Electron-magnon interaction and the electrical resistivity of Tb

Niels Hessel Andersen and Henrik Smith
Physics Laboratory I, H. C. Ørsted Institute, University of Copenhagen, Copenhagen, Denmark
(Received 13 February 1978)

Measured and calculated values of the electrical resistivity of ferromagnetic Tb in the temperature range 1.5–50 K are presented. The theoretical electron-magnon resistivity and the magnon-electron lifetime are expressed in terms of the high-temperature spin-disorder resistivity. When the contribution of electron-phonon scattering is included, the calculated resistivity is shown to agree closely with the measured values.

The interaction between magnons and conduction electrons in a ferromagnetic metal contributes to its electrical resistivity along with electron-phonon and electron-impurity scattering. The magnon-electron interaction also manifests itself as a source of broadening of the magnon states. At temperatures below the magnon energy gap Δ the magnetic resistivity contribution ρ_m falls off exponentially with decreasing temperature, whereas the electron-limited magnon lifetime is independent of temperature. In this paper we present theoretical and experimental results for the electrical resistivity of ferromagnetic Tb in the temperature region from 1.5 to 50 K ($\Delta \sim 19$ K for Tb). We show how the magnon contribution to the resistivity as well as the electron-limited magnon lifetime may be related to the spin-disorder resistivity of the high-temperature paramagnetic phase. These relations eliminate some of the uncertainty about the magnon-electron matrix element, since we use the experimental spin-disorder resistivity in the comparison between calculated and measured values of the electron-magnon resistivity and the magnon-electron lifetime. Our model employs realistic magnon energies as measured by neutron scattering, but treats the conduction electrons in the simplest model possible, that of a spherical Fermi surface.

The electron-magnon resistivity was first measured in several rare earth metals by Legvold and his collaborators.[1] Mackintosh[2] showed that it exhibits the characteristic exponential behavior $e^{-\Delta/k_BT}$, which one expects from the presence of a magnon energy gap, and suggested that the temperature dependence of the resistivity in a metallic ferromagnet with a gap should be given by $\rho_m \sim T^2 e^{-\Delta/k_BT}$. Here we shall demonstrate that a variational solution of the Boltzmann equation leads to the prediction that ρ_m at sufficiently low temperatures is proportional to $Te^{-\Delta/k_BT}$ and determine the prefactor, which depends on the magnon effective mass. We calculate numerically the full temperature dependence of ρ_m in the region from 1.5 to 50 K using realistic magnon energies. In view of the simplified model for the electronic band structure, the result compares well with our measured values, not only as far as the temperature dependence is concerned, but also with regard to the magnitude of the electron-magnon resistivity when compared to the measured high-temperature spin-disorder resistivity. We also give a simple formula for the specific heat derived from the same magnon spectra as were used in the resistivity calculation. The result differs considerably from the measured magnon specific heat reported by Lounasmaa and Sundström[3] and is also somewhat smaller than the calculated values quoted by Wells *et al.*[4]

We now turn to the result of solving the Boltzmann equation which describes the competing effect of the electric field and the electron-magnon scattering processes on the distribution function for the electrons. We treat the scattering within the Born approximation and employ the usual variational principle with a trial function which is energy independent and proportional to the cosine of the angle between the electron wave vector and the electric field. The choice of this trial function is motivated by the fact that it is an exact solution of the Boltzmann equation when only impurity scattering is present (provided the collision probability depends on the momentum transfer only). When the impurity scattering dominates the total resistivity, the inelastic scattering may be treated as a perturbation, and the variational treatment of the Boltzmann equation is therefore exact within our spherical model for the electronic energy bands.

The application of the variational principle leads to the following formula for the resistivity associated with electrons scattering off an arbitrary type of Bose excitation (magnon or phonon)

$$\rho = (m/ne^2)(1/\tau), \tag{1}$$

where $n = k_f^3/3\pi^2$ is the number density and the transport time τ is given by

$$\frac{1}{\tau}=\pi N(0)\int_0^{2k_F}\frac{1}{k_F^4}q^3dq\int\frac{d\Omega_{\hat{q}}}{4\pi}|g_{\vec{q}}|^2\times\frac{\omega_{\vec{q}}/k_BT}{4\sinh^2(\hbar\omega_{\vec{q}}/2k_BT)}\,. \tag{2}$$

Here $2k_F$ represents the maximum wave-vector transfer, $g_{\vec{q}}$ is the electron-boson coupling constant, and $\hbar\omega_{\vec{q}}$ is the boson energy for a given wave vector $\vec{q}$. $N(0)=mk_F/2\pi^2\hbar^2$ is the density of states per spin at the Fermi surface and the electron energies ϵ_k are given by $\epsilon_k=\hbar^2k^2/2m$.

In the electron-phonon case the application of formula (2) at low temperatures leads to the Bloch law $1/\tau\propto T^5$ since $|g_{\vec{q}}|^2\propto q$. For electron-magnon scattering one gets the results that $1/\tau\propto T^5$ for antiferromagnetic magnons (linear dispersion, $|g_{\vec{q}}|^2\propto q$) and $1/\tau\propto T^2$ for ferromagnetic magnons[5] (quadratic dispersion, $|g_{\vec{q}}|^2$ independent of $\vec{q}$). If a gap Δ is present in the magnon spectrum the dispersion is quadratic and one always gets $1/\tau\propto Te^{-\Delta/k_BT}$, as may be seen by the approximation $1/4\sinh^2(\hbar\omega/2k_BT)\simeq e^{-\hbar\omega/k_BT}$ and subsequent integration over q.

We obtain the electron-magnon coupling constant $g_{\vec{q}}$ from the usual electron-ion exchange interaction $\mathcal{H}$, assumed to involve only a δ function in the difference between the position coordinate $\vec{r}$ of the conduction electron and that of the ion $\vec{R}$,

$$\mathcal{H}=-A(g-1)\delta(\vec{r}-\vec{R})\vec{s}\cdot\vec{J}\,. \tag{3}$$

Here $\vec{s}$ and $\vec{J}$ are the spin and total angular-momentum operators for the electron and the ion, respectively ($s_z=\pm\frac{1}{2}$), A is the strength of their interaction, and g is the gyromagnetic factor.

In terms of the Fermi energy ϵ_F and the numbers of ions per unit volume N_{ion} the spin disorder resistivity ρ_{sd} is[6]

$$\rho_{\text{sd}}=\tfrac{3}{8}\pi N_{\text{ion}}A^2(g-1)^2(m/e^2\hbar\epsilon_F)J(J+1)\,, \tag{4}$$

where $J=6$ for Tb. If we assume the magnon dispersion to be isotropic, $\hbar\omega_{\vec{q}}=\Delta+\hbar^2q^2/2m_0$, we may readily find the low-temperature electron-magnon resistivity ρ_m from Eq. (2) and express it in terms of ρ_{sd}, Eq. (4). We get

$$\rho_m=\frac{1}{4(J+1)}e^{-\Delta/k_BT}\frac{T\Delta}{k_B\theta_m^2}\times\left(1+2\frac{k_BT}{\Delta}+\tfrac{1}{2}e^{-\Delta/k_BT}+\dots\right)\rho_{\text{sd}}\,, \tag{5}$$

where $k_B\theta_m=\hbar^2k_F^2/2m_0$. The leading low-temperature behavior is seen to be $Te^{-\Delta/k_BT}$. For simplicity we have here neglected the q dependence of the electron-ion matrix element $g_{\vec{q}}$ caused by the crystal field and magneto-elastic effects (see below). To show the region of validity of this low-temperature expansion we have exhibited the correction term linear in k_BT/Δ as well as the first nonanalytic correction ($\sim e^{-\Delta/k_BT}$).

The magnon dispersion in Tb is strongly anisotropic, differing considerably in the direction of the c axis and in the basal plane. If we represent the spectra by

$$\hbar\omega_{\vec{q}}=\Delta+\hbar^2q_z^2/2m_z+\hbar^2q_\perp^2/2m_\perp\,,$$

where q_z ($q_\perp$) is the component of the momentum along (perpendicular to) the z axis, the result for the directionally averaged ρ_m becomes identical in form to Eq. (5) with the replacement

$$m_0^2\to m_zm_\perp\left[\frac{1}{2}+\frac{1}{2}\frac{m_\perp}{m_z}\frac{1}{(1-m_\perp/m_z)^{1/2}}\times\tanh^{-1}\left(1-\frac{m_\perp}{m_z}\right)^{1/2}\right]. \tag{6}$$

A better representation of the magnon energies is obtained by the form

$$\hbar\omega_{\vec{q}}+\Delta+aq_z^2+bq_\perp^2+cq_\perp\,, \tag{7}$$

with $a=20$ meV Å^2, $b=16$ meV Å^2, $c=10$ meV Å, and $\Delta=1.65$ meV. In the symmetry directions this form reproduces the measured magnon energies[7] well within the energy range of interest. We have numerically calculated the directionally averaged electron-magnon resistivity from Eqs. (2) and (7), the result being exhibited in Fig. 1.

We have included in our numerical calculation the effect on $|g_{\vec{q}}|^2$ of the additional Bogoliubov transformation necessary to remove the part of the crystal field and magneto-elastic interaction,

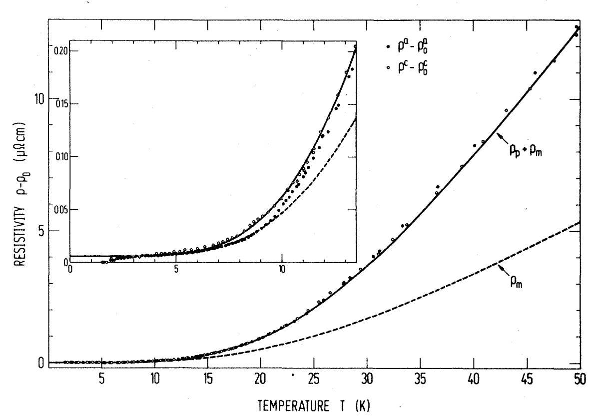

FIG. 1. Measured and calculated resistivity of Tb as a function of temperature. The filled (●) and open (○) circles represent measurements with the current along the a and c axis, respectively, the residual resistivities $\rho_0^a=6.78$ μΩ cm and $\rho_0^c=6.14$ μΩ cm having been subtracted. The solid curve is the sum of the electron-magnon (ρ_m) and electron-phonon (ρ_p) resistivities, calculated as explained in the text, while the dotted curve is the calculated ρ_m only. The inset is a low-temperature blow-up of the same results.

which is not diagonal in the Holstein-Primakoff operators. This causes $|g_{\vec{q}}|^2$ to be multiplied by the factor $[1+(B/\hbar\omega_{\vec{q}})^2]^{1/2}$, where B is the conventional symbol[8] for the magnitude of the nondiagonal part. In our actual calculation we use $B = 2.68$ meV as given by Houmann, Jensen and Touborg.[9] The comparison of theory with experiment will be discussed in detail below.

Turning now to the magnon-electron lifetime we may apply the standard Golden Rule to get the decay rate $1/\tau_{m-e}$ expressed as

$$\frac{1}{\tau_{m-e}} = \frac{\pi}{2}\omega_{\vec{q}}N(0)JA^2(g-1)^2\left[1+\left(\frac{B}{\hbar\omega_{\vec{q}}}\right)^2\right]^{1/2} \times N_{\text{ion}}\frac{m}{k_F q}\frac{1}{\hbar^2} = \frac{1}{3\pi^2}\omega_{\vec{q}}\left[1+\left(\frac{B}{\hbar\omega_{\vec{q}}}\right)^2\right]^{1/2} \times \frac{1}{J+1}\frac{k_F}{q}e^2 k_F\frac{1}{\hbar}\rho_{sd}\,. \quad (8)$$

Here we have again written the result in terms of the spin-disorder resistivity. The expression does not apply for q strictly equal to zero for reasons of energy conservation. With k_F equal to 1.4 Å^{-1} (the free-electron value) and ρ_{sd} given by its experimental value 81 $\mu\Omega$ cm (in contrast with the measurements presented in Ref. 1 we find the same value of ρ_{sd} for the two directions), we obtain $1/\tau_{m-e} = 0.08\omega$ for $q_z = 0.3$ Å^{-1}, $q_\perp = 0$. This estimate is comparable to the temperature-independent broadening which has recently been observed experimentally by Mackintosh and Bjerrum Møller[10] ($1/\tau_{m-e} \simeq 0.1\omega$ at this value of $\vec{q}$). The observed q dependence is quite different from $1/q$, the difference presumably being due to effects of the band structure of the electrons.

The magnon specific heat is at low temperatures proportional to $e^{-\Delta/k_B T}$ just like the resistivity, but the leading term in the prefactor is proportional to $T^{1/2}$. By expanding the exponential we get the molar specific heat c_m in the form

$$c_m = V_0 k_B \frac{1}{4\pi^{3/2}}\left(\frac{\Delta}{a}\right)^{1/2}\frac{\Delta^2}{c^2}t^{1/2}e^{-1/t}I_0(t)\,, \quad (9a)$$

where $t = k_B T/\Delta$ and V_0 is the molar volume. The function $I_0(t)$ is given by

$$I_0(t) = 1 + t(5-6\alpha) + t^2(\tfrac{35}{4} - 42\alpha + 60\alpha^2)\,, \quad (9b)$$

with $\alpha = b\Delta/c^2$. The calculated heat capacity [(9a) and (9b)] differs in the temperature region 0 – 15 K by less than 5% from the exact one which we calculate numerically from the spin wave energies, Eq. (7). It is in this temperature range about four times smaller than the experimental magnetic specific heat obtained by Lounasmaa and Sundström[3] and about a factor of 2 less than the one calculated by Wells *et al.*[4]

Our experimental resistivity results are obtained by use of a four point method where the sample voltage is measured with a lock-in detecting system working at low frequency (20 Hz). With a sample current of 10 mA the relative accuracy is from stability and resolution considerations determined to be about 0.05%. Absolute values are within 5%. The temperature is measured with an Au(+0.03-at.% Fe) versus Chromel thermocouple giving an accuracy in absolute temperature better than 0.4 K (depending on temperature) and a resolution of 0.01 K. The samples (the crystals have been kindly supplied by P. Touborg, Laboratory for Electrophysics, Technical University, Lyngby, Denmark) were spark cut from the same button of single crystalline Tb produced by a recrystallization method[11] and oriented by the x-ray Laue technique. The dimensions are (length, diameter): 16.45 mm, 2.08 mm for the c-axis crystal and 14.75 mm, 1.76 mm for the a-axis crystal.

The experimentally observed resistivity has been plotted in Fig. 1 in the temperature region 1.5-50 K. The region below the gap ($T < 13$ K) has been blown up to exhibit the variation in the region where the exponential temperature dependence is expected to occur. Note that the measured resistivity is only slightly anisotropic, contrary to the situation at higher temperatures ($T > 200$ K). At temperatures below $T \sim 3$ K the resistivity drops slightly below its nearly constant value. We have no explanation for this anomaly, which has been noted before.[12] To compare the data to our theoretical result for ρ_m based on Eqs. (2) and (7) we must also consider the electron-phonon contribution to the resistivity. With the electron-phonon matrix element $g_{\vec{q}}$ given by $|g_{\vec{q}}|^2 = \lambda\hbar\omega_{\vec{q}}/2N(0)$, where λ is a numerical constant of order unity, one obtains from Eq. (2) the well-known Bloch-Grüneisen result

$$1/\tau_{e-p} = \tfrac{1}{2}\pi(k_B\theta_p/\hbar)(T/\theta_p)^5 J_5(2\theta_p/T)\,, \quad (10)$$

with the characteristic temperature θ_p defined from the sound velocity c_s as $k_B\theta_p = \hbar k_F c_s$. [Note that the parameter λ determines the electron-phonon mass enhancement through the factor $(1+\lambda)$ in a model, where the q dependence of the electron-ion pseudopotential is neglected.] The theoretical curve in Fig. 1 represents the sum of the electron-phonon resistivity $\rho_p = (m/ne^2)1/\tau_{e-p}$ obtained from Eq. (10) and the spin-wave contribution ρ_m. The parameters k_F and λ are chosen to be 0.37 Å^{-1} and 0.155, respectively, the longitudinal sound velocity being equal to 3×10^5 cm/sec. The separate contribution of the spin waves has been indicated by a dashed line. The agreement between the calculated and measured resistivity is remarkably good, even considering the fact that we have used k_F and λ as free parameters. The ratio between ρ_m and ρ_{sd} depends very sensitively on k_F ($\sim k_F^{-4}$ at low temperatures). The values of k_F which enable us to make any reasonable fit to the experimental resistivity are therefore very small (0.3 – 0.4 Å^{-1}) in contrast to the free-electron value of 1.4 Å^{-1}. Physically, such a small value of k_F may be understood in a rough sense as a measure of the average radius of curvature of those pieces of Fermi surface which contribute to the conduction. At low temperatures small-angle scattering is dominant, and it becomes crucial for a quantitative estimate to know the distortion of the free electron surface at zone boundaries, that is the details of the Fermi surface. There is very little experimental information available on the Fermi surface of Tb, but de Haas – van Alphen measurements on the neighboring element Gd indicate[13] that our effective k_F value is reasonable. If we had used the free electron value ($k_F = 1.4$ Å^{-1}) in the resistivity it would of course be necessary to explicitly include umklapp processes and take into account the surface distortion at zone boundaries.

We conclude that both the magnitude of the magnon lifetime (which is independent of k_F) and the spin wave resistivity is well described by our simple model, especially with regard to the temperature dependence of the resistivity.

In summary, we have been able to choose a minimum number of parameters, using high-temperature experimental measurements of the spin-disorder resistivity, and account for the observed spin-wave resistivity as a function of temperature, consistent with the overall magnitude of the observed magnon broadening.

ACKNOWLEDGMENTS

It is a pleasure to thank A. R. Mackintosh for stimulating discussions and Jens Jensen for numerous useful comments and generous help.

[1]For a complete list of these measurements, see S. Legvold, *Magnetic Properties of Rare Earth Metals*, edited by R. J. Elliott (Plenum, London and New York, 1972), p. 335.

[2]A. R. Mackintosh, Phys. Lett. 4, 140 (1963).

[3]O. V. Lounasmaa and L. J. Sundström, Phys. Rev. 150, 399 (1966).

[4]P. Wells, P. C. Lanchester, D. W. Jones, and R. G. Jordan, J. Phys. F 6, 11 (1976).

[5]I. Mannari, Prog. Theor. Phys. 22, 335 (1959).

[6]P. G. de Gennes and J. Friedel, J. Phys. Chem. Solids 4, 71 (1958); see also, N. Hessel Andersen, P. E. Gregers-Hansen, E. Holm, H. Smith, and O. Vogt, Phys. Rev. Lett. 32, 1321 (1974).

[7]A. R. Mackintosh and H. Bjerrum Møller, in Ref. 1, p. 187.

[8]N. N. Bogoliubov, Fiz. Zhidk. Sostoyaniya 11, 23 (1947).

[9]J. G. Houmann, J. Jensen, and P. Touborg, Phys. Rev. B 12, 322 (1975).

[10]A. R. Mackintosh and H. Bjerrum Møller (unpublished).

[11]K. A. McEwen and P. Touborg, J. Phys. F 3, 1903 (1973).

[12]D. A. Tindall and M. H. Jericho, J. Phys. F 5, 1359 (1975).

[13]P. G. Mattocks and R. C. Young, J. Phys. F 7, 1219 (1977).