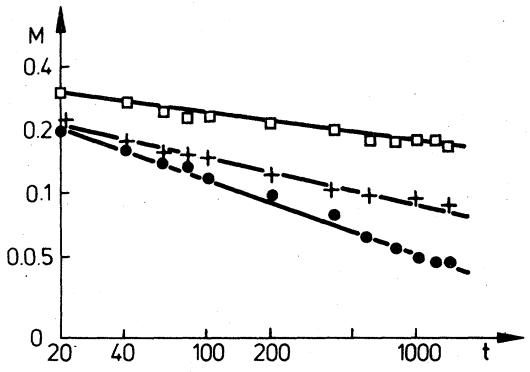

FIG. 8. Remanent magnetization as a function of time on a log-log plot showing a power-law decay $M \sim t^{-a}$. The squares (TRM, $B = \Delta J$) and dots (IRM, $B = 1.5\Delta J$) have the same initial energy, the dots and crosses (TRM, $B = \infty$) have the same initial magnetization ($T = 0.5\Delta J$).

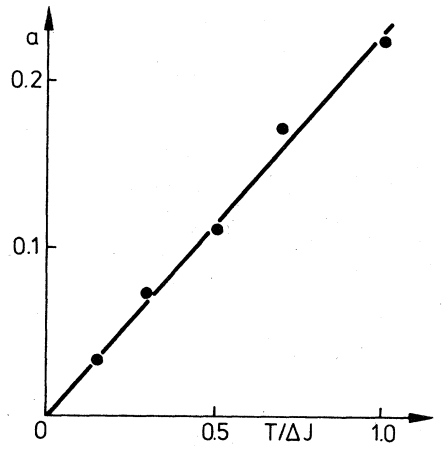

FIG. 9. Temperature dependence of the exponent a of the power law $M \sim t^{-a}$ for the thermoremanent magnetization with initially applied field $B = 1.2\Delta J$.

D. Relaxation of the remanent magnetization

In the preceding sections we discussed the remanent magnetization averaged over a fixed time interval. Now we consider the relaxation process itself. From earlier computer simulations of the same model[32] we know that the infinite field remanence decays with a time dependence which can be fitted to the power law

$$M \propto t^{-a} , \tag{4}$$

with the exponent a increasing linearly with temperature. We found this type of power law for all the temperatures, fields, and different kinds of remanence we have investigated. But now the exponent a also depends on the history of the system. One might guess that a depends on the initial energy of the metastable state, but Fig. 8 shows that $E(t = 20$ MCS/spin) does not determine a uniquely. The squares and dots denote systems with the same initial energy, the crosses and dots systems with the same initial remanent magnetization $M(t = 20$

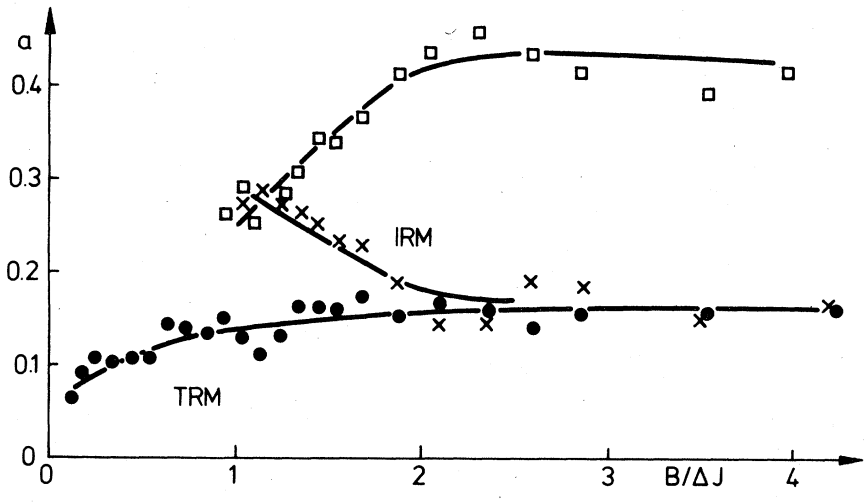

FIG. 10. Field dependence of the exponent a of the power law $M \sim t^{-a}$ which describes the relaxation of different remanent magnetizations (IRM and TRM). The energy difference to the equilibrium value E_0 has the same relaxation law $|E - E_0| \sim t^{-a}$ with a given by the squares ($T = 0.5\Delta J$).

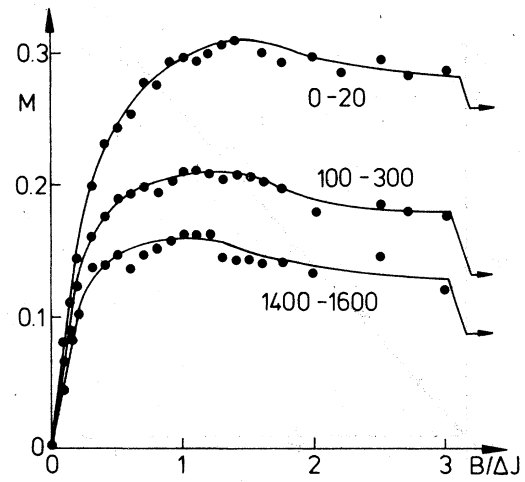

FIG. 11. Thermoremanent magnetization (TRM) as a function of initially applied field. The TRM was measured over three different time intervals ($T=0.5\Delta J$).

MCS/spin). All curves have different slopes a. Thus a depends on both the initial energy and magnetization. We have also investigated the relaxation of two systems with the same initial E and M but produced in different ways. In Fig. 4 one has such initial conditions for IRM and IRM (fc) where both curves intersect in the E-M plane. We found the *same* exponent a, but this, of course, does not exclude other parameters determining a.

Since E and M depend on temperature and previously applied field, a is a function of both T and B. Figure 9 shows that for the TRM at its maximum ($B=1.2\Delta J$) a increases linearly with temperature like $a(B=\infty)$.[32] But more interesting than the temperature dependence is the field dependence, which is shown in Fig. 10. There is a qualitative difference in IRM and TRM. The IRM decays faster than the TRM and faster for small fields, whereas the TRM is slower for small fields. In the conclusion we try to give an explanation for this difference.

We have also investigated the relaxation of the internal energy of the metastable states. The energy difference $\Delta E=E_0-E(t)$, where E_0 is the energy of the thermal equilibrium, also relaxes with a power law t^{-a}. The exponent for the TRM states is shown in Fig. 10 by the squares. One sees that the energy decays much faster than the magnetization.

The field and temperature dependence of the relaxation of the remanent magnetization M must be considered if one investigates $M(T,B)$. For instance, if one considers the TRM as a function of the previously applied field B, the quantitative results change as a function of the measuring time. This is shown in Fig. 11, where the TRM as a function of applied field is calculated for three averaging intervals. According to Fig. 10 the values for higher fields decay faster than those for smaller fields. Thus the maximum shifts toward lower fields and the ratio of the maximum to the saturated TRM increases with increasing time.

This suggests the following question: Is the maximum of TRM vs (B) entirely due to the increase of the exponent a with field? An answer is furnished by the exponent b defined by $\ln M=-b-a\ln t$. b is shown as a function of the previously applied field in Fig. 12. For the TRM one has a clear minimum for field energies of roughly $1.5\ k_B T_f$, whereas in the IRM case the statistical fluctuations do not allow a clear answer. Thus TRM vs (B) would have a maximum even if there were no field dependence of

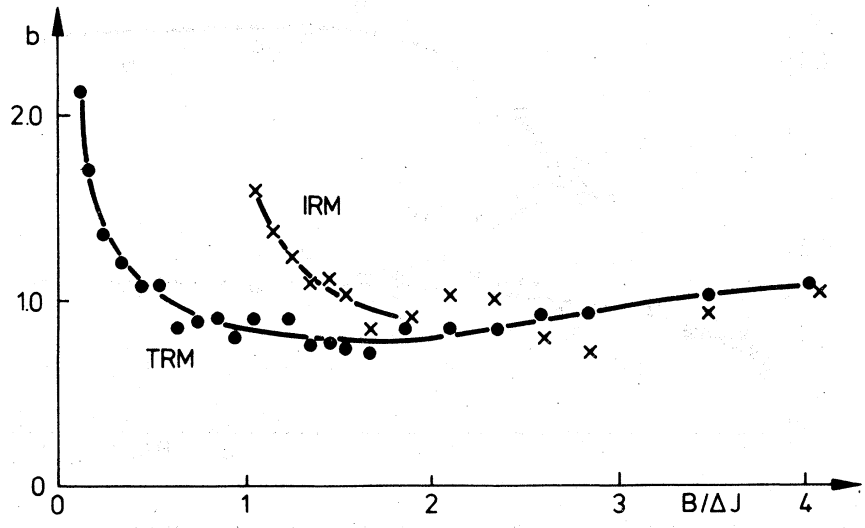

FIG. 12. Exponent b of the fit $\ln M=-b-a\ln t$ for TRM and IRM as a function of initially applied field B. The minimum of $b(B)$ corresponds to a maximum in TRM vs B as in Fig. 11 ($T=0.5\Delta J$).

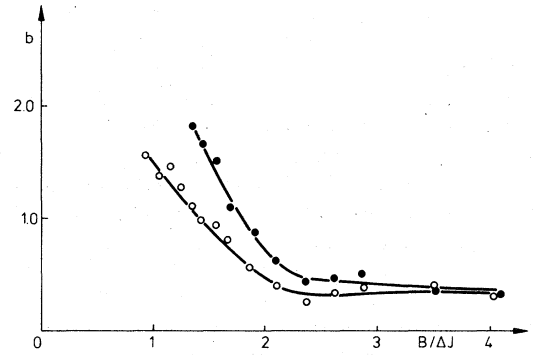

FIG. 13. Same exponent b as in Fig. 12 but for the energy relaxation.

the exponent a.

The corresponding exponent b of the energy relaxation is shown in Fig. 13. There b seems to be monotonically decreasing function of B, meaning that the energy of the metastable state is larger when larger fields are applied.

Up to now only a few experiments dealing with the relaxation of remanence states in spin-glasses have been performed. The heat flow from or to a sample of *Au*Fe (4 at.%) shows a $1/t$ dependence.[14] For the relaxation of the remanent magnetization a logarithmic decay $M=-S\ln t/\tau$ has been observed.[13,15] The prefactor S has a maximum as a function of temperature.[13]

It should be noted that a power law $M\propto t^{-a}$ with a small exponent a (Figs. 9 and 10) can be fitted reasonably well within a limited time interval by a logarithmic decay. Then one has

$$S=\frac{dM}{d\ln t}\propto\frac{d}{d\ln t}e^{-a\ln t}=-at^{-a}\ . \tag{5}$$

With $a(T)=cT$ (Fig. 9) Eq. (5) implies

$$S\propto Tt^{-cT}\ . \tag{6}$$

One sees that S has a maximum as a function of temperature which depends on the measurement time t. Thus the experimental results may not be in disagreement with the present computer simulations. A detailed experimental study of the field dependence of the relaxation, which according to Fig. 10 gives qualitative differences between IRM and TRM, has not yet been carried out as far as we know.

E. Relaxation in an external field

In *Au*Fe, with small fields, Guy has observed the same type of the relaxation for the remanent magnetization and the magnetization in an external field.[13] In our computer simulations we cannot simulate such small fields since the statistical fluctuations due to the finite sample would be much larger than the mean values. For larger fields, the relaxation of the magnetization is shown in Fig. 14. We have plotted the difference ΔM between the magnetization at time t and the magnetization for the field-cooled system which should correspond to the $t=\infty$ magnetization. The log-log plot shows that the relaxation cannot be described by a power law. At best it seems to follow a logarithmic law

$$\Delta M=b-a\ln t \tag{7}$$

as shown by the dashed line in Fig. 14. Figure 15 shows the results for different applied fields. The bars indicate the field-cooled magnetizations. The prefactor a of the fit Eq. (7) is shown as a function

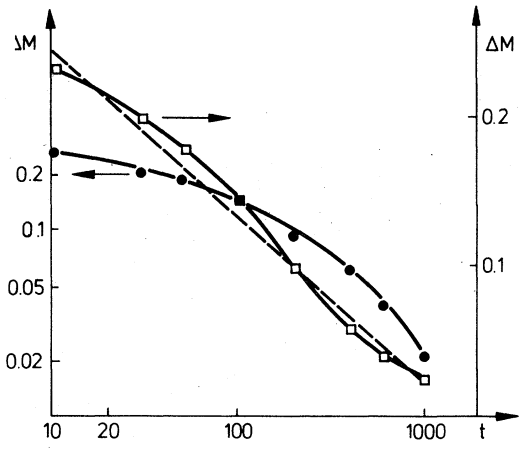

FIG. 14. Relaxation of the magnetization in an external field $B=\Delta J$ for $T=0.5\Delta J$. The difference to the field-cooled value which should correspond to $M(t=\infty)$ is shown in a log-log and a linear-log plot. The dashed line shows $\Delta M\sim\ln t$.

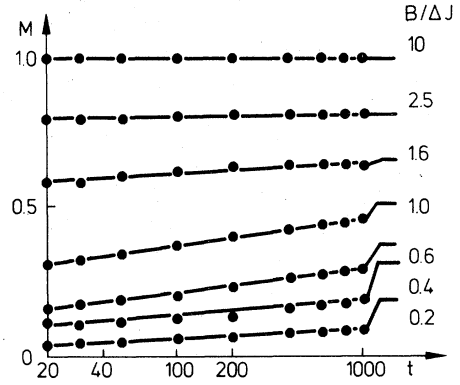

FIG. 15. Relaxation of the magnetization for different applied fields ($T=0.5\Delta J$). The bars show the field-cooled values which should correspond to $M(T=\infty)$.

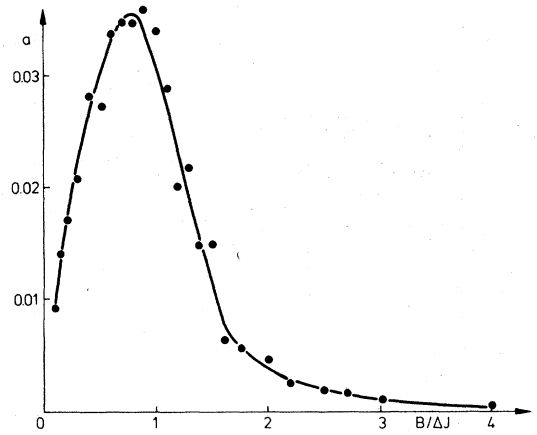

FIG. 16. Coefficient a of the fit $M \sim a \ln t$ for the relaxation of the magnetization in an applied field B ($T = 0.5\Delta J$).

of the applied field in Fig. 16. For small fields the relaxation is slow. For high field the system can reach its saturation value instantly. Thus a is also small. Fields of the order of $k_B T_f$ cannot yet destroy the energy barriers between remanence state and thermal equilibrium but the relaxation is fast. As far as we know such relaxation processes in large fields have not yet been investigated experimentally.

III. METASTABLE STATES IN AN EXACTLY SOLVABLE SPIN-GLASS MODEL

A. Definition of the model

In this section we want to discuss metastable states in spin-glasses in terms of a simple, but exactly solvable model. Two elements are thought to be essential in a model for spin-glasses[2,3]: (a) a symmetric statistical distribution of interactions[18]; (b) the existence of conflicting bonds called frustration.[27,28] The Edwards-Anderson model discussed in Sec. II has these properties, but up to now no analytic description of metastable states has been found. Therefore we restrict ourselves to a mean-field theory which has the properties (a) and (b).

Consider Ising spins with the Hamiltonian

$$H = -\frac{1}{2} \sum_{i,j} J_{ij} S_i S_j - b \sum_i S_i \ , \tag{8}$$

with

$$J_{ij} = (1/N)(J_1 \gamma_i \gamma_j - J_1 \delta_i \delta_j + J_2 \alpha_i \alpha_j) \ , \tag{9}$$

where each of the $\alpha_i, \gamma_i, \delta_i \in \{+1, -1\}$ are randomly distributed over the spin site i, J_1 and J_2 are interaction energies, and N is the number of spins. Thus rather than the bonds J_{ij} some site parameters α_i, γ_i, δ_i are distributed at random. The definition (9) should hold for all pairs $S_i S_j$. For this reason the factor $1/N$ is introduced to make the energy extensive. Thus Eqs. (8) and (9) describe a model with infinite range interactions and the structure of the sites (lattice, space dimension) plays no role. With $J_1 = 0$ one recovers a model investigated by Mattis,[39] which, without field, is equivalent to a pure ferromagnet since it has no frustration.[27] With $J_1, J_2 > 0$ and $\gamma_i = 1$, $\delta_i = 0$ for all i and randomly distributed $\alpha_i = \pm 1$, the model has been considered by Luttinger.[40] (In Ref. 40 a coupling between the two modes was also introduced).

We need three modes to include the spin-glass properties (a) and (b). We consider the model defined by Eqs. (8) and (9) with the restrictions

$$\alpha = \frac{1}{N} \sum_i \alpha_i = 0 \ ,$$

$$1 > \gamma = \frac{1}{N} \sum_i \gamma_i = \frac{1}{N} \sum_i \delta_i > 0 \ , \tag{10a}$$

and

$$2J_1 > J_2 > J_1 (1 - \gamma^4)^{1/2} \ , \tag{10b}$$

which will be explained later. By averaging the bonds J_{ij} over all pairs i,j we obtain

$$\langle J_{ij} \rangle_{\text{av}} = 0 \ ,$$
$$\Delta J = \langle J_{ij}^2 \rangle_{\text{av}}^{1/2} = \frac{1}{N} [J_2^2 + 2J_1^2 (1 - \gamma^4)]^{1/2} \ . \tag{11}$$

Thus the bonds are distributed symmetrically.

B. Ground state and frustration

The energy Eq. (8) can be written in terms of four order parameters

$$q_x = \frac{1}{N} \sum_i x_i S_i, \quad m = \frac{1}{N} \sum_i S_i \ , \tag{12}$$

with $x \in \{\alpha, \gamma, \delta\}$. With these definitions one has

$$-H/N = J_1 q_\gamma^2 - J_1 q_\delta^2 + J_2 q_\alpha^2 + bm \ . \tag{13}$$

Thus H is a quadratic form of the vectors $(x_i S_i)_i$ and $(S_i)_i$. With $(\alpha_i S_i)_i$ orthogonal to all other modes and with the restriction Eq. (10b) one finds that J_2 is the largest eigenvalue (for $b = 0$). Thus the ground state is given by

$$S_i^0 = \alpha_i \ , \tag{14}$$

with energy $-H^0/N = J_2$ and order parameters $q_\gamma = q_\delta = m = 0$. With this ground state one can cal-

culate the number of frustrated bonds $p_f N$ which is defined as the number of bonds with $J_{ij}S_i^0 S_j^0 < 0$. With Eq. (9) this is the case if for $\alpha_i\alpha_j = \pm 1$ the signs of $-\gamma_i\gamma_j$ and $\delta_i\delta_j$ are ± 1. Then one has by definition Eq. (10b) $J_{ij}S_iS_j = J_2 - 2J_1 < 0$. This occurs with probability

$$p_f = \tfrac{1}{4}(1-\gamma^4) \quad . \tag{15}$$

Thus for $\gamma \neq 1$ the model has competing bonds. It should be noted that despite the frustration there is only one ground state.

C. Critical temperature and susceptibility

Since the Hamiltonian can be expressed in terms of just a few order parameters Eq. (13), the model can be solved exactly within the usual mean-field theory as in Ref. 40. Since we are also interested in the relaxation behavior, we give the equations for the time-dependent modes q_α, q_γ, q_δ, and m within the corresponding kinetic Ising model.[41]

$$\left(1+\tau\frac{d}{dt}\right)q_x(t) = \frac{1}{N}\sum_i x_i \tanh\{\beta[\gamma_i J_1 q_\gamma(t) - \delta_i J_1 q_\delta(t) + \alpha_i J_2 q_\alpha(t) + bm(t)]\} \quad , \tag{16a}$$

with $x \in \{\alpha, \gamma, \delta\}$ and

$$\left(1+\tau\frac{d}{dt}\right)m(t) = \frac{1}{N}\sum_i \tanh\{\beta[\gamma_i J_1 q_\gamma(t) - \delta_i J_1 q_\delta(t) + \alpha_i J_2 q_\alpha(t) + bm(t)]\} \quad . \tag{16b}$$

The stationary solutions of these equations are local extrema of the free energy. Thermal equilibrium corresponds to the solution with the lowest free energy. Let us consider the stationary solutions of Eq. (16a) for small q_x. With the definitions [Eqs. (10)] (note that

$$\langle\alpha_i\gamma_i\rangle_{\rm av} = \langle\alpha_i\rangle_{\rm av}\langle\gamma_i\rangle_{\rm av} = 0 \quad ,$$

$$\langle\gamma_i\delta_i\rangle_{\rm av} = \langle\gamma_i\rangle_{\rm av}\langle\delta_i\rangle_{\rm av} = \gamma^2 \quad ,$$

etc.) one obtains

$$\begin{aligned} q_\alpha &= \beta J_2 q_\alpha \quad , \\ q_\gamma &= \beta J_1(q_\gamma - \gamma^2 q_\delta) \quad , \\ q_\delta &= \beta J_1(-q_\delta + \gamma^2 q_\gamma) \quad . \end{aligned} \tag{17}$$

Thus Eq. (16a) has a nonzero solution below

$$T_f = J_2/k_B \tag{18}$$

and because of the restriction Eq. (10b) the system has a second-order phase transition below T_f. Just below T_f or at $T=0$ one has $q_\gamma = q_\delta = 0$ and since

$$q_\alpha = \tanh\beta q_\alpha, \quad q_\gamma = q_\delta = 0 \tag{19}$$

is a solution for all temperatures we expect Eq. (19) to describe the thermal equilibrium for all temperatures. From Eq. (16b) one has zero magnetization in equilibrium. The normalized transition temperature $T_f/\Delta JN$ decreases with increasing frustration p_f Eq. (15) as one would expect,

$$T_f/\Delta J = J_2/(J_2^2 + 8J_1^2 p_f)^{1/2} \quad . \tag{20}$$

Since the distribution of interactions J_{ij} is symmetric the susceptibility is given by[22]

$$\chi = \frac{\partial m}{\partial b} = \beta(1 - \langle\langle S_i\rangle^2\rangle_{\rm av}) = \beta(1-q_\alpha^2) \quad , \tag{21}$$

which of course can also be readily derived from Eqs. (16). Thus $\chi(T)$ has a cusp at the transition temperature T_f.

It should be noted that q_α is just the spin-glass parameter

$$\psi = \frac{1}{N}\sum_i \langle S_i\rangle_{T=0} S_i \quad ,$$

where $\langle S_i\rangle_{T=0}$ is one of the ground states. This order parameter has been introduced and numerically investigated by Binder. The fluctuations of ψ seem to diverge at T_f, and the time dependence shows a critical slowing down at the critical temperature.[34] In the present model we find similar behavior. If we apply the staggered field $h\alpha_i$ varying at each site i, and calculate the staggered susceptibility $\chi = \partial q_\alpha/\partial h$ from Eqs. (16), we find the usual Curie-Weiss divergence at $T_f = J_2/k_B$. This is due to the fact that the staggered field does not influence the modes q_γ and q_δ. Thus the equations are identical to the mean-field equations for a pure ferromagnet. The same holds for the relaxation of q_α. From Eqs. (16) one finds an exponential relaxation for small perturbation of q_α with time constant τ_α given by

$$\frac{\tau}{\tau_\alpha} = 1 - \beta J_2(1-q_\alpha^2) \quad . \tag{22}$$

Thus τ_α diverges like $|T-T_f|^{-1}$, which is the usual mean-field behavior.[41]

D. Remanent magnetization and magnetic–spin-glass transition

By diagonalizing Eq. (17) for $q_\alpha=0$ and from Eqs. (16) we find that below the temperature T_1 given by

$$k_B T_1 = J_1(1-\gamma^4)^{1/2} = 2J_1(p_f)^{1/2} \tag{23}$$

the system has a solution $q_\alpha=0$ and $q_\gamma>0$, $q_\delta>0$. For $T=0$ we find $q_\gamma=1$ and $q_\delta=\gamma^2$. Thus the energy at $T=0$ is given by

$$-\frac{H_1}{N} = J_1(1-\gamma^2) \ . \tag{24}$$

This state does not correspond to thermal equilibrium, but within the present mean-field theory it is stable [see Eqs. (16)]. The state has nonzero magnetization

$$m = \gamma \tanh[J_1(q_\gamma - q_\delta)] \ . \tag{25}$$

In particular $m=\gamma$ at $T=0$. $q_\gamma=1$ means that $S_i=\gamma_i$. Therefore for nonzero magnetic field b one has an additional field energy $b\gamma$, whereas the ground state $S_i=\alpha_i$ has zero-field energy. Thus for sufficiently large field b, the thermal equilibrium is given by $q_\alpha=0$ and q_γ, $q_\delta>0$.

Now let us consider infinite fields $b=\infty$. Then one clearly has $S_i=1$ for all i and according to Eqs. (16) $q_\alpha=0$ and $q_\gamma=q_\delta=\gamma$ in the stationary case. If we now switch off the field, we see from Eqs. (16) that $q_\alpha=0$ is a solution for all times t. Therefore below T_1 the system relaxes into the solution $(q_\gamma,q_\delta)\neq 0$ which has nonzero magnetization Eq. (25). One can easily linearize Eqs. (16) for small perturbation δq_α. One obtains the exponential relaxation $\delta q_\alpha(t)\sim\exp(-t/\tau_\alpha)$ with τ_α given by

$$\frac{\tau}{\tau_\alpha} = 1-\beta J_2\left[1-\frac{1}{N}\sum_i \tanh^2[\beta J_1(\gamma_i q_\gamma - \delta_i q_\delta)]\right], \tag{26}$$

where $(q_\gamma,q_\delta)\neq 0$ is the stationary solution of Eqs. (16). Therefore for sufficiently low temperature τ_α is positive and the remanence state is indeed stable against small perturbations. Thus we have shown that the system can really relax from the ferromagnetic ordered state into a metastable state (which is stable within mean-field theory) with remanent magnetization.

We now show that for certain values of parameters J_1, J_2, and γ when lowering the temperature the system undergoes a second-order phase transition into a magnetic state and then a first-order transition into the spin-glass state. From Eqs. (12) and (14) we know that the spin-glass phase $q_\alpha=1$ and $q_\gamma=q_\delta=0$ is the ground state if

$$J_2 > J_1(1-\gamma^4) \ . \tag{27}$$

On the other side if

$$k_B T_1 = J_1(1-\gamma^4)^{1/2} > J_2 = k_B T_f \tag{28}$$

we know from Eqs. (17), (18), and (23) that just below T_1 there is a unique solution $(q_\gamma,q_\delta)\neq 0$ and $q_\alpha=0$. Since both Eqs. (27) and (28) can be fulfilled simultaneously, at $T=T_1$ we have a transition into a state with nonzero magnetization Eq. (25), and at $T=0$ we have a spin-glass state $S_i=\alpha_i$ with zero magnetization. Since both of the states are solutions of Eqs. (16) for all temperatures below T_1 there must be a first-order transition from the magnetic state to a spin-glass state with decreasing temperature. Such a transition has recently been observed experimentally.[42,43]

III. SUMMARY AND CONCLUSIONS

In the first part of the paper we described numerical simulations of a spin-glass model consisting of the two-dimensional Ising model with random nearest-neighbor interactions. We found that below a characteristic temperature T_f there are reversible and irreversible susceptibilities which deviate from the Curie Law above T_f. Below T_f a remanent magnetization appears which differs for different ways of preparation such as field cooling (TRM) or shortly applying the field for constant temperature (IRM). The TRM increases steeply with the previously applied field and has a well defined maximum while the IRM increases slowly with the field and is smaller than the TRM. The TRM decreases with temperature, whereas the IRM has a maximum. Both the IRM and the TRM decay with a power law as a function of time, but the IRM decay faster than the TRM and is slower the larger the field, whereas the relaxation exponent of the TRM increases with increasing field. Many of these details are in qualitative agreement with experiments. Thus from our results and previous Monte Carlo simulations we can conclude that the investigated model describes many static and dynamic properties of spin-glasses very well. However, in addition to static equilibrium properties and linear response near equilibrium, it is important to consider nonlinear relaxation phenomena after field and temperature changes, just as in the experiments. In general it is difficult to obtain analytic results, but numerical results can be obtained conveniently with Monte Carlo methods.

In the second part of this paper we tried to obtain some understanding of metastable states in spin-glasses with the mean-field analysis of a simple model. We have introduced a model including two

important spin-glass properties such as a symmetric distribution of interactions and frustration. Even with additional simple relaxation processes we could solve this model exactly. We have found a phase transition with a cusp in the susceptibility. For sufficiently low temperatures the system relaxes from a magnetically ordered state into a state with remanent magnetization (for zero field). For a certain range of model parameters the system undergoes a magnetic–spin-glass transition with decreasing temperature even though the distribution of interactions is symmetric.

Our model has infinite range interactions. Thus the remanence states are obtained by a global reordering of spins and not by local turning of the clusters. These states are due to a collective effect. In our mean-field theory these remanence states are stable. In a more realistic model one should include thermal fluctuations which enable the system to reach the state with lowest free energy. In addition, one should treat an infinite number of modes instead of the three considered here. Thus even in the limit of zero field there may exist states which have magnetization for zero field and correspond to thermal equilibrium for small fields. This may be the reason that one observes a reversible susceptibility which measures the fluctuations of the ground state [Eq. (21)], and an irreversible susceptibility, which measures the fluctuation of the remanence states, plus an additional magnetization of these states [Eq. (25)].

Even the different relaxational behavior of TRM and IRM can be qualitatively explained by such considerations. In the IRM case the field energy is larger than the energy barrier V between the ground state and the remanence state. Thus V decreases with decreasing field, and since the relaxation grows faster with decreasing barrier V, the system is faster for smaller fields. In the TRM (field-cooled) case the system starts from $T > T_f$ where there are no energy barriers. Thus even for small fields it can reach remanence states with high-energy barriers to the ground state. Therefore the TRM relaxes much slower than the IRM. This is what was observed in the computer simulations described in the first part of the paper.

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