Magnetic field on a fast charged particle traversing a ferromagnet: Random-phase approximation

J. Bruno and J. Sak
Serin Physics Laboratory, Rutgers University, New Brunswick, New Jersey 08903
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Linear response theory is applied to a calculation of the dynamic magnetic field experienced by a particle of charge Ze when it passes through a ferromagnet with a velocity $v >> Zv_0$ (v_0 is the Bohr velocity). The wave-vector-dependent dielectric function and polarizability of an electron gas as calculated in the random-phase approximation are used in describing the response of the electrons in the ferromagnet to the Coulomb field of the traversing charge. The calculation results in a magnetic field acting on the moment of the charge of $(0.86 \pm 0.01)\ 4\pi M_0 Z(v_0/v)$, where M_0 is the magnetization of the ferromagnet.

I. INTRODUCTION

In 1968, Grodzins and co-workers reported[1] that strong transient magnetic fields acted on ions just prior to their being stopped in a polarized ferromagnetic material. Since then, both experimental and theoretical work has been carried out in order to determine the origin and magnitude of this field. This work is reviewed by Van Middelkoop.[2] Before describing the contents of this paper, some general comments will be made for purposes of orientation.

When an ion of charge Ze and speed v traverses a ferromagnetic material, it is acted upon by both an electric and a magnetic field. The electric field which, on the average, is antiparallel to the velocity, slows down the ion while the magnetic field at the nucleus of the ion causes a precession of its magnetic moment. By measuring this precession for a nucleus of known magnetic moment, information about the magnetic field can be gained. On the other hand, if the field can be calibrated it can be used to measure unknown magnetic moments of nuclear excited states. At present, the origin of the field, and its dependence on various parameters, such as the velocity of the ion and its atomic number Z, are being investigated[3] with a variety of probes.

The problem presents us with two characteristic velocities to which the speed of the ion can be compared. First, the Bohr velocity $v_0 = e^2/\hbar$ gives the order of magnitude of velocities of the valence electrons, both of the ion and the ferromagnetic material. The second characteristic velocity is that of $1s$ electrons of the ion, Zv_0.

For fast ions $v >> v_0$, the processes determining the magnetic field probably involve only electrons. At present there is no hard evidence that nuclear collisions play a major role at high velocities. The region $v >> v_0$ can be subdivided into two domains: moderately high velocities $v_0 << v < Zv_0$, and very high velocities $v >> Zv_0$. At moderately high velocities, the moving ion is partially stripped and its remaining electron shells may become partially polarized by knock-off and pick-up processes during collisions with the polarized electrons in the ferromagnet. Linhard and Winther[4] (LW) developed a model to describe this field, however they neglected the presence of bound states on the ion. Their theory predicted a transient field which depends inversely on the ions velocity for $v > v_p$ and is independent of v for $v < v_p$, where v_p is an average velocity of the polarized electrons in the ferromagnet. The dependence of this field on the ions atomic number Z is predicted to be linear. At first, these predictions seemed to be borne out by experiments[5] but as higher-ion-velocity data became available, deviations from LW predictions became quite large. The magnetic field for ion velocities $v_0 < v < Zv_0$ appears to be of the order of a few megagauss, several times above the LW predictions, and increases approximately linearly with both the velocity of the ion and its atomic number Z. Experiments[6] on light ions (^{12}C, ^{13}C, ^{18}O) at velocities approaching or exceeding Zv_0 suggest that while the observed field increases with velocity for v approaching Zv_0, it becomes negligibly small for $v \approx 2Zv_0$. These effects are thought to be due to a polarization of inner-shell electrons occurring as capture and knock-out processes compete to produce partially unfilled inner shells: as the ions velocity is increased, knock-out processes dominate those of capture and the ion becomes completely stripped of its electrons. At this point, hyperfine interactions can no longer occur and a different mechanism must be invoked to explain the magnetic interactions. We expect that these very-high-velocity totally stripped ions will polarize the medium in their vicinity and that the probability of their capturing electrons into bound states will become negligibly small. In this case, the field will be due to the enhanced magnetization density surrounding the moving ion.

Theoretically, this region of very-high-ion velocities is the easiest to handle with some confidence because a linear-response theory can be developed. The validity of this approach will depend upon how small $Z v_0/v$ is. This is easily demonstrated by looking at the Bohr parameter[7] $K = Z v_0/v_R$, which gives an estimate of the accuracy afforded by a first-order quantum-mechanical perturbation scheme—taking the field of the ion as a perturbation to the almost free electrons in the ferromagnet. In the above, v_R is the relative velocity between the ion and the polarized electrons in the ferromagnet. When v is considerably greater than v_0, then we have $v_R \approx v$ and $K \approx Z v_0/v$. Thus, when we have $K \ll 1$ it is clear that one can treat the presence of the ion as a weak perturbation to the electrons. This first-order treatment is tantamount to letting the $(\vec{q}, \omega)$ Fourier component of the induced charge density, be *linearly* proportional to the $(\vec{q}, \omega)$ Fourier component of the ions charge density, where the constant of proportionality will be related to the dielectric function of the medium.

The purpose of this paper is to develop this linear-response theory to a prediction of the dynamic magnetic field experienced by an ion moving with $v \gg Z v_0$. As mentioned above, this theory will be valid for fast moving ions. In Sec. II we calculate the dynamic magnetic field $\delta\vec{B}$ at the site of the ion, assuming that the dynamic magnetization $\delta\vec{M}(\vec{r},t)$ is known. In Sec. III we relate the dynamic magnetization to the external (ion's) charge density, and obtain an expression for the dynamic field $\delta\vec{B}$ at the site of the ion. The results of numerical computations are given in Sec. IV for different values of v and finally, in Sec. V, a comparison with available data is made.

II. MAGNETIC FIELD ACTING ON THE NUCLEUS

Consider a particle of charge $+Ze$ and speed v moving in the $+z$ direction through a polarized ferromagnetic medium. We will assume that the matter is polarized in the $+x$ direction by a small external field $\vec{H}$ given by $\vec{H} = (H, 0, 0)$ and that, in the absence of the ion, the magnetization density is given by $\vec{M}_0 = (M_0, 0, 0)$. In the above, both H_0 and M_0 are taken to be independent of space and time. The geometrical situation at $t = 0$ is shown in Fig. 1, where the ion is located at the origin of the coordinate system. In the presence of the ion, we take the magnetization density to be given by $\vec{M}(\vec{r},t) = \vec{M}_0 + \delta\vec{M}(\vec{r},t)$, and the associated magnetic field to be given by $\vec{B}(\vec{r},t) = \vec{B}_0 + \delta\vec{B}(\vec{r},t)$. Thus $\delta\vec{M}(\vec{r},t)$ and $\delta\vec{B}(\vec{r},t)$ are the dynamic parts of the fields which we will deal with below, while $\vec{M}_0$ and $\vec{B}_0$ are the static parts. Note that since the Coulomb attraction responsible for the enhanced electron density at the site of the ion (and hence the dynamic field) is spin independent, the fields $\delta\vec{M}$ and $\delta\vec{B}$ will have only x components. In what follows we will deal with these components only. Using classical electrodynamics, we can relate the dynamic magnetization at $t = 0$, $\delta M(\vec{r})$, to the field $\delta B(0)$ at the site of the ion by taking $c\vec{\nabla} \times \delta\vec{M}(\vec{r})$ as the source current density responsible for $\delta B(0)$. This leads to the expression

$$\delta B(0) = -\int \frac{dV}{r^3}\left[y\frac{\partial}{\partial y}\delta M(\vec{r}) + z\frac{\partial}{\partial z}\delta M(\vec{r})\right] , \quad (1)$$

where the integration is carried over all space. After using the vector identity

$$\vec{\nabla}\cdot(\vec{r}\,\delta M/r^3) = (\vec{r}\cdot\nabla\vec{\delta}M)/r^3 + \delta M\,\vec{\nabla}\cdot(\vec{r}/r^3) ,$$

and converting the volume integral of $\vec{\nabla}\cdot(\vec{r}\delta M/r^3)$ to a surface integral (which vanishes as the surface goes to infinity) we obtain

$$\delta B(0) = 4\pi\delta M(0) + \int dV\,\frac{x}{r^3}\frac{\partial}{\partial x}\delta M(\vec{r}) . \quad (2)$$

It is clear physically that $\delta M(\vec{r})$ has cylindrical symmetry about the z axis so we can expand $\delta M(\vec{r})$ in Legendre polynomials in order to simplify expression (2), i.e., we will have

$$\delta M(\vec{r}) = \delta M_0(r) + \delta M'(\vec{r}) , \quad (3a)$$

with

$$\delta M'(\vec{r}) = \sum_{l=1}^{\infty}\delta M_l(r)P_l(\cos\theta) , \quad (3b)$$

where we have $\delta M'(0) = 0$, and θ is the angle between $\vec{r}$ and the $+z$ axis as shown in Fig. 1. After placing Eqs. (3) into Eq. (2), making use of the spherical symmetry of $\delta M_0(r)$, and once again converting a volume integral to a surface integral which vanishes, we obtain

$$\delta B(0) = \tfrac{1}{3}8\pi\delta M_0(0) + \int dV\,\frac{x}{r^3}\frac{\partial}{\partial x}\delta M'(\vec{r}) . \quad (4)$$

We substitute Eq. (3b) into Eq. (4) and do an integration by parts; we obtain after a straightforward calculation

$$\delta B(0) = \tfrac{1}{3}8\pi\delta M_0(0) - \frac{4\pi}{3}\int_0^{\infty}\delta M_2(r)\,\frac{dr}{r} . \quad (5)$$

The orthogonality of the Legendre polynomials was used in arriving at Eq. (5).

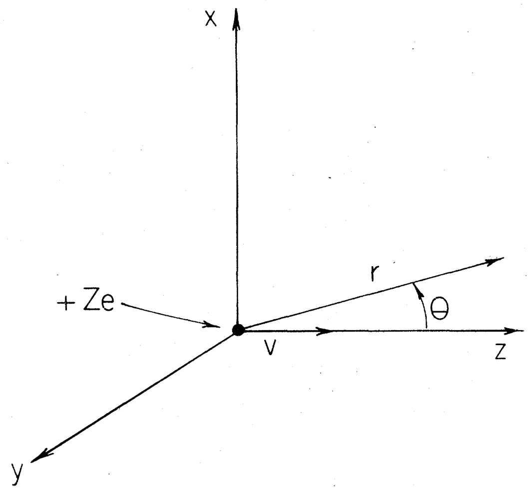

FIG. 1. Particle of charge $+Ze$ located at the origin of the coordinate system at $t=0$.

We have thus obtained an expression for the dynamic field δB at the ion in terms of the dynamic magnetization density in the entire ferromagnet. It is seen from Eq. (5) that actually only the $l=0$ and $l=2$ components of $\delta M(\vec{r})$ are required for a calculation of $\delta B(0)$. Next, a calculation of $\delta M(\vec{r})$ is needed.

III. LINEAR-RESPONSE THEORY

It might be helpful at this point to review the physical ideas which will underlie the coming calculations of the dynamic magnetization $\delta M(\vec{r})$. We are considering an ion whose speed is much greater than any other characteristic velocity in the problem; $v >> Z v_0$. Thus, in the rest frame of the moving, completely stripped ion, one sees an electron wind blowing by. The energies of these electrons, moving at speeds of about v in the negative z direction, are very large compared to their binding energies in the solid and so one can, in this limit, neglect band-structure effects as being insignificant on this energy scale. Also, since v is so large, the Bohr parameter K is much less than unity and we can expect a first-order perturbation calculation to yield results which justify our relating the induced charge density to the external (ion's) charge density in a linear fashion. This relationship will involve the use of a dielectric function and polarizability of the electrons.

Consider then the dynamic magnetization density which can be written

$$\delta M(\vec{r}) = \mu_B[\delta n_\downarrow(\vec{r}) - \delta n_\uparrow(\vec{r})] \quad , \tag{6}$$

where μ_B is the Bohr magneton and

$$\delta n_{\downarrow,\uparrow}(\vec{r}) \equiv n_{\downarrow,\uparrow}(\vec{r}) - n^0_{\downarrow,\uparrow}$$

are the deviations from the mean number density of electrons with spin down and up, respectively. In the above we take $n^0_{\downarrow,\uparrow} = k^3_{F\downarrow,\uparrow}/6\pi^2$ to be the equilibrium value of the number density for a given spin orientation, and we choose the x axis as the axis of spin quantization. Writing the Fourier-transformed version of Eq. (6) in both space and time we obtain

$$\delta M(\vec{k},\omega) = \mu_B[\delta n_\downarrow(\vec{k},\omega) - \delta n_\uparrow(\vec{k},\omega)] \quad . \tag{7}$$

It is clear that we will know the dynamic magnetization, once we can relate $\delta n_{\downarrow,\uparrow}(\vec{k},\omega)$ to the known quantity $\delta n^{\text{ext}}(\vec{k},\omega)$—the external number density (the ion's number density). To this end we assume knowledge of a suitable dielectric function and polarizability to describe the electron system. Then we obtain

$$\delta n_{\downarrow,\uparrow}(\vec{k},\omega) = \frac{4\pi\alpha_{\downarrow,\uparrow}(\vec{k},\omega)}{\epsilon(\vec{k},\omega)}\delta n^{\text{ext}}(\vec{k},\omega) \quad , \tag{8}$$

where $\alpha_{\downarrow,\uparrow}(\vec{k},\omega)$ is the polarizability of the electron gas for spin-down and spin-up electrons respectively, and

$$\epsilon(\vec{k},\omega) = 1 + 4\pi[\alpha_\uparrow(\vec{k},\omega) + \alpha_\downarrow(\vec{k},\omega)] \tag{9}$$

is the dielectric function.

If we assume that, because of its high velocity, the ion's motion is relatively unaffected by the atomic electron collisions, we can write $\delta n^{\text{ext}}(\vec{r},t) = Z\delta(\vec{r}-\vec{v}t)$ and obtain its Fourier transform

$$\delta n^{\text{ext}}(\vec{k},\omega) = 2\pi Z\delta(\omega - \vec{k}\cdot\vec{v}) \quad .$$

Combining this with expressions (8) and (9) and transforming back to $(\vec{r},t)$ space we obtain, for $t=0$,

$$\delta M(\vec{r}) = \frac{4\pi\mu_B Z}{(2\pi)^3}\int d^3k\, e^{i\vec{k}\cdot\vec{r}}\frac{\Delta\alpha(\vec{k},\vec{k}\cdot\vec{v})}{\epsilon(\vec{k},\vec{k}\cdot\vec{v})} \quad , \tag{10}$$

where

$$\Delta\alpha(\vec{k},\omega) \equiv \alpha_\downarrow(\vec{k},\omega) - \alpha_\uparrow(\vec{k},\omega)$$

is the difference between the spin-down and spin-up polarizabilities. From Eq. (10) we can immediately extract the spherically symmetric ($l=0$) part of δM, which is given by

$$\delta M_0(0) = \frac{4\pi\mu_B Z}{(2\pi)^3}\int d^3k\, \frac{\Delta\alpha(\vec{k},\vec{k}\cdot\vec{v})}{\epsilon(\vec{k},\vec{k}\cdot\vec{v})} \quad . \tag{11}$$

The asymmetrical part ($l=2$) is almost as easy to get if we use the relation

$$e^{i\vec{k}\cdot\vec{r}} = \sum_{l=0}^{\infty} (2l+1) i^l j_l(kr) P_l(\hat{k}\cdot\hat{r}) \quad ,$$

along with the addition theorem for spherical harmonics where we can replace $P_l(\hat{k}\cdot\hat{r})$ with

$$P_l(\hat{k}\cdot\hat{r}) = P_l(\hat{r}\cdot\hat{v}) P_l(\hat{k}\cdot\hat{v}) + 2\sum_{m=1}^{l} \frac{(l-m)!}{(l+m)!} P_l^m(\hat{r}\cdot\hat{v}) \times P_l^m(\hat{k}\cdot\hat{v}) \cos[m(\phi_r - \phi_k)] \quad .$$

After placing these expressions into Eq. (10) above, and integrating the second term over ϕ_k, we are left with

$$\delta M_2(r) = -\frac{20\pi\mu_B Z}{(2\pi)^3} \int d^3k\, j_2(kr) \times P_2(\hat{k}\cdot\hat{v}) \frac{\Delta\alpha(\vec{k}, \vec{k}\cdot\vec{v})}{\epsilon(\vec{k}, \vec{k}\cdot\vec{v})} \quad . \tag{12}$$

We can perform the integration of $\delta M_2(r)$ over r as required in Eq. (5) to get

$$\int_0^{\infty} \delta M_2 \frac{dr}{r} = -\frac{20\pi\mu_B Z}{3(2\pi)^3} \int d^3k\, P_2(\hat{k}\cdot\hat{v}) \times \frac{\Delta\alpha(\vec{k}, \vec{k}\cdot\vec{v})}{\epsilon(\vec{k}, \vec{k}\cdot\vec{v})} \quad . \tag{13}$$

When this is combined with Eq. (11) in Eq. (5) we finally obtain

$$\delta B(0) = \frac{2\mu_B Z}{3\pi} \int d^3k \frac{\Delta\alpha(\vec{k}, \vec{k}\cdot\vec{v})}{\epsilon(\vec{k}, \vec{k}\cdot\vec{v})} \times [2 + \tfrac{5}{3} P_2(\vec{k}\cdot\vec{v})] \quad . \tag{14}$$

In order to proceed from this point on, we must decide on a particular choice for ϵ and $\Delta\alpha$. In a recent paper,[8] we chose for ϵ the classical Drude dielectric function and the corresponding polarizability. With this choice, we were forced to cut off by hand an otherwise divergent integral in k space in order to simulate the effects of a quantum-mechanically correct polarizability. The result obtained in that paper for $\delta B(0)$ was

$$\delta B(0) = 4\pi M_0 Z (v_0/v) \quad . \tag{15}$$

We will see in what follows that the use of a polarizability which respects quantum requirements leads to the result (15) modified by a factor of order unity.

Continuing, we choose for our dielectric function, the one calculated in the random-phase approximation (RPA).[9] In the region of high electron densities, when the kinetic energies of the electrons in the ferromagnet are considerably larger than the potential energies of interaction, the RPA dielectric function is known to accurately describe both the single particle and collective behavior of the electrons in the gas.[9] In ferromagnetic media, the conduction electron densities tend to be high and so we expect that the RPA values for ϵ and $\Delta\alpha$ will adequately describe the responsive properties of the electrons in the ferromagnet. We now calculate $\Delta\alpha(\vec{q}, \omega)$ in the RPA. The calculation of $\epsilon(\vec{q}, \omega)$ is well known.[9] The polarizability for spin-up electrons is given by

$$\alpha_\uparrow(\vec{q}, \omega) = -\frac{e^2}{\hbar q^2} \int \frac{d^3k}{(2\pi)^3} \left(\frac{\Theta(|\vec{k}+\vec{q}| - k_F^\uparrow)\Theta(k_F^\uparrow - k)}{\omega + \omega_{\vec{k}} - \omega_{\vec{k}+\vec{q}} + i\eta} - \frac{\Theta(k_F^\uparrow - |\vec{k}+\vec{q}|)\Theta(k - k_F^\uparrow)}{\omega + \omega_{\vec{k}} - \omega_{\vec{k}+\vec{q}} - i\eta} \right) \quad , \tag{16}$$

where we have $\Theta(x) = 1$ for $x > 0$ and 0 for $x < 0$, and $\omega_{\vec{k}} \equiv \hbar k^2/2m$; η is a positive infinitesimal which will be set to zero at the end of the calculation. A similar expression holds for $\alpha_\downarrow(\vec{q}, \omega)$, where $k_F^\uparrow$ is replaced by $k_F^\downarrow$. Here $k_F^\downarrow$ is slightly greater than $k_F^\uparrow$ resulting in a net spin polarization in the negative x direction (corresponding to a nonzero magnetization density M_0 in the positive x direction). These relations for $\alpha_{\downarrow,\uparrow}(\vec{q}, \omega)$ are simply related to the lowest-order polarization insertion—counting only particle-hole excitations above the Fermi sea. Since the expression (14) for $\delta B(0)$ contains $\Delta\alpha$, the difference between down and up spin polarizabilities, it is clear that only particle-hole excitations which originate from states between the two Fermi spheres contribute to $\delta B(0)$—, i.e., only the fraction of electrons which contribute to the equilibrium magnetization are involved.

After a simple transformation of the second term in Eq. (16) is performed, we obtain for $\Delta\alpha$

$$\Delta\alpha(\vec{q}, \omega) = -\frac{e^2}{\hbar q^2} \int \frac{d^3k}{(2\pi)^3} [\Theta(k_F^\downarrow - k)\Theta(|\vec{k}+\vec{q}| - k_F^\downarrow) - \Theta(k_F^\uparrow - k)\Theta(|\vec{k}+\vec{q}| - k_F^\uparrow)] \times \left(\frac{1}{\omega - \omega_{\vec{k}\vec{q}} + i\eta} - \frac{1}{\omega - \omega_{\vec{k}\vec{q}} - i\eta} \right) \quad , \tag{17}$$

where $\omega_{\vec{k}\vec{q}} \equiv \omega_{\vec{k}+\vec{q}} - \omega_{\vec{k}}$. If we assume we have $k_F^{\downarrow} - k_F^{\uparrow} \equiv \delta k << k_F$, we can replace the difference between Θ functions in Eq. (17) by the product of δk with $\delta(k - k_F)$. This simplifies the integrals considerably—the results of which are given in dimensionless variables

$$\mathrm{Re}\Delta\alpha(z,u) = \frac{\chi^2}{32\pi}\frac{\delta k}{k_F}\frac{1}{z^3}\ln\left|\frac{u^2-(z+1)^2}{u^2-(z-1)^2}\right| , \qquad \mathrm{Im}\Delta\alpha(z,u) = \begin{cases} \frac{\chi^2}{64}\frac{\delta k}{k_F}\frac{1}{z^3}[(u-z)^2-1] & \text{for} |1-z| \le u \le 1+z \\ -\frac{\chi^2}{16}\frac{\delta k}{k_F} & \text{for } 0 \le u \le 1-z \\ 0 & \text{otherwise} \end{cases}$$

where $z = k/2k_F$, $u = m\omega/\hbar k_F k$, and $\chi^2 = e^2 k_F / 2\pi\epsilon_F$ with ϵ_F equal to the Fermi energy; χ^2 is a measure of the ratio between potential and kinetic energy of the electrons. The RPA expression for $\epsilon(\vec{k},\omega)$ given by Lindhardt[10] is

$$\epsilon(z,u) = 1 + \chi^2/z^2[f_1(z,u) + if_2(z,u)] ,$$

where

$$f_1(z,u) = \frac{1}{2} + \frac{1}{8z}[1-(z-u)^2]\ln\left|\frac{z-u+1}{z-u-1}\right| + \frac{1}{8z}[1-(z+u)^2]\ln\left|\frac{z+u+1}{z+u-1}\right|$$

and

$$f_2(z,u) = \begin{cases} \frac{1}{2}\pi u & \text{for } z+u \le 1 \\ \frac{\pi}{8z}[1-(z-u)^2] & \text{for } |z-u| \le 1 \le z+u \\ 0 & \text{otherwise} \end{cases} ,$$

where χ^2, z, and u are defined above. One can now change variables in expression (14) to the convenient dimensionless variables to obtain

$$\frac{\delta B}{4\pi M_0} = \frac{128\pi}{6\beta} Z \frac{k_F}{\delta k}\int_0^\infty z^2 dz \int_0^\beta du \frac{\Delta\alpha(z,u)}{\epsilon(z,u)}[1 + \tfrac{5}{6}P_2(u/\beta)] , \tag{18}$$

where $\beta \equiv v/v_F$; v_F is the Fermi velocity. Because the retarded ϵ and $\Delta\alpha$ entered in Eq. (18), the expression is real and can be rewritten

$$\frac{\delta B}{4\pi M_0} = \frac{128\pi}{6\beta} Z \frac{k_F}{\delta k}\int_0^\infty z^2\, dz \int_0^\beta du \frac{\Delta\alpha_1(z,u)\epsilon_1(z,u) + \Delta\alpha_2(z,u)\epsilon_2(z,u)}{\epsilon_1^2 + \epsilon_2^2}[1 + \tfrac{5}{6}P_2(u/\beta)] , \tag{19}$$

where we have

$$\epsilon(z,u) \equiv \epsilon_1(z,u) + i\epsilon_2(z,u)$$

and

$$\Delta\alpha(z,u) = \Delta\alpha_1(z,u) + i\Delta\alpha_2(z,u) .$$

Since an analytic evaluation of Eq. (19) was not possible, numerical calculations were performed.

IV. RESULTS

The numerical evaluation of Eq. (19) was carried out for $\chi = 0.594$ (a value suitable for Fe), $Z = 1$ and for different values of β corresponding to values of v ranging from $v = 5v_F$ to $v = 100v_F$. An asymptotic expression for the integrand in Eq. (19) was obtained for $z >> u$ in order that an anayltic result could be obtained for z ranging from z_0 to ∞. Here z_0 is a value of z, usually four or five times larger than β, which rendered the analytic result accuate to at least 1%. Similarly, the numerical integrations, which were performed with the use of Simpson's Rule, were carried to 1% accuracy. Particular care had to be taken in the numerical work near the region of the plasmon dispersion curve defined by $\epsilon(z,u) = 0$, where the integrand in Eq. (18) has a singularity. The results are plotted in Fig. 2 where we show the ratio of the numerical prediction to the classical one [see Eq. 15] versus particle velocity v/v_0, i.e., we define a coefficient C by

$$(\delta B)_{\mathrm{QM}} = C(\delta B)_{\mathrm{class.}} ,$$

and plot C as a function of v/v_0.

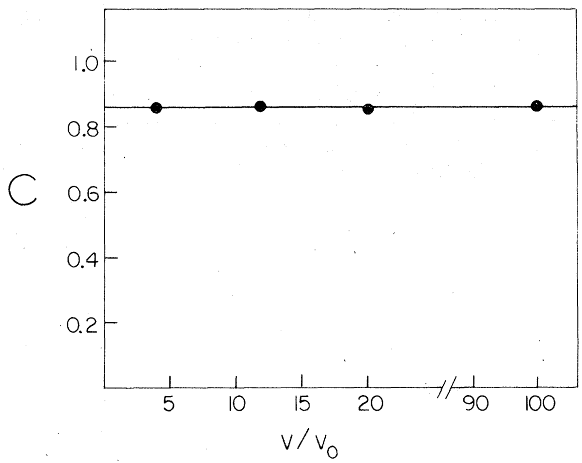

FIG. 2. Plot of C as a function of v/v_0.

V. CONCLUSIONS

We have calculated the dynamic magnetic field at the site of the ion by considering the effect of single-particle collisions and collective oscillations of the electron gas on the dynamic magetization density. From this magnetization density, we then obtained δB by summing the contributions from all over the ferromagnet [expression (19)]. Our numerical results show a reduction in δB from the results of the classical calculation as given by expression (15). Our final result is

$$\delta B(0) = (0.86 \pm 0.01) 4\pi M_0 Z (v_0/v) \quad . \tag{20}$$

The velocity dependence of the dynamic magnetic field is similar to the LW result for $v > v_p$, however, it would be incorrect to extend their results into this velocity region since their calculation of the field at the site of the ion was determined by the magnetization density at the site of the ion—neglecting entirely the anisotropic parts of δM which will contribute to $\delta B(0)$.

Since much of the work described in this paper was dictated by our choice of dielectric function in Eq. (14), one could ask how dependent our results are on this particular choice. As mentioned above, our use of the classical Drude dielectric function in an earlier paper[8] proved adequate only when an otherwise divergent integral was cut off by hand; certainly then, a quantum-mechanically correct dielectric function must be used. On the other hand, we do not expect our results to depend strongly on our choice for $\epsilon(\vec{q}, \omega)$ from among the various quantum-mechanically respectable dielectric functions, since in a recent calculation[11] of the dynamic field we obtained $2\pi^2 M_0 Z v_0/v$. In this calculation we assumed the electrons to be free and independent—responding only to the presence of the ion. The effects of plasmons are absent from such a theory. Nevertheless the result differs from the numerical results above by a factor of about 2 and the functional dependence on parameters Z and v is the same. There is also experimental evidence[3] that the dynamic field is not sensitive to the details of the band structure of the ferromagnet. The measurement on Fe ions traversing Fe and Gd ferromagnets are the same once the magnetization M_0 is factored out, while the band structures of Fe and Gd are quite different.

Data from an experiment recently carried out by the Rutgers-Bell group and collaborators[12] can be compared with the predictions of Eq. (20). In the experiment, the dynamic field was determined, which acted on muons traveling at $v \approx 0.68C$ through magnetized iron. The field was determined to be -0.5 ± 2.6 kG which is consistent with the predicted field of ~ 0.14 kG given by Eq. (20), where $4\pi M_0$ is taken to be about 16 kG and $v/v_0 \approx 100$.

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