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Comment on temperature variations near a surface in He II

Humphrey J. Maris
Department of Physics, Brown University,
Providence, Rhode Island 02912
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An order-of-magnitude estimate is given of the temperature gradients in superfluid helium near a heated surface.

I. INTRODUCTION

In many circumstances superfluid ^{4}He acts like a thermal superconductor.[1,2] Thus if the viscosity of the normal fluid is neglected, the heat flux vector $\vec{j}$ is related to the temperature gradient ∇T by

$$\frac{\partial \vec{j}}{\partial t} \propto -\nabla T \quad , \tag{1}$$

whereas the heat flow in normal systems obeys Fourier's law

$$\vec{j} \propto -\nabla T \quad . \tag{2}$$

A consequence of Eq. (1) is that, in the steady state, no temperature gradients can exist within the liquid. However, near to a boundary across which heat is entering the liquid, Eq. (1) does not hold,[3,4] and it has been predicted theoretically that temperature gradients can exist in the liquid near the surface. These temperature gradients occur within a distance from the surface of the order of the mean free path of an excitation (phonon or roton). To calculate the temperature distribution in this boundary layer it is necessary to use a microscopic theory based on a Boltzmann equation for the excitations in the liquid. For a boundary surface lying in the plane $z=0$, the theory predicts that the temperature in the liquid should be of the form

$$T(z) = T_0 + \sum_{i=1}^{\infty} A_i e^{-z/\delta_i} \quad . \tag{3}$$

The sum is over a set of "modes," each having an amplitude A_i and a decay length δ_i. T_0 is the temperature *in the liquid* far from the surface. Twerdochlib and Kirk[5] have performed an experiment to look for temperature variations of the form of Eq. (3). To within the accuracy of their experiment they detected no temperature variations within the liquid. They pointed out that their experiment did not prove the impossibility of surface modes, but only gave an upper limit on the magnitudes of the $\{A_i\}$ coefficients. In the present note we present an order-of-magnitude theoretical estimate of the size of the expected temperature variations.

II. CALCULATION OF AMPLITUDES

For a review of the relevant background theory see Refs. 3, 4, and 6. At the temperatures of interest ($T < 0.5\,^\circ$K) the only excitations in liquid helium are long-wavelength phonons. Their distribution function n_p obeys the linearized Boltzmann equation

$$\frac{\partial n_p}{\partial t} = \left(\frac{\partial n_p}{\partial t}\right)_{\text{coll}} - \vec{v}_p \cdot \nabla n_p \quad , \tag{4}$$

$(\partial n_p/\partial t)_{\text{coll}}$ is the collision term and $\vec{v}_p$ is the phonon-group velocity. The equation is linearized by setting

$$n_p = n_p^0 + \delta n_p \quad , \tag{5}$$

where n_p^0 is the equilibrium Bose-Einstein distribution function corresponding to temperature T_0, and δn_p is small. In the steady state, Eq. (4) becomes

$$0 = \int C(\vec{p},\vec{p}\,')\,\delta n_{p'} - \vec{v}_p \cdot \nabla \delta n_p \quad , \tag{6}$$

where C describes the collisions. One solution of this equation is[7]

$$\psi_D(\vec{p}) = p_z n_p^0 (n_p^0 + 1) \quad . \tag{7}$$

This solution corresponds to a perturbation in which the gas of excitations has a uniform drift velocity away from the surface. There is no spatial temperature variation associated with this solution. There are also solutions of the form

$$\psi_i(\vec{p},z) = f_i n_p^0 (n_p^0 + 1) e^{-z/\delta_i}, \quad i = 1,2,\ldots, \tag{8}$$

where f_i is some function of $\vec{p}$. In general, the distribution function $\psi_i(\vec{p},z)$ for the ith solution will contain a part that corresponds to a temperature. By this remark it is meant that for these solutions

$$\int \epsilon_p \psi_i(\vec{p})\, d\tau_p \neq 0 \quad , \tag{9}$$

and so the energy density of the excitations, and hence the temperature, varies with z. It is these solutions that correspond to the modes that enter in Eq. (3).

The solution of the Boltzmann equation can be written

$$\delta n_p = B_D \psi_D(\vec{p}) + \sum_i B_i \psi_i(\vec{p},z) \ , \tag{10}$$

where B_D and the $\{B_i\}$ are some coefficients. These coefficients are determined by a boundary condition at the surface. The precise form of this condition is not known, and must depend on the details of the surface (composition, smoothness, etc.). We do not attempt to go into these details. Our approach is as follows: There is no reason to believe that ψ_D satisfies the boundary conditions by itself. Thus we expect that $B_D\psi_D$ and $\sum_i B_i\psi_i(\vec{p},z=0)$ must be of the same order of magnitude. We can relate B_D to the rate of flow of heat $\dot{Q}$ per unit area of the surface. It can be shown, in a straightforward manner, that the heat current density j for the distribution (7) is

$$j = C_v T_0 / 3\beta \ , \tag{11}$$

where C_v is the specific heat per unit volume at temperature T_0, and $\beta = 1/kT_0$. Thus we have

$$\dot{Q} = B_D C_v T_0 / 3\beta \ , \tag{12}$$

or

$$B_D = 3\beta \dot{Q} / C_v T_0 \ . \tag{13}$$

Thus the order of magnitude of $B_i f_i$ must be

$$3\beta \dot{Q} \langle p \rangle / C_v T_0 \ , \tag{14}$$

where $\langle p \rangle$ is the average magnitude of the momentum of a phonon. Hence the phonon energy density in the modes i at $z=0$ is of the order of

$$\int (3\beta\dot{Q}\langle p\rangle / C_v T_0)\epsilon_p n_p^0 (n_p^0+1)\, d\tau_p$$

$$\sim (3\beta\dot{Q}/C_v T_0 c) \int \epsilon_p^2 n_p^0(n_p^0+1)\, d\tau_p \tag{15}$$

$$= 3\dot{Q}/c \ , \tag{16}$$

where c is the phonon velocity. Thus the order of magnitude of the temperature due to the modes i at the surface is

$$\Delta T = 3\dot{Q}/C_v c \ . \tag{17}$$

Hence, this is the order of magnitude of the spatial variation of the temperature in the liquid.

III. RELATION TO EXPERIMENT

These temperature variations should be experimentally observable only over a restricted range of ambient temperatures. This is because of the lengths δ_i change very rapidly with temperature. Above about 0.4 K, for example, the *longest* of the $\{\delta_i\}$ becomes less than a millimeter, and so the whole temperature variation occurs very close to the surface. Below 0.2 K, on the other hand, the longest of the $\{\delta_i\}$ becomes several centimeters ($\sim$6 cm as estimated in Ref. 3). To observe this one clearly needs a large volume of helium.

Twerdochlib and Kirk[5] have carried out experiments at several temperatures down to 0.1 °K. Their helium sample was held at a pressure of about 2 bars. If we insert experimental values for C_v (see Ref. 8) and c at 2 bars we obtain

$$\Delta T = 5 \times 10^{-5} \dot{Q} / T_0^3 \ , \tag{18}$$

where $\dot{Q}$ is measured in mW cm^{-2} sec^{-1}, and ΔT and T_0 are in °K. In the measurements of Twerdochlib and Kirk two methods were used to determine ΔT. However, even for their more sensitive detection method the magnitude of the ΔT they could resolve was always at least one order of magnitude *larger* than the ΔT predicted by Eq. (18). Thus the conclusion is that an even more sensitive detection scheme will be needed before these temperature gradients can be observed.

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