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Correct form of energy-balance equation for intervalley and intersubband scattering in semiconductors

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Two different methods for deriving the energy-balance equation for hot electrons in semiconductors are described in the literature. The first method uses the Boltzmann equation, while in the second method the equation is obtained from the rates of scattering out of a state by absorption and emission processes. It is shown that for the general case of intervalley and intersubband scattering in semiconductors the use of method 2 is incorrect, and only in a special case the two methods are identical. It is pointed out that in method 2 the rate of change of energy due to intervalley exchange of electrons is neglected. Because of this, the two methods should give different expressions and different values of transport coefficients. The nature of this difference is discussed from the general point of view, and illustrated in the case of bulk and of quantized inversion layer by assuming a Maxwellian distribution.

I. INTRODUCTION

In a theoretical study of hot-electron transport in semiconductors,[1] the distribution function for the electrons is usually assumed to be Maxwellian. The transport coefficients of the electrons are then calculated by solving the energy- and momentum-balance equations obtained from the Boltzmann equation. The energy-balance equation is obtained by multiplying the field and collision terms by energy and integrating over the momentum space of the valley. Thus one gets

$$\int E_i \left.\frac{\partial f_i}{\partial t}\right|_F d\vec{k} = -n_i e \mu_i F^2 = \int E_i \left.\frac{\partial f_i}{\partial t}\right|_{\mathrm{coll}} d\vec{k}, \qquad (1)$$

where $\partial f_i/\partial t|_F$ and $\partial f_i/\partial t|_{\mathrm{coll}}$ are, respectively, the rate of change of the distribution function f_i, for the ith valley electrons due to field and collision, $\vec{k}$ is the wave vector, E_i is the energy, n_i and μ_i are, respectively, the number density and mobility of the electrons, e is the electron charge, and F is the field. The above method of expressing the energy-balance equation was first proposed by Fröhlich and Paranjape[2] and will hereafter be referred to as method 1.

A different method (hereafter mentioned as method 2) was proposed by Conwell.[3] In this, the rate of loss of energy to the lattice is calculated from the rate of scattering from a state $\vec{k}$ in the ith valley. The rate of loss is given by

$$-\frac{dE}{dt} = \hbar\omega_0 \left[\left(\frac{1}{\tau}\right)_e - \left(\frac{1}{\tau}\right)_a\right], \qquad (2)$$

where $\hbar\omega_0$ is the phonon energy, $(1/\tau)_e$ is the rate of scattering by emission of phonons, and $(1/\tau)_a$ is the same for absorption.[4] The average rate of energy loss is obtained by multiplying dE/dt by f_i and integrating over momentum space. When this average rate of loss is equated with the rate of gain of energy from the field, one gets

$$-n_i e \mu_i F^2 = \int f_{0i} \left(\frac{dE}{dt}\right) d\vec{k}. \qquad (3)$$

The above method has been utilized by several workers[5-9] to solve the hot-electron problems in bulk semiconductors. Recently the same method has been followed[10-13] to calculate the energy loss due to intersubband and intervalley scattering in quantized inversion layer.

Although both these methods have been in continuous use[5-19] in the study of hot electrons, as far as the author is aware, no attempt has yet been made to examine whether the two methods are equivalent in all respects. Such an attempt is made in the present paper. We have found that for the general case of intervalley and intersubband scattering, method 2 is incorrect, and only in a special case may its use be justified. We will give a proof of this statement in Sec. II and discuss how the expressions and calculated results for transport parameters are expected to differ in two methods. We will consider three examples: (i) equivalent intervalley scattering, (ii) nonequivalent intervalley scattering in the bulk, and (iii) intervalley and intersubband scattering in quantized inversion layer in Sec. III to illustrate the conclusions drawn from Sec. II. By using a Maxwellian distribution for carriers, we will compare the expressions and numerical results obtained from the two methods. Finally, in Sec. IV we will discuss some of the cases where the two methods may give nearly equal numerical values for transport coefficients.

II. GENERAL CONSIDERATIONS

We will first give a direct proof regarding the limited validity of method 2. The Boltzmann equation in the presence of intervalley scattering is given as

$$\frac{e}{\hbar}\vec{F}\cdot\nabla_{\vec{k}}f_i=\sum_j\sum_{\vec{k}}[f_j(\vec{k}')W^{ji}(\vec{k}',\vec{k})-f_iW^{ij}(\vec{k},\vec{k}')]\,, \tag{4}$$

where f_i and f_j are the distribution functions in the ith and jth valley, respectively, $W^{ji}(\vec{k}',\vec{k})$ is the probability of transition from the state $\vec{k}'$ in the jth valley to the state $\vec{k}$ in the ith valley, and $W^{ij}(\vec{k},\vec{k}')$ is the same for the reverse process. The energy-balance equation following method 1 is obtained by multiplying both sides of the above by $E_i(\vec{k})$ and summing over $\vec{k}$. This yields

$$\frac{e}{\hbar}\sum_{\vec{k}}\vec{F}\cdot\nabla_{\vec{k}}f_iE_i(\vec{k})=\sum_{\vec{k},\vec{k}',j}[f_j(\vec{k}')W^{ji}(\vec{k}',\vec{k})-f_i(\vec{k})W^{ij}(\vec{k},\vec{k}')]E_i(\vec{k})\,. \tag{5}$$

The final expression is obtained once W^{ij}'s are expressed in terms of the deformation-potential constants and other physical constants of the material, the form of the distribution function is introduced, and the summation over $\vec{k}$ and $\vec{k}'$ is performed. We will deal with the expressions for a few cases in Sec. III. In the following, we will show how, from Eq. (5), the balance equation according to method 2 follows as a special case. For this purpose we write Eq. (5) in an altered form

$$\frac{e}{\hbar}\sum_{\vec{k}}\vec{F}\cdot\nabla_{\vec{k}}f_iE_i(\vec{k})=\sum_{\vec{k}}f_i(\vec{k})\sum_{j,\vec{k}'}W^{ij}(\vec{k},\vec{k}')[E_i(\vec{k})-E_j(\vec{k}')]+\Bigg(\sum_{j,\vec{k},\vec{k}'}[f_i(\vec{k})W^{ij}(\vec{k},\vec{k}')E_j(\vec{k}')-f_j(\vec{k}')W^{ji}(\vec{k}',\vec{k})E_i(\vec{k})]\Bigg)\,. \tag{6}$$

Interchanging the variables $\vec{k}$ and $\vec{k}'$ in the last term of the above, one gets

$$\frac{e}{\hbar}\sum_{\vec{k}}\vec{F}\cdot\nabla_{\vec{k}}f_iE_i(\vec{k})=\sum_{\vec{k}}f_i(\vec{k})\sum_{j,\vec{k}'}W^{ij}(\vec{k},\vec{k}')[E_i(\vec{k})-E_j(\vec{k}')]+\Bigg(\sum_{j,\vec{k},\vec{k}'}[f_i(\vec{k})W^{ij}(\vec{k},\vec{k}')E_j(\vec{k}')-f_j(\vec{k})W^{ji}(\vec{k},\vec{k}')E_i(\vec{k}')]\Bigg)\,. \tag{7}$$

Now for intervalley or nonpolar-optical phonon scattering the transition probability W is isotropic, independent of $\vec{k}$ and $\vec{k}'$. Therefore, one gets $W^{ij}=W^{ji}$. Furthermore, if we consider the scattering between equivalent valleys as in n-Ge or n-Si, we have $E_j(\vec{k}')=E_i(\vec{k}')$. If in addition, we put $f_i=f_j$, the two terms within large parentheses in Eq. (7) cancel each other. Under this condition we have

$$\frac{e}{\hbar}\sum_{\vec{k}}\vec{F}\cdot\nabla_{\vec{k}}f_iE_i(\vec{k})=\pm\hbar\omega_0\sum_{\vec{k}}f_i(\vec{k})\sum_{j,\vec{k}'}W^{ij}(\vec{k},\vec{k}')\,, \tag{8}$$

since we have $E_j(\vec{k}')-E_i(\vec{k})=\pm\hbar\omega_0$, the phonon energy. Using the definition of relaxation time, one may rewrite Eq. (8),

$$-n_ie\mu_iF^2=\sum_j\int f_i(\vec{k})\hbar\omega_0\left[\left(\frac{1}{\tau}\right)_e-\left(\frac{1}{\tau}\right)_a\right]d\vec{k}\,, \tag{9}$$

which is the balance equation derived in accordance with method 2.

The above arguments point out that only for the specific case when the two valleys are equivalent and are characterized by identical distribution function, the two balance equations given by the two methods are identical. In all other cases, i.e., when we have (i) $E_j(\vec{k}')=E_i(\vec{k}')$, but $f_i(\vec{k})\neq f_j(\vec{k})$; (ii) $E_j(\vec{k}')\neq E_i(\vec{k}')$, but $f_i(\vec{k})=f_j(\vec{k})$; and (iii) $E_j(\vec{k}')\neq E_i(\vec{k}')$ and also $f_i(\vec{k})\neq f_j(\vec{k})$,[20] the above two terms in Eq. (7) do not cancel each other. Since in method 2 this difference of terms in large parentheses is always ignored, the method is liable to give incorrect expressions.

It may be concluded by examining Eq. (9) that the energy-balance equation for the ith valley obtained from method 2, contains f_i only, but is independent of f_j. In other words, the final expressions are decoupled. Method 1, on the contrary always gives coupled equations as can be seen from Eq. (5).

To obtain more information about the nature of the expressions derived, we may write all the terms in Eq. (7) as

$$\left\langle\frac{dE}{dt}\right\rangle_F=\left\langle\frac{dE}{dt}\right\rangle_{\text{lattice}}+\left(\left\langle\frac{dE}{dt}\right\rangle_{ij}-\left\langle\frac{dE}{dt}\right\rangle_{ji}\right)=\left\langle\frac{dE}{dt}\right\rangle_{\text{lattice}}+\left\langle\frac{dE}{dt}\right\rangle_{\text{ivx}} \tag{10}$$

for the ith valley electrons. Here $\langle dE/dt\rangle_F$ denotes the average rate of gain of energy from the field. When the electrons make a transition from the ith valley to the jth, energy is drained out of the valley. Part of this energy is given to the lattice at a rate of $\langle dE/dt\rangle_{\text{lattice}}$ and the rest is brought to the jth valley at the rate $\langle dE/dt\rangle_{ij}$. Similarly the lost energy in the ith valley is replenished at the rate $\langle dE/dt\rangle_{ji}$ by the electrons coming from the jth valley. The difference, $\langle dE/dt\rangle_{ij} - \langle dE/dt\rangle_{ji}$, therefore, represent the rate of change of energy in the valley due to intervalley exchange of electrons, $\langle dE/dt\rangle_{\text{ivx}}$. This exchange contribution is completely ignored in method 2. If the total energy of the jth valley electrons is higher, it is only natural to expect that the electrons coming to the ith valley will bring more energy than that carried away by outgoing electrons. In other words the difference, $\langle dE/dt\rangle_{\text{ivx}}$ will be nonzero and negative. It may be similarly argued that for intravalley scattering, or that between two valleys having the same distribution, the above difference will be zero. In such a case, both the methods will give identical expressions.

Another observation can be made. If one sums the two sides of Eq. (5) over all valleys, one gets

$$-\sum_i n_i e\mu_i F^2 = \sum_{ij} \int f_i(\vec{\mathrm{k}})\hbar\omega_0\left[\left(\frac{1}{\tau}\right)_e - \left(\frac{1}{\tau}\right)_a\right] d\vec{\mathrm{k}}\,, \quad (11)$$

which is also obtained from Eq. (9).

Physically, this means that in a two-valley system the exchange contribution for one valley is equal and opposite to that of the other. Hence, while considering the total rate of change of energy of the system, the gain is due to the field and the loss is due to collisions.

Now in method 1, the appropriate share of the total loss for each valley is correctly evaluated by taking into account the exchange contribution. In method 2, however, the loss is assumed to be entirely due to collision to the lattice. From the foregoing discussions it is evident that when method 1 is employed, there is more equal distribution of energy amongst the valleys than what is obtained from method 2. In other words, if the term "electron temperature" is used to characterize the distribution function in a valley, method 1 will lead to a less difference of temperature between two valleys, than that calculated following method 2. In Sec. III we will consider three practical situations to illustrate how, by assuming a Maxwellian distribution of carriers, the above conclusions drawn from a general point of view may be found to be correct.

III. EXAMPLES USING A MAXWELLIAN DISTRIBUTION

A. Bulk: Equivalent intervalley scattering

The energy-balance equation for equivalent intervalley scattering in bulk semiconductor, as occurs in n-Ge or n-Si, under the assumption of a Maxwellian distribution of carriers has been derived by employing both the methods. The equation for ith valley electrons making a transition to the jth valley is[17]

$$-n_i e\mu_i F^2 = \frac{D^2 m_{Di}}{2^{1/2}\pi\hbar^3\rho\omega_0}\left[2n_j\left(\frac{k_B T_{ei}}{\pi}\right)^{1/2} k_B T_{ej}\left[(N_0+1)e^{-x_{ej}} I_{31}(x_{ej}) + N_0 I_{13}(x_{ej})\right] - 2n_i\left(\frac{k_B T_{ei}}{\pi}\right)^{1/2} k_B T_{ei} + \left[(N_0+1)e^{-x_{ei}} I_{13}(x_{ei}) + N_0 I_{31}(x_{ei})\right]\right], \quad (12)$$

where we have

$$x_{ei}(j) = \hbar\omega_0/k_B T_{ei(j)}\,,$$

$$I_{mn}(y_0) = \int_0^\infty [y^m(y+y_0)^n]^{1/2} e^{-y}\,dy\,.$$

In the above equations, $T_{ei(j)}$ is the temperature of the $i(j)$th valley, N_0 is the phonon number, D is the deformation-potential constant, m_D is the density-of-states effective mass, and ρ is the density of the material.

When method 2 is employed to calculate the rate of energy loss to the lattice, one obtains for both the intervalley and the intravalley optical-phonon scattering[3]

$$e\mu_i F^2 = \left(\frac{2}{\pi}\right)^{1/2} \frac{D^2 m_{Di}}{\pi\hbar^2\rho}(k_B T_{ei})^{1/2} \times \left[(N_0+1)\exp(-x_{ei}) - N_0\right](\tfrac{1}{2}x_{ei}) \times \exp(\tfrac{1}{2}x_{ei})K_1(\tfrac{1}{2}x_{ei})\,, \quad (13)$$

K_1 being the modified Bessel function of second kind.[21]

Equation (13) shows that the balance equation for ith valley depends only on the temperature of that valley, but is independent of that of jth valley, to which the electrons are scattered. Equation

(12), on the other hand is coupled, i.e., it involves the number and temperature of both the valleys.

It may easily be verified that for optical-phonon scattering or intervalley scattering when we have $T_{ei}=T_{ej}$ and $n_i=n_j$, Eq. (12) reduces to Eq. (13).[21]

It is interesting to examine how the numerical values of different parameters calculated by the two methods differ. This has been done in Ref. 17, in which the temperature of the two sets of valleys in bulk silicon, when the field is along [100] direction, are calculated.[22] Figure 3 of that paper shows that the temperature of the hot valleys is less, and that of the cool valleys is higher than the same calculated with method 2. In other words, the temperatures tend to equalize when calculated with method 1, as a consequence of including the intervalley exchange contribution.

B. Bulk: Nonequivalent intervalley scattering

The nonequivalent intervalley scattering, such as the transition from the central to the satellite valleys in GaAs, or from L-point minima to X-point minima in Ge, has been treated by several workers.[5,9,14–16,18,19] We will consider here the transition from a lower valley, denoted by subscript 1 to a upper one, denoted by 2, both the valleys being assumed to have parabolic E-$\vec{k}$ relationship and scalar-effective mass. Following method 1 the expression becomes[19]

$$-n_1e\mu_1F^2=\frac{D^2}{2^{1/2}\pi\rho\hbar^2\omega_{12}}\Bigg[2n_1(m_2)^{3/2}\left(\frac{k_BT_1}{\pi}\right)^{1/2}k_BT_1\ (N_{12}+1)e^{-\Delta_{1+}}\phi(\tfrac{1}{2},\tfrac{3}{2},\Delta_{1+})+N_{12}e^{-\Delta_{1-}}\phi(\tfrac{1}{2},\tfrac{3}{2},\Delta_{1-})]$$
$$-2n_2(m_1)^{3/2}\left(\frac{k_BT_2}{\pi}\right)^{1/2}k_BT_2[(N_{12}+1)\phi(\tfrac{1}{2},\tfrac{3}{2},\Delta_{2-})+N_{12}\phi(\tfrac{1}{2},\tfrac{3}{2},\Delta_{2+})]\Bigg]\,, \qquad (14)$$

where $\hbar\omega_{12}$ is the phonon energy, N_{12} is the phonon number and we have

$$\phi(m,n,a)=\int_0^\alpha e^{-x}x^m(x+a)^n\,dx\,,$$

$$\Delta_{i\pm}=(E_{12}\pm\hbar\omega_{12})/k_BT_i\,,$$

E_{12} being the energy separation between 1 and 2.

Following the procedure described in Ref. 5, the equation on the other hand becomes

$$-n_1e\mu_1F^2=n_1\frac{D^2(m_2)^{3/2}}{2^{1/2}\pi\rho\hbar^2}$$
$$\times\left[\left(\frac{k_BT_1}{\pi}\right)^{1/2}k_BT_1[N_{12}e^{-\Delta_{1-}}\phi(\tfrac{1}{2},\tfrac{1}{2},\Delta_{1-})\right.$$
$$\left.-(N_{12}+1)e^{-\Delta_{1+}}\phi(\tfrac{1}{2},\tfrac{1}{2},\Delta_{1+})]\right]\,. \qquad (15)$$

Equations (14) and (15) are evidently different. Equation (15) is decoupled while Eq. (14) is not. It may be noticed, however, that even by making $T_1=T_2$ and also $m_1=m_2$, which yields further

$$n_2=n_1\exp(-E_{12}/k_BT_1)\,,$$

Eq. (14) does not reduce to Eq. (15), unlike the result in the case of interequivalent scattering. The reason is, by making $m_1=m_2$ the condition $E_1(\vec{k})=E_2(\vec{k})$ may be realized, but we have $f_1(\vec{k})\neq f_2(\vec{k})$ even if we have $T_1=T_2$, because of different normalization factors [cf. Eq. (7) and case (iii) in Sec. II].

C. Quantized inversion layer

The rate of energy loss of electrons due to scattering between two valleys within the same subband in a quantized inversion layer,[23,24] has been obtained by Hess and Sah[10] following method 2. More recently, Ferry[11,13] has derived the results for scattering between two valleys belonging to two different subbands involving a zero-order and a first-order coupled phonon. For the zero-order case the expression is

$$e\mu_iF^2=\frac{D^2m_{Di}\beta_b}{\rho_m\hbar\omega_j}[(N_0+1)e^{-x_{ei}}-N_0]e^{-\Delta_{ij}}\,, \qquad (16)$$

where D, m_D, and x_{ei} have the same meaning as before. β_b is the number of equivalent valleys. ρ_m is the mass density, w_j is the width of jth subband, and we have $\Delta_{ij}=\epsilon_{ij}/k_BT_{ei}$, T_{ei} being the temperature of the valley and ϵ_{ij} being the energy gap between two subbands i and j.

To derive the energy-balance equation following method 1 we may first write the collision operator for the symmetric part of the distribution function for the ith subband as

$$\left.\frac{\partial f_{0i}}{\partial t}\right|_{\text{coll}}=\frac{2\pi}{\hbar}\frac{D^2\hbar\beta_b}{2A\rho_m\omega_0}\sum_{\vec{q}}\Bigg(\frac{1}{w_j}\{(N_0+1)f_{0i}(E+\hbar\omega_0)[1-f_{0i}(E)]\delta(E_{\vec{k}}-E_{\vec{k}+\vec{q}}+\hbar\omega_0-\epsilon_{ij})$$
$$+N_0+f_{0j}(E-\hbar\omega_0)[1-f_{0i}(E)]\delta(E_{\vec{k}}-E_{\vec{k}-\vec{q}}-\hbar\omega_0-\epsilon_{ij})\}$$
$$-\frac{1}{w_i}\{(N_0+1)f_{0i}(E)[1-f_{0i}(E-\hbar\omega_0)]\delta(E_{\vec{k}-\vec{q}}-E_{\vec{k}}+\hbar\omega_0-\epsilon_{ij})$$
$$+N_0f_{0i}(E)[1-f_{0i}(E+\hbar\omega_0)]\delta(E_{\vec{k}+\vec{q}}-E_{\vec{k}}-\hbar\omega_0-\epsilon_{ij})\}\Bigg)\,, \qquad (17)$$

where A is the area of the surface. We have retained in Eq. (17) the degeneracy factors. Replacing the summation over q by intergration in the usual manner,[25] one gets

$$\left.\frac{\partial f_{0i}}{\partial t}\right|_{\text{coll}} = \frac{D^2 m_{Di}\beta_b}{2\hbar^2\rho_m\omega_0}\left(\frac{1}{w_j}\{(N_0+1)f_{0j}(E+\hbar\omega_0)[1-f_{0i}(E)]u(E+\hbar\omega_0-\epsilon_{ij})+N_0 f_{0j}(E-\hbar\omega_0)\right.$$
$$\times[1-f_{0i}(E)]u(E-\hbar\omega_0-\epsilon_{ij})\} - \frac{1}{w_i}\{(N_0+1)f_{0i}(E)[1-f_{0j}(E-\hbar\omega_0)]u(E-\hbar\omega_0-\epsilon_{ij})$$
$$\left.+N_0 f_{0i}(E)[1-f_{0j}(E+\hbar\omega_0)]u(E+\hbar\omega_0-\epsilon_{ij})\}\right), \quad (18)$$

u being a step function.

The energy-balance equation is obtained by multiplying Eq. (18) by E and integrating over the momentum space of ith subband electrons. The integration cannot, however, be performed analytically if one assumes we have for $f_{0i(j)}$ a Fermi distribution characterized by an electron temperature $T_{ei(j)}$. It is evident that the energy-balance equation contains two temperatures corresponding to the two levels. Some simplification results when f_0 is assumed to be Maxwellian. Neglecting then the $[1-f_0(E)]$ factors, one may obtain

$$-n_i e\mu_i F^2 = \frac{D^2\beta_b}{2\hbar^2\rho_m\omega_0}\left(\frac{m_{Dj}n_j k_B T_{ej}}{w_j}[(N_0+1)e^{-x_{ej}}+N_0(x_{ej}+1)]\right.$$
$$-\frac{m_{Di}n_i k_B T_{ei}}{w_i}[(N_0+1)(\Delta_i+x_{ei}+1)e^{-x_{ej}}$$
$$\left.+N_0(\Delta_i+1)]e^{-\Delta_i}\right), \quad (19)$$

where we have

$$\Delta_{i(j)} = \epsilon_{ij}/k_B T_{ei(j)} ,$$

$$x_{ei(j)} = \hbar\omega_0/k_B T_{ei(j)} ,$$

and

$$n_{i(j)} = \int f_{0i(j)}\, d\vec{k} ,$$

are the electron concentration in the respective subband.

Equation (19) is different from Eq. (16). Even when one puts $w_i = w_j$ and $T_{ei} = T_{ej}$ [in Eq. (19)] and consider transitions between subbands arising from the same type of valleys, (such as between E_0 and E_1 in n-channel silicon inversion layer[24]) for which we have $m_i = m_j$ and $n_j = n_i \exp(-\Delta_{ij})$ with $\Delta_{ij} = \Delta_i = \Delta_j$, it is not possible to arrive at Eq. (16). When we have $\Delta_{ij} = 0$, $T_{ei} = T_{ej}$, $n_i = n_j$, and $w_i = w_j$, Eq. (19) reduces to the form for optical-phonon or intervalley scattering as derived by Hess and Sah.[10] Such a situation arises for the $E_{0'}$ subband in n-channel Si, when the field is along the [110] direction.

It may, therefore, be concluded that Eq. (16) derived from method 2 is incorrect. It is of interest to note that Eq. (16) gives values of temperature numerically different from those calculated from Eq. (19) even if the same scattering model is used. To illustrate this we consider scattering amongst the four valleys in the $E_{0'}$ subband in a Si inversion layer with the field along the [100] direction. Such a situation has been considered by Hess and Sah.[10] In this case, two valleys having major axis along [100] will be less heated than the other two valleys. We consider the scattering between the two cool valleys to be first-order coupled and that between a cool and a hot valley to be zero-order coupled in accordance with Ferry.[11] In addition, intravalley acoustical-phonon scattering is taken into account for the calculation of momentum-relaxation time. Phonon temperatures and deformation-potential constants are of the same values as used by Ferry and the width of the $E_{0'}$ layer is assumed to be 40 Å.

Figure 1 shows the electron temperatures in the hot and cool valleys for different values of electric field calculated by using expressions (16) and (19). It is found that the valley temperatures tend to equalize when calculated with method 1. The values of drift velocity calculated by the two methods are different as can be found from Fig. 2. Although the effect of first-order coupled phonon is included in the rate of energy loss, it is negligibly small so that the difference is entirely due to the two approaches.

IV. DISCUSSIONS

We have found that the use of method 2 to derive the energy-balance condition is not justified. We have also demonstrated that the expressions and

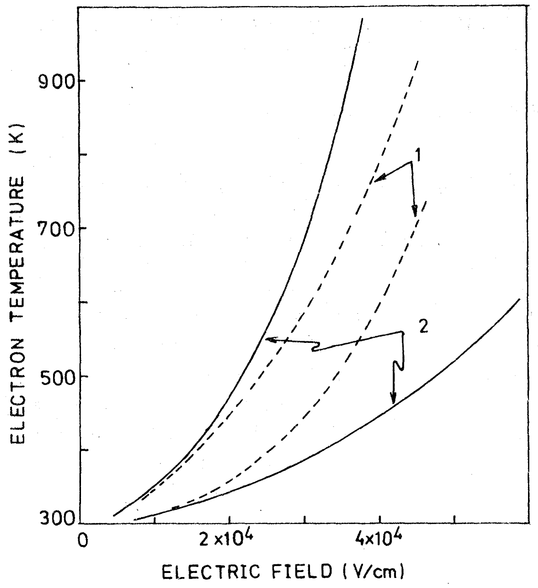

FIG. 1. Electron temperatures in the hot and cool valleys in the $E_{0'}$ subband in n-channel silicon inversion layer with the field along the [100] direction. Curves labeled 1 and 2 are the results calculated with methods 1 and 2, respectively, at 300 K.

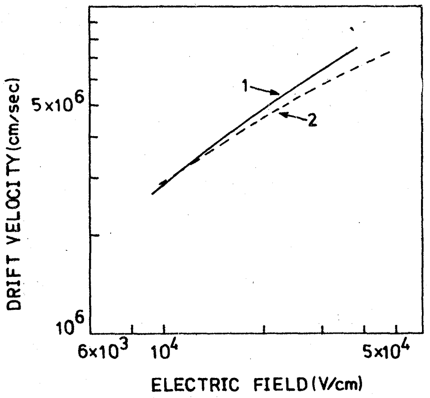

FIG. 2. Drift velocity of electrons in the $E_{0'}$ subband at 300 K for field along the [100] direction. Solid and dashed curves are obtained by using methods 1 and 2, respectively.

the numerical results for the electron temperature and drift velocity are quite different for the cases of equivalent and nonequivalent intervalley scattering in bulk, and the intersubband scattering in inversion layers when the above two methods are employed.

The intervalley scattering in GaAs (see Ref. 5) has been considered within the framework of method 2. However, in Ge or GaAs the mobility or electron temperature in a valley is primarily determined by intravalley processes, such as acoustical, nonpolar, or polar optical-phonon scattering. The effect of intervalley scattering is to introduce a small perturbation and to change the valley population. Therefore, in these cases use of method 2 may not lead to a serious deviation of the results from those calculated by method 1. However, in Si, in both bulk and inversion layer, intervalley scattering is a dominant process and the two methods give divergent results. Ferry[11] has recently calculated the drift velocity in a silicon inversion layer by considering E_0, E_1, and $E_{0'}$ subbands. His work is based on method 2 and gives good agreement with experimental results. In the light of the above discussion, it may be worthwhile to examine whether the agreement is still close when calculations are based on method 1.

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[1] (a) For a general review see B. R. Nag, *Theory of Electrical Transport in Semiconductors* (Pergamon, London, 1972); (b) E. M. Conwell, *High Field Transport in Semiconductors*, Solid State Phys. Suppl. 9, edited by F. Seitz, D. Turnbull, and H. Ehrenreich (Academic, New York, 1967). (c) Recent methods are described in P. J. Price, *Solid State Electron.* 21, 9 (1978).

[2] H. Fröhlich and B. V. Paranjape, Proc. Phys. Soc. London 69, 21 (1956).

[3] E. M. Conwell, see Ref. 1(b), p. 154.

[4] The sum $(1/\tau)_e + (1/\tau)_a$ is equal to the reciprocal of the momentum-relaxation time for optical or intervally scattering. In the derivation of the momentum-relaxation time the in-scattering terms become zero for the case of randomizing collisions.

[5] E. M. Conwell and M. O. Vassell, IEEE Trans. Electron Devices, 13, 22 (1966).

[6] H. Heinrich and M. Kriechbaum, J. Phys. Chem. Solids, 31, 927 (1970).

[7] D. K. Ferry, Phys. Rev. B 12, 2361 (1975).

[8] P. Das and D. K. Ferry, Solid State Electron. 19, 857 (1976).

[9] M. Sweid, K. Hess and K. Seeger, J. Phys. Chem. Solids 39, 373 (1978).

[10] K. Hess and C. T. Sah, J. Appl. Phys. 45, 1254 (1974).

[11] D. K. Ferry, Phys. Rev. B 14, 5364 (1976).

[12] K. Nakamura, Surf. Sci. 58, 48 (1976).

[13] D. K. Ferry, Solid State Commun. 22, 127 (1977).

[14] P. N. Butcher and W. Fawcett, Proc. Phys. Soc. London 86, 1205 (1965).

[15] P. Das and R. Bharat, Appl. Phys. Lett. 11, 386 (1967).

[16] W. Heinle, Phys. Rev. 178, 1319 (1969).

[17] P. K. Basu and B. R. Nag, Phys. Rev. B 1, 627 (1970).

[18] K. Blotekjaer, IEEE Trans. Electron Devices 17, 38 (1970).

[19] D. Chattopadhyay and B. R. Nag, Phys. Rev. B 6, 1399 (1972).

[20] These conditions may be realized in Ge or Si. Case (i) represents interequivalent valley scattering in n-Si with field in any direction other than [111]. Case (ii) and (iii) may be for nonequivalent intervalley scattering in n-Ge or that between light- and heavy-hole bands in p-Ge or p-Si. The condition $f_i = f_j$ is, however, difficult to realize.

[21] See *Handbook of Mathematical Functions*, edited by M. Abromowitz and I. A. Stegun (Dover, New York, 1965), Chap. XIII.

[22] The calculations were performed by including acoustical-and g-phonon scattering, the contribution of which to the energy-loss process is almost negligible.

[23] For a review see F. Stern, CRC Crit. Rev. Solid State Sci. 4, 499 (1974); G. Landwehr, *Festkörperprobleme* XV (Pergamon, New York, 1975), p. 49.

[24] F. Stern, Phys. Rev. B 5, 4891 (1972).

[25] C. Y. Wu and G. Thomas, Phys. Rev. B 9, 1724 (1974).