

Heisenberg exchange in heavy rare-earth alloys

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It is shown that the highest ordering temperatures of heavy rare earths and alloys among the heavy rare earths follow the Heisenberg spin dependence $S(S + 1)$. The so-called anomalous behavior of alloys of heavy rare earths with Sc and Y is explained in terms of a combination of enhanced conduction-electron polarization and conduction-electron scattering by solute atoms.

For the rare-earth elements the indirect-exchange interaction between the "localized" $4f$ electrons and the itinerant conduction electrons leads to ferromagnetic and/or antiferromagnetic ordering. In studies of the highest ordering temperatures of the pure heavy elements and alloys of these elements Koehler¹ found that the ordering temperatures departed from the de Gennes law $G = (g - 1)^2 J(J + 1)$. Subsequently Bozorth and Gambino² found that the Curie-Néel points followed the empirical law $\bar{G}^{2/3}$, where $\bar{G} = C_1 G_1 + C_2 G_2$ with C_1 and C_2 the concentrations of the constituents and G_1 and G_2 their respective de Gennes factors. In the course of preparing a review of magnetism in rare-earth metals and alloys it seemed appropriate to reexamine this phenomenon. It was at once discovered that T_N (or T_C) of the elements has the $S(S + 1)$ dependence which Heisenberg exchange predicts. Having found this dependence for the elements the next step was to examine the readily accessible alloy data. It turned out that the intra-heavy-rare-earth data of Fujiwara *et al.*³ and of Koehler⁴ fell right along the line for the elements when the spin factor was calculated by use of $C_1 S_1(S_1 + 1) + C_2 S_2(S_2 + 1)$, where C_1 is the concentration of the element with spin S_1 and C_2 is the concentration of the element with spin S_2 . The data are shown in Fig. 1, where the (+) shows the data for the elements and the round dots the intra-heavy-rare-earth alloy data. The real problem can be seen in the figure to be the data of Child *et al.*^{5,6} for Y and Sc in Tb, the data of Nigh *et al.*⁷ for Sc in Gd, and the data of Thoburn *et al.*⁸ for Y in Gd which all obviously depart markedly from the line fixed by the transitions of the elements.

In order to understand the Y and Sc results qualitatively it is necessary to take into account (a) the reduction in the ordering temperature caused by conduction-electron scattering from the solute atoms (the indirect-exchange interaction is affected by the conduction-electron mean free path) and (b) the enhanced $5d$ -band polarization which affects the saturation magnetic moment. These have been discussed in a recent paper by Legvold *et al.*⁹

In the inset of Fig. 1 the residual resistivities for Y, Lu, and Sc solutes in Gd and Tb host metals are shown for concentrations ranging up to 15 at.%. The data are typically the start of parabolic Nordheim plots¹⁰ and this should be borne in mind in what follows. It can be seen that the conduction-electron scattering caused by Sc is considerably greater than that caused by Lu and that scattering caused by Y is the lowest of the three. Certainly Y atoms are the most compatible with the lattice. This is in keeping with the position Y has in the ion-exchange elution process where it falls between Tb and Dy. Further-

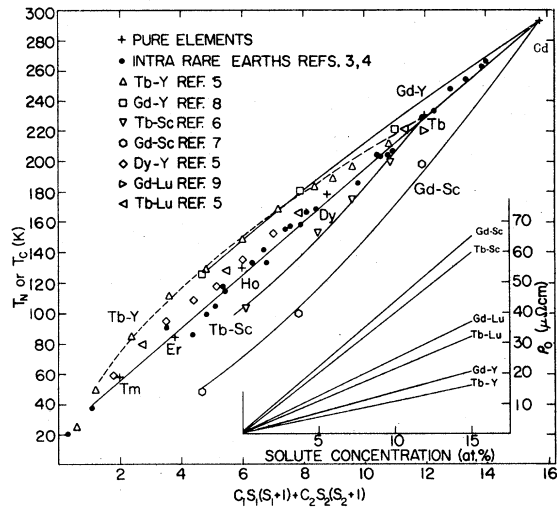


FIG. 1. Plot of the highest ordering temperature vs the average spin factor squared for the heavy rare earths and alloys of the heavy rare earths. The filled circles from Ref. 3 are data for one sample each of Gd-Tb, Tb-Dy, and three samples (25, 50, 75 at.%) for each of Gd-Dy, Gd-Ho, Gd-Er, Dy-Ho, Dy-Er, Ho-Er; the other filled-circle points are from the tables in Ref. 4. The inset of the figure shows the residual resistivity vs solute concentration for Gd alloyed with Sc, Lu, and Y and for Tb alloyed with Sc, Lu, and Y.

TABLE I. Surplus moments for Sc, Lu, and Y solutes in Gd.

Sample	Gd	0.155 Sc	0.15 Lu	0.148 Y	0.301 Sc	0.297 Lu	0.306 Y
$\mu_{\text{sat}}/(\text{Gd atom}) (\mu_B)$	0.555	0.61	0.64	0.67	0.68	0.75	0.78

more, the atomic volume of Sc is $15.03 \text{ cm}^3/\text{mole}$ as compared with 19.88 for Gd, 19.86 for Y, and 17.8 for Lu. In view of this aspect of the problem, one should certainly expect the most incompatible element Sc to cause a greater decrease in the ordering temperature as compared with Y and Lu. For the intra-heavy-rare-earth alloys the scattering effect is small.

In order to understand the Y alloys it is necessary to invoke the $5d$ -band polarization-enhancement effect. In Ref. 9 it was reported that the surplus magnetic moment of an alloy (the moment in excess of $7\mu_B$ expected per Gd atom) is proportional to the c/a ratio of the alloy. Y and Lu both gave favorable c/a ratios with Sc somewhat less effective in this respect. In Table I the surplus moments found for the mother Gd and for the different samples are

shown. All three elements give some enhancement, but Y has the most favorable effect.

The combination of both influences places the Tb-Lu alloy data just slightly above the line for the elements. For Y the $5d$ wave-function polarization-enhancement effect gives higher than expected ordering temperatures while the incompatibility effect almost counteracts the effect for Lu. In the case of Sc the conduction-electron scattering dominates.

This paper gives an alternate explanation for the observed data to that given recently by Lindgård.¹¹

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