Self-consistent many-body theory: Application to spin waves in itinerant ferromagnets

R. Kishore
Instituto de Física, Universidade Federal da Bahia, 40.000, Salvador, Bahia, Brazil
and Departamento de Engenharia Espacial INPE, São José dos Campos, SP, Brazil

(Received 25 April 1978)

A self-consistent many-body theory for the many-particle Green's function is developed and applied to derive an exact microscopic formula for the transverse dynamical susceptibility. As a concrete application of the above general results we consider the Hubbard Hamiltonian and obtain the spin-wave stiffness constant for an itinerant ferromagnet. It is found that earlier expressions for the stiffness constant obtained by Edwards and Hertz and Edwards are special cases of our general expression.

I. INTRODUCTION

Recently Fedro and Wilson[1] have developed a self-consistent many-body theory for the single-particle Green's function by using a commutation projection operator introduced by Kim and Wilson.[2] They derived the Dyson equation which differs from that of the usual many-body theory in the sense that the self-energy is self-consistent in every order of perturbation. Thus it is very promising to the problems where there exists a possibility of phase transition. However, in its present form it cannot be applied to calculate quantities such as susceptibility, resistivity, or many-particle correlation functions which are expressed in terms of two or more particle Green's functions. The purpose of this paper is to make the theory applicable to these cases by extending it to the many-particle Green's functions. This is done by defining a generalized projection operator which for the one-particle Green's function reduces to that of Fedro and Wilson.[1]

We have applied our general theory to calculate an exact formula for the transverse dynamical susceptibility and thereby the spin-wave stiffness constant for itinerant ferromagnets, described by the Hubbard Hamiltonian. This exact formal expression for the spin-wave stiffness constant is expressed in terms of the one-electron self-energy. Earlier, similar expressions have been obtained by Edwards[3] and Hertz and Edwards,[4] but their expressions are special cases of our general result.

In Sec. II we give a formal many-body theory for the many-particle Green's function. By a special choice of projection operator, Dyson's equation is derived. In Sec. III the formal theory of Sec. II is used to obtain an exact formula for the transverse dynamical susceptibility. In Sec. IV the spin-wave stiffness constant is calculated and discussed.

II. FORMAL THEORY

We consider the conventional many-particles retarded Green's function[5]

$$G_{ij}(t) \equiv i\theta(t)\langle[A_i, B_j(t)]_\eta\rangle , \tag{1}$$

where the subscripts i and j may denote spin, lattice sites, Wannier states or Bloch states, etc. and $\theta(t)$ is the Heaviside unit-step function. A_i and B_j are any one-, two-, or many-particle Heisenberg operators defined as

$$B_j(t) \equiv e^{iHt} B_j e^{-iHt} , \tag{2}$$

where H is the Hamiltonian operator and $B_j \equiv B_j(t=0)$. These operators are assumed to satisfy the relations

$$\langle[A_i, B_j]_\eta\rangle = \langle[A_i, B_i]_\eta\rangle \delta_{ij} , \tag{3}$$

where in Eqs. (1) and (3) angular brackets $\langle\ \rangle$ denote the grand canonical ensemble average, and square brackets correspond to a commulator for $\eta = -1$ and anticommulator for $\eta = +1$. Here and hereafter we work in a system of units where $\hbar = 1$. Differentiation of Eq. (1) with respect to time t gives the equation of motion

$$-i\frac{\partial}{\partial t} G_{ij}(t) = \langle[A_i, B_i]_\eta\rangle \delta_{ij}\delta(t) + i\theta(t)\langle[A_i, LB_j(t)]_\eta\rangle , \tag{4}$$

where for any arbitrary χ, the Liouville operator L is defined as

$$L\chi \equiv [H, \chi]_- . \tag{5}$$

Now following the projection-operator formalism

of Zwanzig and Mori,[6] we break the operator $B_j(t)$ into two parts in the following manner:

$$B_j(t) \equiv PB_j(t) + (1-P)B_j(t)\,, \tag{6}$$

and choose the projection operator P as

$$P \equiv \sum_j P_j \tag{7}$$

with

$$P_j\chi \equiv B_j\langle[A_j,\chi]_\eta\rangle/\langle[A_j,B_j]_\eta\rangle\,. \tag{8}$$

From Eqs. (3) and (8), it is easy to see that the P_j's form a set of orthogonal projection operators, i.e.,

$$P_iP_j\chi = \delta_{ij}P_i\chi\,, \tag{9}$$

and thus P satisfies the basic projection-operator conditions $P^2=P$. On substituting Eq. (6) in the equation of motion (4) and using Eqs. (7) and (8) for the projection operator we get

$$-i\frac{\partial}{\partial t}G_{ij}(t) = \langle[A_i,B_i]_\eta\rangle\delta_{ij}\delta(t) + \sum_l \Omega_{il}G_{lj}(t) + i\theta(t)\langle[A_i,L(1-P)B_j(t)]_\eta\rangle\,, \tag{10}$$

where

$$\Omega_{il} = \equiv \langle[A_i,LB_l]_\eta\rangle/\langle[A_l,B_l]_\eta\rangle. \tag{11}$$

The third term on the right-hand side of Eq. (10) can be related to the Green's function $G_{ij}(t)$ at another time τ. To see how it can be done, let us define an operator

$$\bar{B}_j(t) \equiv \theta(t)B_j(t). \tag{12}$$

Then from Eqs. (2) and (5) we get

$$\frac{\partial}{\partial t}\bar{B}_j(t) = \delta(t)B_j + iL\bar{B}_j(t). \tag{13}$$

On multiplying this equation from the left by the operator $(1-P)$ and using the easily verified facts that the projection operator P commutes with the differential operator and $(1-P)\,B_j=0$, we obtain

$$\frac{\partial}{\partial t}(1-P)\bar{B}_j(t) = i(1-P)L\bar{B}_j(t). \tag{14}$$

It can be shown that the solution of the above equation is[1]

$$(1-P)\bar{B}_j(t) = i\int_0^t d\tau\, e^{i\tau(1-P)L}(1-P) \times LP\bar{B}_j(t-\tau) \tag{15}$$

which, on using our choice of projection operator [see Eqs. (1) and (8)] and the Green's-function definition (1), becomes

$$(1-P)\bar{B}_j(t) = \sum_l \int_0^\infty d\tau\, e^{i\tau(1-P)L}(1-P)L \times (B_l/\langle[A_l,B_l]_\eta\rangle)G_{lj}(t-\tau). \tag{16}$$

The upper limit in the integral goes from t to ∞ because of the $\theta(t-\tau)$ functions contained in the Green's function $G_{lj}(t-\tau)$. Substitution of Eq. (16) in Eq. (10) gives a closed equation for the Green's function $G_{ij}(t)$,

$$-i\frac{\partial}{\partial t}G_{ij}(t) = \langle[A_i,B_i]_\eta\rangle\delta_{ij}\delta(t) + \sum_l \Omega_{il}G_{lj}(t) + \sum_l \int_{-\infty}^\infty \gamma_{il}(\tau)G_{lj}(t-\tau)\,d\tau, \tag{17}$$

where

$$\gamma_{il}(\tau) = \frac{i\theta(\tau)\langle[A_i,Le^{i\tau(1-P)L}(1-P)LB_l]_\eta\rangle}{\langle[A_l,B_l]_\eta\rangle}\,, \tag{18}$$

and the $\theta(\tau)$ function contained in $\gamma_{il}(\tau)$ allows to stretch the lower limit in the integral in Eq. (17) from 0 to $-\infty$. Equation (17) can be rewritten in a simpler form by introducing the Fourier transform

$$F_{ij}(t) = \frac{1}{2\pi}\int_{-\infty}^\infty d\omega\, e^{i\omega t}F_{ij}(\omega), \tag{19}$$

where $F_{ij}(t)$ corresponds to either $G_{ij}(t)$, $\gamma_{ij}(t)$, or Ω_{ij}. In terms of this Fourier transform Eq. (17) becomes

$$\omega G_{ij}(\omega) = \langle[A_i,B_i]_\eta\rangle\delta_{ij} + \sum_l \Omega_{il}G_{lj}(\omega) + \sum_l \gamma_{il}(\omega)G_{lj}(\omega)\,. \tag{20}$$

The above equation will be the starting point of all further calculations. For a special case, where subscripts i, j, or l correspond to lattice sites or Wannier states and the system possesses translational invariance, Eq. (20) can be Fourier transformed into momentum space by introducing the Fourier transform

$$F_{ij}(\omega) = \frac{1}{N}\sum_{\vec{k}} e^{i\vec{k}\cdot(\vec{R}_i-\vec{R}_j)}F_{\vec{k}}(\omega), \tag{21}$$

where N is the number of lattice sites and as before F_{ij} stands for $G_{ij}(\omega)$, $\gamma_{ij}(\omega)$, or Ω_{ij}. Substitution of Eq. (21) in Eq. (20) gives the Dyson equation

$$G_{\vec{k}}(\omega)=\frac{\frac{1}{N}\sum_i \langle[A_i,B_i]_\eta\rangle}{\omega-\Omega_{\vec{k}}-\gamma_{\vec{k}}(\omega)} \quad (22)$$

In the denominator on the right-hand side of Eq. (22), the quantity $\Omega_{\vec{k}}$ can be found directly by evaluating the commutator $LB_l\equiv[H,B_l]_-$ appearing in Eq. (11). But the quantity $\gamma_{\vec{k}}(\omega)$, in general, is not obtainable exactly. To evaluate it approximately one writes its chain of equations of motion[1] in a manner similar to that of Zubarev[5] for the Green's function and decouples the higher-order terms in some approximate way. Alternatively, one can break the Liouville operator L appearing in the exponential of Eq. (18) into two parts L_0 and L_1 and write a perturbation series by using the operator identity

$$\begin{aligned} e^{it(1-P)L}&\equiv e^{it(1-P)L_0}+i\int_0^t d\tau\, e^{i\tau(1-P)L_0}\\ &\quad\times(1-P)L_1e^{i(t-\tau)(1-P)L}\\ &\equiv e^{it(1-P)L_0}+i\int_0^t d\tau\, e^{i\tau(1-P)L_0}\\ &\quad\times(1-P)L_1e^{i(t-\tau)(1-P)L_0}+\cdots . \end{aligned} \quad (23)$$

The operator L_0 is chosen such that

$$\gamma^0_{il}(t)=i\theta(t)\langle|A_i,Le^{it(1-P)L_0}(1-P)LB_l|_\eta\rangle,$$

corresponding to the first term in (23), can be evaluated exactly. In the Sec. III, we shall follow this later approach to evaluate the spin-wave stiffness constant.

It should be noted that self-consistency in the theory is achieved by keeping the full ensemble average in all the averaged quantities. Apart from a few differences, our many-particle Green's-function theory is similar to the single-particle Green's-function theory developed by Fedro and Wilson.[1] We have used the retarded Green's function[7] instead of the Green's function without the $\theta(t)$ function used by Fedro and Wilson. Because of this to obtain $G_{ij}(\omega)$ we used a Fourier instead of a Laplace transform. Their choice of projection operator is a special case of our more general choice. For single-particle fermion operators, both choices are the same.

III. TRANSVERSE DYNAMICAL SUSCEPTIBILITY

Now we apply the formal theory of Sec. II to a translationally invariant system to calculate the wave vector $\vec{q}$ and the ω-frequency-dependent transverse dynamical susceptibility[8]

$$\chi(\vec{q},\omega)=-g^2\mu_B^2\sum_{ij}\chi_{ij}(\omega)e^{-i\vec{q}\cdot(\vec{R}_i-\vec{R}_j)}, \quad (24)$$

where subscripts i and j correspond to lattice sites, g is the Landé splitting factor, μ_B is the Bohr magneton, $\chi_{ij}(\omega)$ is the Fourier transform of the retarded Green's function,

$$\chi_{ij}(t)\equiv i\theta(t)\langle[S_i^+,S_j^-(t)]_-\rangle. \quad (25)$$

Here $S_i^\pm=S_i^x\pm iS_i^y$ are the spin-raising and lowering operators. On putting $A_i=S_i^+$ and $B_j(t)=S_j^-(t)$, the formulation of Sec. II immediately gives an exact formula for the transverse dynamical susceptibility as

$$\chi(\vec{q},\omega)=2g^2\mu_B^2N\langle S^z\rangle/[\Omega_{\vec{q}}+\gamma_{\vec{q}}(\omega)-\omega], \quad (26)$$

where

$$\langle S^z\rangle=\frac{1}{N}\sum_i\langle S_i^z\rangle, \quad (27)$$

$$\Omega_{\vec{q}}=(1/2N\langle S^z\rangle)\langle[S_{\vec{q}}^+,LS_{-\vec{q}}^-]_-\rangle, \quad (28)$$

and

$$\begin{aligned}\gamma_{\vec{q}}(\omega)&=\frac{i}{2N\langle S^z\rangle}\int_{-\infty}^{\infty}dt\,e^{-i\omega t}\theta(t)\\ &\quad\times\langle[S_{\vec{q}}^+,Le^{it(1-P)L}(1-P)LS_{-\vec{q}}^-]_-\rangle.\end{aligned} \quad (29)$$

Here it is assumed that the system is ferromagnetic so that $\langle S_i^z\rangle=\langle S^z\rangle$. The operators $S_{\vec{q}}^\pm$ are

$$S_{\vec{q}}^\pm=\sum_i e^{-i\vec{q}\cdot\vec{R}_i}S_i^\pm, \quad (30)$$

and the projection operator P from Eqs. (7) and (8) is given by

$$P\chi=\frac{1}{2N\langle S^z\rangle}\sum_q S_{-\vec{q}}^-\langle[S_{\vec{q}}^+,\chi]_-\rangle. \quad (31)$$

In obtaining Eqs. (28) and (29) we have made use of Eqs. (11), (18), (19), (21), and (22).

For the one-band Hubbard Hamiltonian

$$H=\sum_{\vec{k}\sigma}\epsilon_{\vec{k}}a_{\vec{k}\sigma}^\dagger a_{\vec{k}\sigma}+\frac{I}{N}\sum_{\vec{q},\vec{k}\vec{k}'}a_{\vec{k}+\vec{q}+}^\dagger a_{\vec{k}+}+a_{\vec{k}'-\vec{q}-}^\dagger a_{\vec{k}'-}, \quad (32)$$

where $\epsilon_{\vec{k}}$ is the band energy, I is the intra-atomic interaction, and $a_{\vec{k}\sigma}, a_{\vec{k}\sigma}^\dagger$ are the annihilation and creation operators for an electron of spin σ and

momentum $\vec{k}$. Equations (28) and (29) can be further simplified by use of the following identities:

$$LS^{\pm}_{\vec{q}} \equiv \sum_{\vec{k}} (\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}}) a^{\dagger}_{\vec{k}+\vec{q}\pm} a_{\vec{k}\mp}, \tag{33}$$

and

$$\langle [X, LY]_- \rangle \equiv - \langle [LX, Y]_- \rangle, \tag{34}$$

for any X and Y. In deriving Eq. (33) we have used the fact that the interaction part of the Hamiltonian (32) commutes with the operator $S^{\sigma}_{\vec{q}} \equiv \sum_{\vec{k}} a^{\dagger}_{\vec{k}+\vec{q}\,\sigma} a_{\vec{k}-\sigma}$, and in Eq. (34) the cyclic invariance property of the trace, implied in the ensemble average, is exploited. Use of Eqs. (33) and (34) in Eqs. (28) and (29) gives

$$\Omega_{\vec{q}} = \frac{1}{2N\langle S^z \rangle} \sum_{\vec{k}\sigma} (\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}}) \langle a^{\dagger}_{\vec{k}\sigma} a_{\vec{k}\sigma} \rangle \tag{35}$$

and

$$\gamma_{\vec{q}}(\omega) = - \frac{i}{2N\langle S^z \rangle} \sum_{\vec{k}\vec{k}'} (\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}})(\epsilon_{\vec{k}'-\vec{q}} - \epsilon_{\vec{k}'}) \times \int_{-\infty}^{\infty} dt\, e^{-i\omega t} \theta(t) \times \langle [a_{\vec{k}+\vec{q}+} a_{\vec{k}-}, e^{it(1-P)L}(1-P) a^{\dagger}_{\vec{k}'-\vec{q}-} a_{\vec{k}'+}]_- \rangle. \tag{36}$$

In Sec. IV, the above equations will be used to calculate the spin-wave stiffness constant.

IV. SPIN-WAVE STIFFNESS CONSTANT

The spin-wave stiffness constant D is defined as the coefficient of the q^2 term in the long-wavelength limit of the spin-wave energy given by the solution of the equation

$$\omega - \Omega_{\vec{q}} - \gamma_{\vec{q}}(\omega) = 0, \tag{37}$$

obtained by making the denominator of the transverse dynamical susceptibility (26) equal to zero.

In the long-wavelength limit, using Eqs. (35) and (36) for $\Omega_{\vec{q}}$ and $\gamma_{\vec{q}}(\omega)$, respectively, we get the spin-wave stiffness constant as

$$\omega = Dq^2 = \frac{1}{4N\langle S^z \rangle} \sum_{\vec{k}ij\sigma} q_i q_j \frac{\partial^2 \epsilon_{\vec{k}}}{\partial k_i \partial k_j} \langle a^{\dagger}_{\vec{k}\sigma} a_{\vec{k}\sigma} \rangle + \frac{i}{2N\langle S^z \rangle} \sum_{\vec{k}\vec{k}'} (\vec{q}\cdot\nabla_{\vec{k}} \epsilon_{\vec{k}})(\vec{q}\cdot\nabla_{\vec{k}} \epsilon_{\vec{k}'}) \times \int_{-\infty}^{\infty} dt\, \theta(t) \times \langle [a^{\dagger}_{\vec{k}+} a_{\vec{k}-}, e^{it(1-P)L}(1-P) a^{\dagger}_{\vec{k}'-} a_{\vec{k}'+}]_- \rangle, \tag{38}$$

where the subscripts i and j correspond to Cartesian coordinates.

Now we divide the Liouville operator L into a sum of two parts, L_0 and L_1, defined by Eq. (5) with Hamiltonians H_0 and H_1, respectively, such that the total Hamiltonian $H = H_0 + H_1$. On substituting the perturbation expansion (23) for the exponential $e^{it(1-P)L} \equiv e^{it(1-P)(L_0+L_1)}$ in Eq. (38) and using the equation

$$\sum_{\vec{k}'} \nabla_{\vec{k}'} \epsilon_{\vec{k}'} P a^{\dagger}_{\vec{k}'-} a_{\vec{k}'+} = 0 \tag{39}$$

obtained by the use of Eq. (31) and the translational invariance property of the system, we get

$$Dq^2 = \frac{1}{4N\langle S^z \rangle} \sum_{\vec{k}ij\sigma} q_i q_j \frac{\partial^2 \epsilon_k}{\partial k_i \partial k_j} \langle a^{\dagger}_{\vec{k}\sigma} a_{\vec{k}\sigma} \rangle + \frac{i}{2N\langle S^z \rangle} \sum_{\vec{k}\vec{k}'} (\vec{q}\cdot\nabla_{\vec{k}} \epsilon_{\vec{k}})(\vec{q}\cdot\nabla_{\vec{k}} \epsilon_{\vec{k}'}) \times \int_{-\infty}^{\infty} dt\, \theta(t) \langle [a^{\dagger}_{\vec{k}+} a_{\vec{k}-}, e^{it(1-P)L_0} a^{\dagger}_{\vec{k}'-} a_{\vec{k}'+}]_- \rangle + \frac{1}{2N\langle S^z \rangle} \sum_{\vec{k}\vec{k}'} T(\vec{k}, \vec{k}')(\vec{q}\cdot\nabla_{\vec{k}} \epsilon_{\vec{k}})(\vec{q}\cdot\nabla_{\vec{k}'} \epsilon_{\vec{k}'}), \tag{40}$$

where

$$T(\vec{k}, \vec{k}') = - \int_{-\infty}^{\infty} dt\, \theta(t) \int_0^t d\tau \langle [a^{\dagger}_{\vec{k}+} a_{\vec{k}-}, e^{it(1-P)L_0}(1-P) L_1 e^{i(t-\tau)(1-P)L} a^{\dagger}_{\vec{k}'-} a_{\vec{k}'+}]_- \rangle. \tag{41}$$

The second term on the right-hand side of Eq. (40) can be written in terms of the self-energy of the electron by taking the Hamiltonian H_0 as

$$H_0 = \sum_{\vec{k}\sigma} (\epsilon_{\vec{k}} + M_{\vec{k}\sigma}) a^{\dagger}_{\vec{k}\sigma} a_{\vec{k}\sigma}, \tag{42}$$

where $M_{\vec{k}\sigma}$ is the self-energy of an electron with

momentum $\vec{k}$ and spin σ. For the above Hamiltonian (42), the use of Eqs. (5) and (31) and the translational-invariant property of the system give

$$\sum_{\vec{k}'} \nabla_{\vec{k}'}\epsilon_{\vec{k}'}(1-P)L_0 a^{\dagger}_{\vec{k}'-}a_{\vec{k}'+} = \sum_{\vec{k}'} (\nabla_{\vec{k}'}\epsilon_{\vec{k}'})(M_{\vec{k}'-} - M_{\vec{k}'+})a^{\dagger}_{\vec{k}'-}a_{\vec{k}'+} . \quad (43)$$

In the same way one arrives at the nth-order term

$$\sum_{\vec{k}} \nabla_{\vec{k}'}\epsilon_{\vec{k}'}[(1-P)L_0]^n a^{\dagger}_{\vec{k}'-}a_{\vec{k}'+} = \sum_{\vec{k}'} (\nabla_{\vec{k}'}\epsilon_{\vec{k}'})(M_{\vec{k}'-} - M_{\vec{k}'+})^n a_{\vec{k}'-}a_{\vec{k}'+}. \quad (44)$$

Using this result in the second term of the right-hand side of Eq. (40) and performing the integral, we get the stiffness constant as

$$Dq^2 = \frac{1}{4N\langle S^z\rangle}\sum_{\vec{k}ij\sigma} q_i q_j \frac{\partial^2\epsilon_{\vec{k}}}{\partial k_i\,\partial k_j}\langle a^{\dagger}_{\vec{k}\sigma}a_{\vec{k}\sigma}\rangle + \frac{1}{2N\langle S^z\rangle}\sum_{\vec{k}} (\vec{q}\cdot\nabla_{\vec{k}}\epsilon_{\vec{k}})\frac{\langle a^{\dagger}_{\vec{k}+}a_{\vec{k}+}\rangle - \langle a^{\dagger}_{\vec{k}-}a_{\vec{k}-}\rangle}{M_{\vec{k}+} - M_{\vec{k}-}} + \frac{1}{2N\langle S^z\rangle}\sum_{\vec{k}\vec{k}'} T(\vec{k},\vec{k}')(\vec{q}\cdot\nabla_{\vec{k}}\epsilon_{\vec{k}})(\vec{q}\cdot\nabla_{\vec{k}'}\epsilon_{\vec{k}'}). \quad (45)$$

Equation (45) is an exact expression for the stiffness constant at finite temperature for an itinerant ferromagnet described by the Hubbard Hamiltonian. Usually the Hamiltonian H_0, given by Eq. (42), is a good approximation to the total Hamiltonian H, and hence the contribution of the nondiagonal third term on the right-hand side of Eq. (45) to the stiffness constant is quite small because of its dependence on the weakly interacting part H_1 of the Hamiltonian H. In all practical calculations, one can neglect this nondiagonal term and thus a good estimate of the spin-wave stiffness constant depends on how accurately the self-energy of the electron is calculated. It is apparent that Edwards' exact expression[3] obtained for strong ferromagnets at absolute zero can be obtained from (45) by just putting $M_{\vec{k}+}=0$ and $\langle a^{\dagger}_{\vec{k}-}a_{\vec{k}-}\rangle = 0$. Hertz and Edwards[4] also obtained a similar expression at absolute zero except that they gave an approximate form for the nondiagonal term by exploiting the requirement of spin conservation in Ward-identity form.

ACKNOWLEDGMENTS

We are grateful to Financiadora de Estudos e Projetos and Conselho Nacional de Desenvolvimento Cientifico e Tecnológico, Rio de Janeiro, Brazil, for financial support.

[1] A. J. Fedro and R. S. Wilson, Phys. Rev. 11, 2148 (1975).

[2] S. K. Kim and R. S. Wilson, Phys. Rev. A 7, 1396 (1973).

[3] D. M. Edwards, Proc. R. Soc. London Ser. A 300, 373 (1967); J. Appl. Phys. 39, 481 (1968).

[4] J. A. Hertz and D. M. Edwards, J. Phys. F 3, 2174 (1973).

[5] D. N. Zubarev, Usp. Fiz. Nauk. 71, 71 (1960) [Sov. Phys. Usp. 3, 320 (1960)].

[6] R. Zwanzig, J. Chem. Phys. 33, 1338 (1960); H. Mori, Prog. Theor. Phys. 33, 423 (1965).

[7] One can also use advanced Green's functions instead of the retarded one without any change in formalism.

[8] T. Izuyama, D. J. Kim, and R. Kubo, J. Phys. Soc. Jpn. 18, 1025 (1963).