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Electric field depinning of charge density waves

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The pinning of charge-density waves by impurities is considered in systems that exhibit at least short-range order in three dimensions. Impurities are classified into strong and weak with quite different pinning properties. The pinning of spin-density waves is weak and the phase values at impurity sites are almost random, in agreement with a recent experiment. The electric field required to depin the charge-density wave is estimated. The coupling between a drifting charge-density wave and carriers either from a remnant Fermi surface or thermal excitation is considered. Attention is focused on umklapp scattering of carriers by phasons as a coupling mechanism at finite temperature. The conductivity in the high-electric-field depinned limit can be large. Dislocations in the charge-density-wave lattice are examined with particular emphasis on the piecewise motion of the charge-density wave through the motion of dislocations. We also discuss the generation of dislocations by the analog of Frank-Read sources. The unusual nonlinear conductivity observed in $NbSe_3$ is interpreted in terms of depinning of charge-density waves. The possibility of observing similar effects in other systems is briefly examined.

I. INTRODUCTION

One of the most fascinating aspects of charge-density waves (CDW) is the possibility of carrying a current by drifting the electron fluid and the CDW, a possibility first recognized at the outset by Frohlich.[1] It is generally accepted[2-4] that because of impurity pinning a finite-strength electric field is required to dislodge the CDW. The oscillator strength in the linear electric conductivity is shifted to a finite frequency and the resulting optically active phase mode has been observed experimentally.[5,6] However, the competition between impurity pinning and the electric-field energy has not been examined in detail. Further impetus for research in this direction has been provided by the observation of nonlinear conductivity associated[7] with CDW formation[8,9] in $NbSe_3$. It is tempting to interpret the nonlinear conductivity as evidence of depinning of CDW. In this paper we study nonlinear conductivity associated with the depinning of the CDW.

We begin by elucidating the nature of impurity pinning in CDW systems. An extension of these ideas to the spin-density-wave state enables us to explain a recent observation[10] of a distribution of phases at impurity sites in Cr. In Sec. III we estimate the characteristic electric field to dislodge the CDW as a whole. In Sec. IV we consider the coupling of the drifting CDW and the free Fermi surface or thermally excited carriers. Some of these questions have been studied recently by Boriack and Overhauser.[4,11] We emphasize umklapp scattering by phasons as an effective coupling between the two systems, and write the phenomenological equation for the drifting of the coupled system. In Sec. V we consider the situation below the characteristic field and study the possibility of moving one part of the CDW relative to the rest. This naturally leads to the study of dislocations in the CDW lattice. We consider various mechanisms for the generation of such dislocations, especially the analogs of Frank-Read sources.[12] In Sec. VI we discuss the experimental observations on $NbSe_3$ in the light of our results.

There has been a considerable amount of work on nonlinear excitations in one-dimensional systems pinned by a periodic potential. These excitations are solitons and can be excited thermally[13,14] or created by quantum-mechanical tunneling in a large electric field.[15] However the soliton conduction mechanism does not permit one to get around the impurity-pinning problem.[16] As we shall see, dislocations in the CDW lattice may be thought of as generalization of the nonlinear excitation to three dimensions. Such excitations cannot be thermally generated in the three-dimensional system that we discuss in this paper and we consider the most likely source to be extrinsic, e.g., Frank-Read sources.

For the purpose of this paper it suffices to treat the CDW phenomenologically. The charge density is given by

$$\rho(\vec{r}) = \bar{\rho} + \rho_1 |\psi| \cos[Qz + \phi(\vec{r})], \tag{1.1}$$

where we have considered a single $\vec{Q}$ state in three-dimensional space and $\psi(= |\psi| e^{i\phi})$ is the CDW order parameter normalized to unity at $T = 0$. The phase variable $\phi(\vec{r})$ denotes the location of the CDW relative to the lab frame. Phenomenologically we think of the CDW as a charged lattice (e.g., a Wigner lattice). Therefore when it moves it carries a current

$$J=\rho_c\bar{\rho}e\dot{\phi}/Q\,. \tag{1.2}$$

The collective density ρ_c equals unity at $T=0$ for a Peierls insulator and not the amplitude of the charge modulation. It is progressively reduced for higher temperature in the manner discussed in Sec. IV. As the CDW moves the lattice distortion must move with it. As a result we can associate an effective mass m^* with the CDW.[1,2] The temperature dependence of the effective mass as well as ρ_c is calculated microscopically in a companion paper.[17] An electric field $\mathcal{E}$ in the z direction couples to the phase via the following additional term in the Hamiltonian

$$H'=\int d\vec{r}\,\frac{e\rho_{eff}\bar{\rho}\mathcal{E}_z\phi}{Q}\,, \tag{1.3}$$

where ρ_{eff} is an effective density to be discussed in Sec. IV. It is to be understood that the $\mathcal{E}_z$ field couples to the free-carrier density in the normal way.

II. COUPLING TO IMPURITIES

An impurity atom located at $\vec{r}_i$ can be described by a potential $v(\vec{r}-\vec{r}_i)$ which is the difference between the potential at the impurity site and the potential of the host atom. The interaction energy is

$$\begin{aligned}H_{\text{imp}}&=\int d\vec{r}\,v(\vec{r}-\vec{r}_i)\rho(\vec{r})\\&=\rho_1|\psi|\,\mathrm{Re}\int d\vec{r}\,v(\vec{r})e^{i\vec{Q}\cdot\vec{r}}e^{i(\vec{Q}\cdot\vec{r}_i+\phi)}\\&=\rho_1|\psi|\,v(Q)\cos[\vec{Q}\cdot\vec{r}_i+\phi(\vec{r}_i)]\,.\end{aligned} \tag{2.1}$$

In a Peierls system $\rho_1/\bar{\rho}\approx\Delta_0/\lambda\epsilon_F$, where Δ_0 is the energy gap at $T=0$ and λ is the electron-phonon coupling constant. In a typical case $\rho_1/\bar{\rho}$ may be 0.1. For charged impurities, i.e., impurity from a different column of the Periodic Table, $v(Q)=[4\pi e^2/\epsilon_\infty(Q)]Q^{-2}$, where $\epsilon_\infty(Q)$ is the dielectric function which includes excitation across the Peierls gap. At a large wave vector Q such screening as well as carrier screening is small. Thus we may estimate that $\rho_1 v(Q)$ for charged impurities may be of the order of several tenths of eV. For isoelectronic impurities or impurities located away from the conducting chains [such as Br disorder in $K_2Pt(CN)_4Br_{0.3}\cdot nH_2O$ (KCP)] the impurity potential is considerably smaller.

The impurity coupling (2.1) has two consequences. First, the local phase $\phi(r_i)$ has certain preferred value; second, the linear coupling to the order parameter implied by Eq. (2.1) leads to a local enhancement of the ordered phase. This effect has been discussed by McMillan.[18] Let us consider a Ginzburg-Landau expansion for the order parameter

$$\begin{aligned}F=f_0\int d\vec{r}\,\mathcal{E}\Big(&-t|\psi|^2+\tfrac{1}{2}|\psi|^4\\&+\xi_x^2\left|\frac{\partial\psi}{\partial x}\right|^2+\xi_y^2\left|\frac{\partial\psi}{\partial y}\right|^2+\xi_z^2\left|\frac{\partial\psi}{\partial z}\right|^2\Big),\end{aligned} \tag{2.2}$$

where $t=-(T-T_c)/T_c$ and ξ_x, ξ_y, ξ_z are the coherence lengths. In all CDW's observed to date, with the notable exception of the $4Hb$ layered compounds,[19] the low-temperature state consists of CDW's with transverse ordering greater than the interchain or interlayer spacing. Such system must be considered as three dimensional as far as the Ginzburg-Landau expansion is concerned. Indeed by rescaling the length scale in the transverse directions, $x'=(\xi_z/\xi_x)x$ and $y'=(\xi_z/\xi_y)y$, Eq. (2.2) can be treated as an isotropic system:

$$\begin{aligned}F=&f_0(\xi_x\xi_y/\xi_z^2)\\&\times\int dx'\,dy'\,dz(-t|\psi|^2+\tfrac{1}{2}|\psi|^4+\xi_z^2|\nabla\psi|^2)\,.\end{aligned} \tag{2.3}$$

To this we add the pinning term due to a single impurity at the origin. From Eq. (2.1) we obtain

$$F_{\text{pin}}=-\mathrm{Re}\int d\vec{r}\,\rho_1 v|\psi|e^{i[\phi(\vec{r})-\tilde{\phi}]}\delta(\vec{r})\,. \tag{2.4}$$

Let us choose the overall phase such that the solution far away from the impurity is real. Then $\tilde{\phi}$ is the preferred phase at the pinning site. For small v we can linearize

$$\psi(\vec{r})=t^{1/2}+\psi'$$

so that ψ' obeys

$$f_0(\xi_x\xi_y/\xi_z^2)[-\xi_z^2\nabla^2\psi'+2t(\psi'+\psi'^*)]=\rho_1 v e^{-i\tilde{\phi}}\delta(\vec{r})\,. \tag{2.5}$$

The solution of this equation is

$$\mathrm{Re}\psi'=\epsilon\cos\tilde{\phi}(\xi_z/r)e^{-r/\xi(T)} \tag{2.6}$$

and

$$\mathrm{Im}\psi'=\epsilon\sin\tilde{\phi}(\xi_z/r)\,, \tag{2.7}$$

where

$$\epsilon=\rho_1 v/f_0\xi_x\xi_y\xi_z \tag{2.8}$$

and $\xi(T)=\xi_z t^{-1/2}$. The solution must be cut off at small r because the pinning potential is in reality not a δ function and more importantly because the length scale of the variation of ψ cannot be smaller

than ξ_z, i.e., $\xi_z^2|\nabla\psi|^2$ must be less than or equal to $|\psi|^2$. This latter condition implies that Eqs. (2.6) and (2.7) are valid only for $r > \xi_z$. The condition of validity of the linearized solution is that $|\psi'(0)| < t^{1/2}$ or

$$\epsilon < t^{1/2}. \tag{2.9}$$

For $\epsilon \ll t^{1/2}$ we find that the phase at the impurity site is largely determined by the phase at infinity. The pinning potential is approximately $v\rho_1\cos\tilde{\phi}$ with corrections of order $\epsilon v\rho_1$. In three dimensions the elastic energy cost increases with the size of the spatial variation about the impurity. As a result there is a minimum elastic energy that one must pay to interpolate the phase between $\tilde{\phi}$ at the origin and zero at infinity, and that minimum energy is of order $f_0 t \xi_x\xi_y\xi_z\tilde{\phi}^2$. On the other hand the energy to be gained from the impurity is $\rho_1 v t^{1/2}$. When Eq. (2.9) is satisfied the gain in impurity energy is simply not sufficient to overcome the elastic energy and the phase assumes its value at infinity everywhere.

While an individual weak impurity is unable to pin the phase as the preferred value, a collection of these weak impurities can still pin the overall phase of the CDW.[20,21] Such pinning is described as weak pinning by Fukuyama and Lee.[20] Basically the phase varies on a scale L much greater than the impurity spacing $n_i^{-1/3}$, where n_i is the impurity concentration. It gains energy from the fluctuation in the impurity potential of the order of

$$-v\rho_1|\psi|[(\xi_x\xi_y/\xi_z^2)L^3 n_i]^{1/2}.$$

It pays an elastic energy equal to $f_0|\psi|^2\xi_x\xi_y L$. The length L can be obtained by minimizing the free energy per unit volume:

$$L^{1/2} = \frac{4}{3}\left(\frac{f_0\xi_x\xi_y\xi_z|\psi|}{v\rho_1}\right) n_i^{-1/2}(\xi_x\xi_y)^{-1/2}. \tag{2.10}$$

On the other hand, if $\epsilon > t^{1/2}$ the linear solution described earlier breaks down. The order parameter then assumes an enhanced value at the origin which will saturate at some value which is relatively independent of t. At the same time the phase at the origin will be pinned at $\tilde{\phi}$ and interpolates smoothly to the value at infinity. The pinning behavior of these strong impurities is quite different from the weak impurities.

Let us now make some estimates on the criterion given by Eq. (2.9). We will consider a quasi-one-dimensional problem. In this case $f_0 = \Delta_0^2/\epsilon_F\Omega$, where $\Omega = a_x a_y a_z$ is the volume of the unit cell. For a quasi-one-dimensional problem, if $\xi_x < a_x$, $\xi_y < a_y$, $f_0\xi_x\xi_y\xi_z$ should really be replaced by $f_0 a_x a_y \xi_z$. Using the relation $\xi_z/a_z \approx \epsilon_F/\Delta_0$, we obtain $f_0 a_x a_y \xi_0 \approx \Delta_0$ and the criterion (2.9) becomes simply $v\rho_1 > \Delta(T)$. Since $v\rho_1$ is estimated to be tenths of eV, for most systems we see that charged impurities will qualify as strong impurities whereas isoelectronic impurities will generally be weak except very near T_c.

The situation is quite different for spin-density waves (SDW) such as in chromium. In this case a charged impurity will not couple directly to the SDW but only to the second-order harmonic CDW that coexists with the SDW. Clearly this will lead to a much smaller coupling and to values which will be in the weak pinning regime. Furthermore the coupling will be of the form $\cos 2(\phi - \tilde{\phi})$ for the CDW harmonic and this is analogous to the random-anisotropy problem,[22] rather than the random-field problem[20] discussed above. For both these reasons, it is to be expected that the SDW will not be strongly pinned to individual impurity sites even for dilute impurities. If so, the phase will not attain its preferred value $\tilde{\phi}$ at individual sites. Instead, the values of the phase of the SDW will be random at individual impurity sites. This explains the rather unexpected observation of Teisseron *et al.*[10] that in the SDW phase of Cr doped with Ta to one part in 10^3, the spin density at individual Ta sites was random and did not take a unique value. This is exactly what happens in weak pinning where the individual impurities do not maximize (or minimize) the local density as in strong pinning, but rather the phase only pins to large-scale fluctuations in the impurity density.

III. ESTIMATES OF DEPINNING FIELD

Let us consider a large volume of CDW and ask what is the electric field required to dislodge the entire volume from the impurity pinning and move it bodily. An upper bound can be obtained by keeping the CDW rigid and simply comparing the total electric field energy with the pinning energy per unit volume. We have to treat the strong and weak impurities separately. From the arguments leading to Eq. (2.10) we see that for weak impurities the pinning energy per unit volume is given by

$$f_{\rm pin} \approx -f_0|\psi|^2\xi_x\xi_y L/(L^3\xi_x\xi_y/\xi_z^2)$$
$$\approx |\psi|^2 f_0(\xi_z\xi_x\xi_y n_i)^2(\epsilon/|\psi|)^4. \tag{3.1}$$

The last factor $\epsilon/|\psi|$ is by definition less than unity. The quantity $\xi_z\xi_x\xi_y n_i$ is extremely small. For a quasi-one-dimensional system, taking $\xi_x = a_x$, $\xi_y = a_y$, and $\xi_z \approx 100\,a_z$, and an impurity concentration of one part in 10^5, $\xi_z\xi_x\xi_y n_i \approx 10^{-3}$. On the other hand, from Eq. (1.3) the electric-field energy per unit volume when the phase is advanced by 2π is given by

$$f(\mathcal{E}_z) = e\bar{\rho}\rho_{\rm eff}\mathcal{E}_z(2\pi/Q). \tag{3.2}$$

Equating Eqs. (3.1) and (3.2), we obtain at low temperature ($\rho_{eff} \approx 1$)

$$e\mathcal{E}_z(2\pi/Q) \approx (\Delta_0^2/\epsilon_F)10^{-6}\epsilon^4 . \tag{3.3}$$

Taking $\Delta_0^2/\epsilon_F \approx 0.1\ \Delta_0 \approx 10^{-3}$ eV and $\epsilon \approx 1$, this translates into a field $\approx 10^{-9}$ eV on the atomic scale, or 10^{-2} eV/cm. Based on these estimates we conclude that weak impurities can be depinned by extremely weak fields. The reason is that in three dimensions the domain size L is extremely large and a weak electric field provides an energy proportional to the large volume. The same reasoning leads us to believe that thermal depinning is unlikely in three-dimensional systems. As the temperature is raised the CDW remains pinned up to the CDW onset. There is no separate transition analogous to the spin-glass transition above which the local value of the phase becomes random.

The strong impurity has quite different pinning properties. In the presence of an electric field the phase ϕ_∞ far away from the impurity site will increase while the phase at the impurity site remains pinned at $\tilde{\phi}$. This will cost elastic energy of the order of $|\psi(0)|^2 f_0 \xi_x \xi_y \xi_z (\phi_\infty - \tilde{\phi})^2$. However the energy of the lowest-energy state must be periodic in $\phi_\infty - \tilde{\phi}$. As ϕ_∞ continues to increase, the solution near the impurity site is metastable and eventually jump to the stable solution, providing phase slippage of 2π between the pinned phase and ϕ_∞. Such phase slippage may proceed by tunneling, thermal activation over barrier, or directly for a sufficiently large electric field. As an estimate for the depinning field we may balance the elastic energy per impurity with the electric field energy, setting

$$f_0 \xi_x \xi_y \xi_z n_i = e\mathcal{E}\rho_{eff}\bar{\rho}(2\pi/Q) . \tag{3.4}$$

In a quasi-one-dimensional problem for n_i of the order of one part in 10^5, we obtain at low temperature $e\mathcal{E}_z(2\pi/Q) \approx \Delta_0 n_i \approx 10^{-7}$ eV or 1 eV/cm which is much larger than the depinning field for weak impurities.

Near T_c, as we remarked earlier, a local $|\psi|$ is induced at the impurity site. The elastic energy required for phase slippage is presumably a complicated function of Δ which is somewhere between a function linear in Δ (if we ignore the local enhancement of $|\psi|$) and a function independent of Δ. As we shall see in Sec. IV ρ_{eff} is linear in Δ near T_c. Thus the depinning field $\mathcal{E}_z$ should diverge as $\Delta^{-\eta}$ where $0 < \eta < 1$ as $T \to T_c$.

IV. INTERACTION BETWEEN THE DRIFTING CDW AND NORMAL CARRIERS

In the presence of a remnant Fermi surface, or of thermally excited carriers across the gap, it is necessary to clarify the relation between the drifting CDW and the quasiparticle contribution to the current. Let us go to a frame moving with the drift velocity $\vec{D}$ of the CDW so that the CDW is stationary (referred to as the CDW frame below). In this frame the single-particle energy $E(\vec{k})$ is the standard one, i.e.,

$$E(\vec{k}) = \zeta_{\vec{k}} \pm E_0(k) ,$$

where $\xi_k = \frac{1}{2}(\epsilon_{\vec{k}+\vec{Q}} - \epsilon_{\vec{k}})$ and $\zeta_k = \frac{1}{2}(\epsilon_{\vec{k}+\vec{Q}} + \epsilon_{\vec{k}})$ and $E_0^2(k) = \xi_k^2 + \Delta^2$. The occupation of the states will be determined by the balance between the external field on the one hand, and the relaxation to the lab frame on the other, as well as possible relaxation to the moving CDW frame. We follow Boriack and Overhauser[11] and make the simplifying approximation that the occupation (in the extended-zone scheme) is given by

$$f_{\vec{k}} = f(E_{\vec{k}}) - m(\vec{K} - \vec{D}) \frac{\partial E}{\partial \vec{k}} \frac{\partial f}{\partial E} , \tag{4.1}$$

where $\vec{K}$ and $\vec{D}$ are in the z direction and m is the band mass. This can be viewed as an expansion of $f(E(\vec{k} - m(\vec{K} - \vec{D}))$, i.e., the single-particle distribution is centered at $\vec{K} - \vec{D}$ in the CDW frame. Since k transforms like a momentum under a Galilean transformation we see that $\sum_k k_z f_k = m(\vec{K} - \vec{D})$ in the CDW frame and is equal to $\vec{K}$ in the lab frame. Equation (4.1) is the usual approximation for the Boltzmann equation and leads to Matthiessen's rule, i.e., the additivity of various scattering processes.

The average velocity carried by such a state is given by (in the following we suppress the N^{-1} factor in front of $\sum_k$)

$$\vec{V} = \sum_k \frac{\partial E}{\partial k_z} f_k = -m(\vec{K} - \vec{D}) \sum_k \left(\frac{\partial E}{\partial k_z}\right)^2 \frac{\partial f}{\partial E} \tag{4.2}$$

in the CDW frame. In the lab frame we have

$$\vec{V} = \vec{D}\rho_c + \vec{K}\rho_n , \tag{4.3}$$

where

$$\rho_n = -m \sum_k \left(\frac{\partial E}{\partial k_z}\right)^2 \frac{\partial f}{\partial E} \tag{4.4}$$

and $\rho_c = 1 - \rho_n$. This expression for ρ_n/m is the usual one that determines the plasma frequency of carriers in semiconductors. This must be the case because if $D = 0$, i.e., if the CDW is pinned, this problem becomes identical to the usual semiconductor or semimetal.

It is instructive to look at Eq. (4.4) in a different way. Suppose there is no scattering. In the presence of an external electric field $\mathcal{E}$ in the z direction the crystal momentum is accelerated by $m\dot{\vec{K}} = e\vec{\mathcal{E}}$. Therefore the gain in momentum by the single particle after a time t is given by

$$m\sum_{\vec{k}}\frac{\partial E}{\partial \vec{k}}f(\vec{k}-e\vec{\mathcal{E}}t)=-me\vec{\mathcal{E}}t\sum_{\vec{k}}\left(\frac{\partial E}{\partial \vec{k}}\right)^2\frac{\partial f}{\partial E}$$

$$=e\vec{\mathcal{E}}t\rho_n .$$

The remainder of the momentum density fed into the system is then $e\vec{\mathcal{E}}t\rho(1-\rho_n)$. This remainder must have gone into accelerating the CDW as a whole. Therefore the force field on the CDW is given by $(1-\rho_n)e\mathcal{E}$. It is natural to interpret $e\rho_c = e(1-\rho_n)$ as the fractional charge density associated with the condensate. To study the behavior of ρ_c near T_c it is convenient to use the relation

$$\frac{1}{mv_F^2}+\sum_{\vec{k},\sigma}\frac{\xi^2}{E_0^2(k)}\frac{\partial f}{\partial E}$$

$$=\sum_{\vec{k},\sigma}\frac{\Delta^2}{E_0^3(k)}\left[1-f(-\zeta+E_0)-f(\zeta+E_0)\right].$$

For $\Delta \ll T$ we can expand

$$1-f(-\zeta+E)-f(\zeta+E)\approx \tfrac{1}{2}\beta E_0$$

for regions in k space where $\zeta \ll T$. The remaining region gives negligible contribution and we find that ρ_c is linear in Δ near T_c.

Our expression for ρ_c is in agreement with Boriack and Overhauser's γ in the proper limit but it is in disagreement with Allender, Bray, and Bardeen[23] who obtained an answer analogous to the superfluid density in He and in which $\rho_c \sim \Delta^2$. The difficulty with their argument is that they work in the lab frame with a time-dependent CDW potential. They obtain an eigenvalue spectrum $\lambda'(\vec{E}) = E(\vec{k}-m\vec{D})+\vec{D}\cdot\vec{k}$, which does not have the proper Galilean transformation properties of an energy. As Boriack and Overhauser[24] pointed out, the energy as defined by the mean value of $i\partial/\partial t$ is not λ' but $E(\vec{k}-m\vec{D})+m\vec{D}\cdot\partial E/\partial\vec{k}$. Once this is properly taken into account, their result can be brought into agreement with Eq. (4.4).

In the presence of scattering there is an additional force on the condensate arising from the relative motion of the single particle and the CDW. Boriack and Overhauser[11] have considered the problem at low temperatures when impurity scattering is the dominant mechanism. At higher temperatures when the conductivity is temperature dependent we have to consider additional scattering mechanisms. This leads us to consider umklapp scattering of the single particles by phasons of the CDW. We assume that the phasons are in thermal equilibrium in the CDW frame. This is a good approximation for $kT \ll \omega_0$ (where ω_0 is the bare phonon frequency at Q) so that the mixing between the phason and the ordinary phonons is not strong. This problem is quite similar to ordinary umklapp scattering in polyvalent metals where Lawrence and Wilkins[25] have shown that umklapp scattering is dominant. The time rate of change in momentum in the single-particle system (which must go into accelerating the CDW) is given by

$$\frac{\Delta p}{\Delta t}=\sum_{k,k'}m(v_{k'}-v_k)W_{k,k'}\{\delta(E_k-E_{k'}-\omega_q)[f_k(1-f_{k'})(1+n_q)-(1-f_k)f_{k'}n_q)]$$

$$+(E_k-E_{k'}+\omega_q)[f_k(1-f_{k'})n_q-(1-f_k)f_{k'}(1+n_q)]\}, \tag{4.5}$$

where it is understood that $\vec{k}'=\vec{k}+\vec{q}+\vec{G}$ and $\vec{G}$ is a reciprocal-lattice vector associated with the CDW. The transition probability is given by

$$W_{k,k'}=(1-m/m^*)(\omega_0/\omega_q)M^2 \tag{4.6}$$

and

$$M=gv_F q\Delta/E_0^2(k). \tag{4.7}$$

In Eq. (4.6) the scattering rate $W_{k,k+q}$ contains the spectral weight[2] of the phase mode $(1-m/m^*)$; ω_0 and ω_q are the bare phonon frequency and the phase mode frequency, respectively, and M is the matrix element between the phonon and the quasiparticle. It can easily be worked out in terms of the electron-phonon coupling constant g via the Bogoliubov operator and is given by Eq. (4.7).

Using the ansatz Eq. (4.1) in Eq. (4.5) we obtain the time rate of change in momentum per unit volume

$$\Delta p/\Delta t=(\bar{\rho}\rho_n/\tau)m(K-D), \tag{4.8}$$

where

$$\frac{\rho_n}{\tau}=\sum_{kk'}(v_k-v_{k'})^2W_{kk'}$$

$$\times\left(\delta(E_k-E_{k'}-\omega_q)\frac{\partial f}{\partial E}(1+n_q-f_{k'})\right.$$

$$\left.+\delta(E_k-E_{k'}+\omega_q)\frac{\partial f}{\partial E'}(1+n_q-f_k)\right). \tag{4.9}$$

The order of magnitude of $1/\tau$ is estimated in the

Appendix. Since the phason is defined only for $q < \xi_0^{-1}$ it is clear that k must be restricted to be within ξ_0^{-1} of k_F as well. In a quasi-one-dimensional situation where part of the Fermi surface remains at low temperature, only the part of the Fermi surface that is within Δ of the gap will contribute to τ^{-1}. In this case τ^{-1} is estimated to be $\rho_n\tau^{-1} \sim \lambda(T^2/\omega_0)F$, where $\lambda = g^2/\omega_0\epsilon_F$ is the dimensionless electron-phonon coupling constant and F is a geometric factor related to the fraction of the Fermi surface within Δ of the energy gap. Near T_c, $\rho_n\tau^{-1}$ goes to zero linearly with Δ.

The total force per unit volume accelerating the CDW is then given by

$$\vec{F}_D = e\bar{\rho}\rho_c\vec{\mathcal{E}} + \bar{\rho}\rho_n\tau^{-1}m(\vec{K} - \vec{D}) . \tag{4.10}$$

Since total momentum is conserved in the umklapp scattering, a similar term must appear in the acceleration of the quasiparticle,

$$\dot{\vec{K}} = -\tau_k^{-1}\vec{K} - \tau^{-1}(\vec{K} - \vec{D}) + e\vec{\mathcal{E}}/m , \tag{4.11}$$

where we have included a term which relaxes the momentum to the lab. The source of this relaxation may be impurity scattering or scattering by the ordinary phonons which are in equilibrium in the lab frame.

We can now study two limiting cases: (i) the CDW is pinned, so that $\vec{D} = 0$, and (ii) the high-field limit in which the CDW is depinned and the conductivity is again linear. In the first case we set $D = 0$ and solve Eq. (4.11) for $\vec{K}$. Inserting the result in Eq. (4.10), we obtain

$$\vec{F}_D = e\bar{\rho}\rho_{eff}\vec{\mathcal{E}} , \tag{4.12}$$

where $\rho_{eff} = \rho_c + \rho_n/(1 + \tau/\tau_k)$. Thus we see the coupling of the CDW to the electric field depends on the ratio τ/τ_k. Near T_c τ^{-1} approaches zero and $\vec{F}_D \approx e\rho_c\vec{\mathcal{E}}$, where ρ_c itself is linear in Δ. At low temperature, τ_k and τ may be comparable. If τ^{-1} dominates, F_D approaches $e\vec{\mathcal{E}}$. At very low temperature, τ^{-1} vanishes like T^2 and $\rho_{eff} \approx \rho_c$.

In the opposite limit we assume that the CDW is depinned. Then we can write the following equation of motion for the change in the collective contribution to the momentum $m^*\rho_c\vec{D}$:

$$\dot{\vec{D}} = -\frac{1}{\tau_D}\vec{D} - \frac{1}{\tau}\frac{\rho_n}{\rho_c}\frac{m}{m^*}(\vec{D} - \vec{K}) + \frac{e\vec{\mathcal{E}}}{m^*} , \tag{4.13}$$

where we have added a phenomenological decay time τ_D for the damping of the CDW to the lab frame. The source of damping may be mixing of the phason with the ordinary phonons which have a finite lifetime,[2] or it may be radiation of phasons at impurity sites, or it may be the kind of impurity damping discussed by Boriack and Overhauser.[11] Equations (4.11) and (4.13) can be solved to obtain

$$\frac{D - K}{K} = \left(\frac{1}{\tau_k}\frac{1}{m^*} - \frac{1}{\tau_D}\frac{1}{m}\right)\Big/\left[\frac{1}{m}\left(\frac{1}{\tau^*} + \frac{1}{\tau_D}\right) + \frac{1}{m^*}\frac{1}{\tau}\right] , \tag{4.14}$$

$$\vec{K} = \frac{e\vec{\mathcal{E}}}{m}\left(\frac{1}{\tau^*} + \frac{1}{\tau_D} + \frac{m}{m^*}\frac{1}{\tau}\right)\Big/\left[\frac{1}{\tau_k}\left(\frac{1}{\tau^*} + \frac{1}{\tau_D}\right) + \frac{1}{\tau_D}\frac{1}{\tau}\right] , \tag{4.15}$$

where $\tau^{*-1} = \tau^{-1}(\rho_n m/\rho_c m^*)$. The conductivity can be readily obtained by combining these equations with Eq. (4.3). It is interesting to point out here that the qualitative nature of the solution depends on whether the phason scattering is the dominant process. If $\tau_k^{-1} \gg \tau^{-1}$, i.e., the relaxation to the lab is the dominant process, e.g., when the conductivity is impurity dominated,[11] the conductivity is dominated by normal carriers if $\tau_k^{-1} \ll (m^*/m)\tau_D^{-1}$ and by the drifting CDW otherwise. A more interesting situation obtains if $\tau_k^{-1} \ll \tau^{-1}$ and $\tau_D^{-1} \ll \tau^{*-1}$. In this case we have $|\vec{D} - \vec{K}|/K \ll 1$ and

$$\vec{K} \approx \frac{e\vec{\mathcal{E}}\tau_k}{m}\left(\frac{1 + \rho_c/\rho_n}{1 + \tau_k\rho_c m^*/\tau_D\rho_n m}\right) . \tag{4.16}$$

The CDW and the normal carriers are drifting at similar rates. In particular the expression in parentheses in Eq. (4.16) may be of order unity leading to a total conductivity of the order of the normal conductivity in the absence of the CDW.

V. DISLOCATION IN THE CHARGE-DENSITY-WAVE LATTICE

Suppose the electric field is smaller than that required to move the CDW as a whole as discussed in Sec. IV. It may still be possible to move part of the CDW relative to the rest. This will require the presence of dislocations in the CDW lattice.

We shall restrict ourselves to the single-$\vec{Q}$ CDW state. The Burgers vector can only be parallel to the $\vec{Q}$ vector, i.e., in the z direction. Suppose we ignore amplitude fluctuations and consider a region in space far away from strong impurities. The free energy reduces to

$$F = f_0\xi_x\xi_y|\psi|^2\int dx'\,dy'\,dz\,(\nabla\phi)^2 . \tag{5.1}$$

For a straight line dislocation the solution is simply $\phi = \theta$, where θ is the angle in the plane normal to the dislocation. Then energy per unit length is

$$T = f_0\xi_x\xi_y|\psi|^2\ln(R/\xi_0) , \tag{5.2}$$

where R is a large-distance cutoff typically equal to the distance to the nearest dislocation with the opposite Burgers vector. It is instructive to examine two examples of dislocations. The first is a dislocation loop lying in the x-y plane. Since

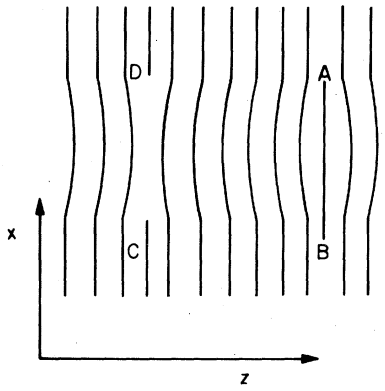

FIG. 1. Projection of two dislocation loops in the x-z plane. Solid lines are the contour of constant phase in units of 2π.

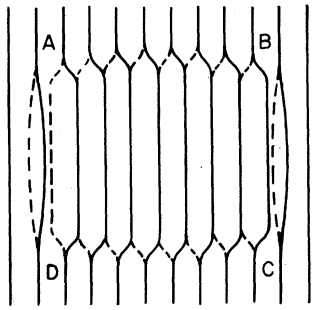

FIG. 2. Solid and dashed lines are the contours of constant phase for the layer above and below the plane in which the dislocation loop *ABCD* lies. The segments *AB* and *CD* are screw dislocations whereas *BC* and *AD* are edge dislocations.

the Burgers vector is in the z direction this is a purely edge-type dislocation. Its projection in the x-z plane is shown in Fig. 1. The line AB is the projection of a disk of an extra wavelength of the CDW. If we consider two loops with opposite Burgers vectors as shown in Fig. 1, clearly the phase within the volume bound by the two dislocation loops (the area $ABCD$ in the projection) has slipped by 2π relative to the outside. An electric field will tend to pull the two loops apart in the z direction. Suppose the radius of the loop is R (in the scaled isotropic space) and the distance between the loops is z. The gain in energy from the electric field equals

$$\rho_{\text{eff}}\bar{\rho}\mathcal{E}_z\pi R^2 z(2\pi/Q)(\xi_x\xi_y/\xi_z^2)\,.$$

This is opposed by the elastic energy which causes an attraction between the two loops. This is analogous to the force between two parallel dislocations in a crystal. It can be evaluated in a straightforward way similar to the calculation in the limit when the crystal is replaced by an elastic continuum.[26] The attractive energy has a logarithmic dependence on the distance z, as in the solid, and takes the form

$$f_0\xi_x\xi_y 2\pi R|\psi|^2\ln(z/\xi_0)\,.$$

Minimizing the total energy we find that it has a maximum at a value z_0,

$$z_0=f_0\xi_z^2|\psi|^2/\rho_{\text{eff}}\bar{\rho}\mathcal{E}_z(2\pi/Q)R\,. \tag{5.3}$$

For values of $z>z_0$ the electric field energy dominates and the disks will run away from each other whereas for $z<z_0$ the disk will collapse and annihilate.

A second example is a rectangular dislocation loop in the x-z plane such as $ABCD$ shown in Fig. 2. The segments AB and CD parallel to z are pure screw dislocations. If we consider two loops spaced by y_0 in the y directions with opposite Burgers vectors, it is easy to see that the volume bound by the two loops has slipped by 2π relative to the rest of the CDW. It is particularly interesting to consider a layered system such that the CDW is weakly coupled in the y direction, i.e., $\xi_y<a_y$. In that case we can imagine the dislocation loop to be between layers. Then the dislocation loop describes the slippage of some layers relative to the bulk. The dislocation loop plays the role of a domain wall. Insofar as the stationary layer provides a periodic pinning potential to the slipped layer, the dislocation picture can be considered as a three-dimensional generalization of the soliton (or domain-wall) idea discussed by Rice *et al.*[13] in one dimension. Again, it is easy to show that for a sufficiently strong field the dislocation loop will expand in both the x and the y directions, thereby causing phase slippage and carrying a current.

However, examination of Eq. (5.3) shows that for electric fields too weak to depin the CDW as a whole, z_0 and R have to be enormous before the dislocations will grow and run away. Such large-scale objects preclude the possibility of thermally nucleating dislocation loops. In a real system there may be dislocation loops that were created in the process of condensation of the CDW. Generally, however, these dislocations can be used only once and are eliminated as they run into the surface of the sample or grain boundaries. This same problem arises in considering the shear in ordinary crystals. An ingenious proposal for dislocation sources was proposed by Frank and Read[12] and should be applicable to the CDW lattice as well. Let us consider a rectangular dislocation loop with sides x_0' and y_0' located in the x'-y' plane as shown in Fig. 3(a). We assume that the corners of the loop are pinned. A pinning mechanism may be regions in space where the CDW has reduced

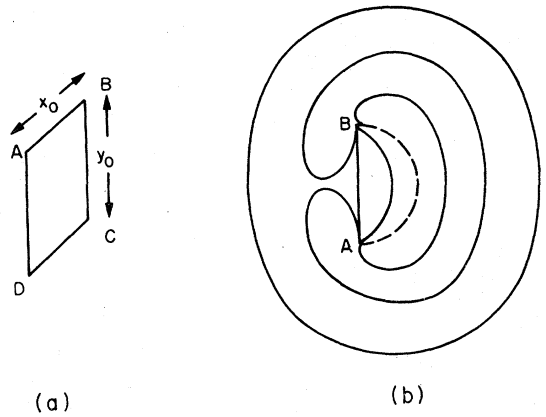

FIG. 3. (a) Dislocation loop in the x-y plane, segments of which can act as a Frank-Read source. (b) Shows the operation of a Frank-Read source. The dashed line is the semicircle that separates stable and unstable solutions.

amplitude which will attract the core of the dislocation line. Let us focus our attention on the segment AB lying along the x direction. Upon application of an electric field the segment will bow out as shown in Fig. 3(b). The balance is between the energy gained from the electric field and the elastic energy which is proportional to the length of the arc. The problem is analogous to that in crystals where the driving force is an applied stress and as in that case,[12,26] the boundary of the region of stable bowing of the segment AB occurs when the segment is a semicircle and the radius of the circle is just equal to one-half the length of the segment x_0', separating the stable and unstable solutions. The electric field required to produce an unstable solution is found[12,26] by balancing the energy gained from the field

$$\rho_{eff}\bar{\rho}\,\mathcal{E}_z\left(\frac{2\pi}{Q}\right)\pi\left(\frac{x_0'}{2}\right)^2 y_0'\left(\frac{\xi_x\xi_y}{\xi_z^2}\right)$$

and the dislocation energy

$$f_0\xi_x\xi_y|\psi|^2\pi\,(\tfrac{1}{2}x_0')\ln(x_0'/\xi_z)\,.$$

In terms of the length in real space $x_0=(\xi_x/\xi_z)x_0'$ and $y_0=(\xi_y/\xi_z)y_0'$, and using the quasi-one-dimensional expression for f_0 and ξ_z, we obtain the characteristic field

$$e\mathcal{E}_z\left(\frac{2\pi}{Q}\right)\approx\frac{f_0|\psi|^2}{\rho_{eff}\bar{\rho}}\,\frac{\xi_x\xi_y}{x_0y_0}\ln\left(\frac{x_0\xi_x}{\xi_z}\right). \tag{5.4}$$

For a quasi-one-dimensional system $f_0\approx\Delta_0^2/\epsilon_F\Omega$, ξ_x, ξ_y may be considerably less than a_x, a_y and for $x_0/a_x, y_0/a_y\approx 100$, the characteristic electric field may be quite small, ~1 eV/cm. For fields exceeding Eq. (5.4) the bowing out continues in the manner shown in Fig. 3(b), and the segment can operate as a source of dislocation loops expanding outward. At the same time the segment DC on the opposite side of the rectangle operates as a Frank-Read source in an identical manner. The two expanding loops then define an expanding domain of slipped phase.

As the dislocation loop expands it may encounter a strong impurity. The dislocation can get around the impurity in several ways. If the dislocation is screwlike it can cross-slip to a different glide plane. For dislocations that are edgelike it will have to climb (motion normal to the Burgers vector). In ordinary crystal, climbing requires migration of vacancies or interstitials. In a CDW, a climb can be accomplished by converting collective density to normal carriers. Such a conversion will take place when an edge dislocation hits a wall or grain boundary or when it is stopped by a strong impurity that it is unable to get around. The conversion of a normal electron to a collective density at one end of the sample and its transport by dislocation motion some distance down the sample constitutes a parallel channel of transport and the conductivity should be additive. The magnitude of the conductivity that this mechanism can provide is very complicated even to estimate, and Eq. (5.4) is to be understood as the minimum electric field required for the operation of a source of a particular dimension and hence the opening up of a new channel for conduction. As the electric field is increased, more and more Frank-Read sources operate, until a field is reached when the CDW is depinned and drifts as a whole, we then arrive at the situation described in Sec. III.

We should remark on an additional restriction for the Frank-Read source to operate, namely, that there must be no strong impurities within a radius of $\frac{1}{2}x_0'$ of the source. In the z direction this means a distance in real space of $z_0=(\xi_z/\xi_x)x_0$. This is the reason we restricted our attention to the segments AB and DC of the initial loop that lies along the x direction. In a layered system $\xi_y<\xi_x$ and hence there is a more severe restriction on the segments BC and AD to operate as Frank-Read sources.

VI. DISCUSSION

We would like to discuss the experimental results on $NbSe_3$ in light of the above theoretical considerations. It is found[7] that the nonlinear conductivity can be very well fitted by the form

$$\sigma(\mathcal{E})=\sigma_0+\sigma_1 e^{-\mathcal{E}_0/\mathcal{E}}. \tag{6.1}$$

Furthermore $\mathcal{E}_0$ is temperature dependent, behaving like $t^{-1/2}$ just below each of the two transition temperatures.[2] The minimum $\mathcal{E}_0$ is of the order of

1 eV/cm for the 144-K transition and 0.1 eV/cm for the 59-K transition. Below about 25 K, $\mathcal{E}_0$ increases abruptly and due to heating effects a voltage greater than 0.5 eV/cm cannot be applied. X-ray diffraction has been performed[9] with a current running through the sample, and no change in the superlattice period or intensity was observed. The extremely small electric field (10^{-8} eV on an atomic scale) indicates that we must be dealing with a phenomenon on a large scale. The natural explanation is that the CDW is being depinned by the electric field. Since the x ray is an instantaneous snapshot, it will not detect the small drift velocity of the CDW. The observation[7] that conductivity at microwave frequency approximates $\sigma_0+\sigma_1$ further confirms this view, as ac conductivity is not expected to be so strongly affected by the pinning. In a strictly one-dimensional problem the frequency-dependent conductivity is shown to be of the form $\sigma(\omega)\sim\exp(-\omega_0/\omega)$. In three dimensions the problem is more complicated and a detailed experimental determination of $\sigma(\omega)$ would be very interesting. Our study shows that the impurities can be divided into weak and strong ones and estimates made in Sec. III indicate that the weak impurities are easily depinned. The depinning field for the strong impurities was estimated to be of the order of 1 eV/cm, not too different from the measured value. (The higher temperature transition is expected to require a larger depinning field because Δ_0 is larger and ρ_{eff} is smaller, since a large portion of the Fermi surface survives the first transition.) However, we would expect a more sudden onset of extra conductivity, and there is no obvious way to understand the $\exp(-\mathcal{E}_0/\mathcal{E})$ dependence. One possibility may be that the CDW is broken up into grains (which may or may not be associated with grain boundaries of the real lattice). Each grain has its own depinning field, and an averaging over a distribution of depinning fields results in a more gradual nonlinear behavior, as discussed at the end of Sec. III.

In the dislocation model it is more natural to expect a distribution in the size x_0y_0 of the Frank-Read sources. A Poisson distribution of the size (resulting from a random distribution of dislocation pinning sites, for instance) can nicely account for the $\exp(-\mathcal{E}_0/\mathcal{E})$ behavior since x_0y_0 goes like $\mathcal{E}^{-1}$ according to Eq. (5.4). For a fixed density of strong impurities, Eq. (5.4) predicts an $\mathcal{E}_0$ that goes like $|\psi|$. However, from Eq. (2.9) we see that more and more impurities become strong impurities near T_c. The increase in strong impurity concentration will render some Frank-Read sources inoperative because the expanding dislocation line may run into the impurity before it becomes unstable. This will have the effect of increasing $\mathcal{E}_0$ beyond a linear dependence on $|\psi|$.

Another interesting experimental observation is that the saturated conductivity $\sigma_0+\sigma_1$ approximates the conductivity one would expect if the CDW did not form. As we discussed in Sec. IV, the umklapp scattering with phasons is the dominant relaxation mechanism, in which case the normal electron and the CDW are drifting at similar velocity. In the dislocation model there is no particular reason to expect this behavior and indeed one expects σ_1 to be much less than the observed value.

The temperature dependence of $\mathcal{E}_0$ can be understood qualitatively in the depinning model. As discussed at the end of Sec. III, $\mathcal{E}_0$ is expected to diverge near T_c even though the precise exponent is not known. Experimentally a $t^{-1/2}$ divergence is reported.[27] Below the 144-K transition $\mathcal{E}_0$ is observed to rise slowly even after the order parameter has apparently saturated. This can be explained by a temperature-dependent ρ_{eff} as given by Eq. (4.12) arising from the temperature dependence of the ratio τ/τ_k.

One of the most puzzling features of the experiment is the abrupt rise in $\mathcal{E}_0$ around 25 K. An explanation we would like to speculate upon here is that around 25 K there is a lock-in between the two apparently independent CDWs. The two CDW's have wave vectors $\vec{q}_1=(0, 0.243, 0)$ and $\vec{q}_2=(0.5, 0.263, 0.5)$. The harmonics $2q_1$ and $2q_2$ are almost identical up to a reciprocal-lattice vector and indeed a weak harmonic has been observed experimentally.[28] If $2q_1=2q_2$ there will be a term in the free energy $\Delta_1^2\Delta_2^{*2}$ which will tend to lock the relative phase of the two CDW's. Our speculation is that this lock-in occurs at around 25 K below which Δ_2 will have to move against Δ_1. Since Δ_1 is pinned by a much larger electric field we expect the depinning field to be dramatically increased. This model will predict a weak anomaly in the harmonic superlattice reflection around 25 K and that a field of several eV/cm which is big enough to depin the 144-K CDW will depin the CDW below 25 K. One unsatisfactory feature of this picture is that it is difficult to explain why the lock-in occurs only at 25 K when the order parameter has more or less saturated and does not occur closer to the onset of the second CDW.

If impurity pinning plays a strong role in the conductivity of $NbSe_3$ as we have suggested, there should be a correlation between the characteristic depinning field $\mathcal{E}_0$ and the concentration of strong impurities. In the depinning model we would predict that $\mathcal{E}_0$ should increase linearly with the strong impurity concentration. Recently it has been noted that $\mathcal{E}_0$ are different for samples with different room-temperature to low-temperature re-

sistance ratios.[29] However, the resistance ratio is sensitive to both strong and weak impurity concentration. As a result, a quantitative correlation is not possible. One possible test is to dope the system with a small amount of charged impurities, such as Ti. Our picture predicts that even a small amount, of the order of one part in 10^4, will greatly reduce the nonlinear conductivity.

To summarize we have studied the possible nonlinear conductivity mechanisms for moving CDW in the presence of impurities. Those considerations should be applicable to all CDW systems, including layered compounds.[30] However, to exhibit an effect for modest electric fields requires very pure samples. So far only one system, $NbSe_3$, exhibits nonlinear conductivity. An electric field depinning of CDW is, to our knowledge, the only viable explanation for the unusual behavior in $NbSe_3$. As we see in this paper the qualitative features and the order of magnitude of the depinning fields are reasonably accounted for. It will be most interesting if the condensate can be made to move in other CDW systems. We have already seen that there is evidence that the SDW in chromium should be quite mobile; the problem is, of course, that one has to find means to couple to it. In other CDW systems, tetrathiafulvalene-tetracyanoquinodimethane (TTF-TCNQ) is not a suitable candidate, except perhaps for the narrow temperature region between 48 and 54 K,[29,31] because the CDW on oppositely charged chains provide a periodic pinning potential to each other. In KCP we expect a substantial amount of pinning due to disorder in the bromine sites, which leads to the surprisingly high pinned phase-mode frequency of ≈ 2.5 meV.[32] The depinning field is expected to be extremely large. A promising class of systems is the TTF halides and tetrathiafulvalene-thiocyanate [TTF-$(SCN)_{0.588}$].[33] However, disorder in the halides and in the SCN may also lead to a large depinning field. For the CDW in layered compounds our dislocation picture needs to be generalized to the state with three coexisting Q vectors. The basic physics is expected to remain the same and the layered compounds should be promising candidates for drifting CDW. We should mention the very interesting 4*Hb* layered compounds,[19] where the CDW are apparently uncorrelated from layer to layer. In this case dislocation pairs may be created thermally within each layer.[34]

Note added in proof. Recent developments of note are (i) the report by Ong and collaborators [N. P. Ong, Bull. APS 24, 294 (1979) and N. P. Ong, J. W. Brill, J. C. Eckert, J. W. Savage, S. K. Khanna, and R. B. Somoano (unpublished)] that ϵ_0 scales as n_i^2 in samples doped with Ta in agreement with Eqs. (3.1) and (3.2); (ii) the observation of a threshold field in relatively pure $NbSe_3$ by Fleming and Grimes [R. M. Fleming and C. C. Grimes, Bull. APS 24, 386 (1979) and unpublished] which is in accord with the simple breakaway of the CDW as a whole from the pinning centers discussed in Sec. VI; and (iii) the observation of similar nonlinear conductivity below the structural phase transition in ZrV_2 and HfV_2 by V. M. Pan, I. E. Bulakh, A. L. Kasatkin, and A. D. Shevchenko, Pis'ma Zh. Eksp. Teor. Fiz. 27, 629 (1978) [JETP Lett. 27, 594 (1978)]. If our model is to be applicable to ZrV_2, an incommensurate CDW must be present also in this material, but to date no such CDW has been reported to our knowledge.

APPENDIX

In Eq. (4.9) the first term corresponds to emission of phonons and the second to absorption of phonons upon a transition from k to k'. By a transformation $k \rightleftarrows k'$ and we can show that the two terms are equal. Hence

$$\frac{\rho_n}{\tau} = 2\sum_{kk'} (v_k - v_{k'})^2 W_{kk'} \delta(E_k - E_{k'} - \omega_q) \times \frac{\partial f}{\partial E}(1 + n_q - f_{k'}) . \tag{A1}$$

To estimate the order of magnitude of τ^{-1} we observe that $\omega_q = sq$, where $s = (m/m^*)^{1/2} v_F$; the phason velocity is much lower than v_F and we may assume that for most of the scattering that contributes to the sum, $v_{k'} \approx -v_k \approx \xi/E_0$ and that $q \approx 2k$. The k sums can be converted to integrals over E and E', and we obtain upon using Eq. (4.7)

$$\frac{\rho_n}{\tau} = \frac{2}{\epsilon_F}\sum_{k_\perp} \int dE \frac{g^2\xi^2\Delta^2}{E_0^4} \frac{\omega_0}{\omega_{2k}} \frac{\partial f}{\partial E} \times [1 + n_{2k} - f(E - \omega_{2k})] . \tag{A2}$$

Let us first examine the low-temperature limit $T \ll \Delta$. Noting that $\partial f/\partial E$ restricts E to be of order zero, we find the thermal factor $[1 + n_{2k} - f(E - \omega_{2k})]$ of order kT/ω_{2k} for $\omega_{2k} < kT$ and $\exp(-\omega_{2k}/T)$ for $\omega_{2k} \gg kT$. Further we note that phase mode q is restricted to less than ξ_0^{-1} and $\omega_q < \omega_0$. Thus if $kT < \omega_0$, only a fraction kT/ω_0 of the region in k space will contribute to the sum. Even for $\omega_0 > kT$, only regions of the Fermi surface within Δ of the gap will contribute, i.e., $E_0 \lesssim 2\Delta$. Equation (A2) is then estimated to be

$$\rho_n/\tau = (2g^2/\epsilon_F\omega_0) F T \min(1,\ kT/\omega_0) , \tag{A3}$$

where F is a geometric factor which is roughly the area of the Fermi surface that is within Δ of the energy gap

$$F = \sum_{k_\perp} \int dE \, \frac{\xi^2\Delta^2}{E_0^4} \frac{\omega_0^2}{\omega_q^2} \Theta(2\Delta - E_0) \frac{\partial f}{\partial E}$$
$$\approx \sum_{k_\perp} \int dE \, \frac{\Delta^4}{E_0^4} \Theta(2\Delta - E_0) \frac{\partial f}{\partial E} \qquad \text{(A4)}$$

where Θ is the step function. At low temperature $\rho_n/\tau \sim \lambda F T^2/\omega_0$.

Next we consider the case near T_c, when $\Delta \ll T$. For the regions of the Fermi surface where $\zeta \ll T$, $\partial f/\partial E \approx \beta E_0$ after we account for contribution for the electron and the hole pockets. Just as the estimate for ρ_c near T_c, it is clear that ρ_n/τ is linear in Δ just by scaling the integrals (A2).

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