# Effects of anisotropy on ultrasonic cyclotron resonance

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A general expression for the wavelength-dependent magnetoconductivity, valid for arbitrary $\vec{k}$-dependent electron energy and relaxation time, is given. The effects of a small cubic anisotropy in the relaxation time $1/\tau = (1/\tau_0)(1 + w K_4)$ and in the energy band $E = ak^2 - bk^4 - \eta k^4 K_4$, where $K_4$ is the Kubic harmonic of the fourth order, on the transverse-acoustical Doppler-shifted cyclotron resonance are studied. The acoustical attenuation is found to be dependent on a strictly anisotropic contribution term $L_{\pm}$, in addition to the usual dependence on the generalized conductivity $G_{\pm}$, which contain implicit dependence of anisotropy. Special attention is paid to the anisotropic effects on the absorption edge. The relaxation-time anisotropy on the magneto-acoustical dispersion is significant only in the domain where $ql$ is small. An expression for the helicon-phonon dispersion relation is obtained. A graphical presentation of the results of numerical computations for the sound attenuation and dispersion versus magnetic field in potassium is given.

## I. INTRODUCTION

The attenuation of ultrasonic waves in metals increases with decreasing temperature. At high temperature, dislocations are the dominant cause, while at low temperatures interaction with conduction electrons may dominate. The mechanical motion of the lattice produces an internal electric field, which drives conduction electrons into collective motion. This electron current tends to nullify the very cause producing it. Due to the short but finite response time of the conduction electrons, a small residual electric field is necessary to maintain the quasibalance of currents. That part of the self-consistent interaction represented by this electric field, therefore, enters into the picture of irreversible energy transfer from the acoustical wave to the electron system, as is the case for instance in ultrasonic cyclotron resonance. At the superconducting temperature, the attenuation decreases suddenly if the metal undergoes transition to superconducting state, and otherwise the attenuation continues to increase. The magnetic field dependence of the attenuation of ultrasonic and electromagnetic waves in metals at low temperature is valuable in studying the Fermi surface. Many papers have been published along this line or on closely related subjects.[1-10] However, a free-electron model, or an isotropic energy band, and a constant relaxation time were adopted in most of the explicit theoretical treatments. The effects of anisotropy on cyclotron resonance have been of some interest recently.[11,12] In the present paper we examine the anisotropic effects of the energy band and the relaxation time on the ultrasonic cyclotron resonance. The attenuation of the shear ultrasonic wave, for an isotropic energy band and a constant relaxation time, is given by

$$\alpha(q, \omega, H) \propto \mathrm{Re}[(1/G) - 1], \tag{1}$$

where $G = \sigma/\sigma_0$, $\sigma$ is a Fourier component of the conductivity tensor, $\sigma_0$ is the dc zero-magnetic-field conductivity, $H$ is the applied static magnetic field, and $q$ and $\omega$ are the wave number and the frequency of the acoustical wave. In the present work, we take a system of normal nonmagnetic cubic metal at low temperature. A shear ultrasonic wave is propagating along the [001] direction, and a static magnetic field is applied along the direction of wave propagation. This geometry is appropriate for the case of Doppler-shifted cyclotron resonance and helicons. Throughout the work we assume (i) a magnetic-field-independent relaxation time, (ii) an effective single-band Hamiltonian based on Bloch scheme, and (iii) closed cyclotron orbits on the Fermi surface. We take the classical linear approach. Acoustical and electric fields are treated as classical vectors. The conduction electrons are handled by a linearized Boltzmann equation.

We assume the relaxation time in the form

$$1/\tau(\vec{k}) = (1/\tau_0)(1 + wK_4), \tag{2}$$

where $1/\tau_0$ is a constant, $K_4$ is the Kubic harmonic of the fourth order

$$K_4 = (1/k^4)(k_x^4 + k_y^4 + k_z^4) - \tfrac{3}{5} \tag{3}$$
$$= \tfrac{1}{4}(3 + \cos 4\varphi)\sin^4\theta + \cos^4\theta - \tfrac{3}{5}$$

and $w$ is a small parameter. We take the effective Hamiltonian[13] derived by WKB method in momentum representation

$$H = E(\vec{k}) + H^{(s)}$$
$$= E(\vec{k}) + \left(\frac{\partial}{\partial t} + \frac{d\vec{r}}{dt}\cdot\nabla\right)\left(\hbar\vec{k} - m\frac{d\vec{r}}{dt}\right)\cdot\vec{u} - i\sum_{i,j}\frac{\partial u_j}{\partial r_i}C_{ij}, \tag{4}$$

 

where $\hbar\vec{k}$ is $\hbar\vec{K}+(e/c)(\vec{A}+\vec{a})$, $\hbar\vec{K}$ is the crystal momentum, and $-e$ is the charge of the electron, and $\vec{u}$ is the lattice displacement vector. The deformation potential $C_{ij}$ is

$$C_{ij}=\int_\Omega d^3r\,|u_{\vec{k}}|^2 r_i\left(\frac{\partial V}{\partial R_0^j}\right)_{\vec{R}_{\vec{g}}=0} - \frac{i}{2}\int_\Omega d^3r\left(\frac{\partial u_{\vec{k}}^*}{\partial k_i}u_{\vec{k}}-u_{\vec{k}}^*\frac{\partial u_{\vec{k}}}{\partial k_i}\right)\frac{\partial V_0}{\partial r_j}\,, \tag{5}$$

where $V_0$ is the unperturbed periodic potential, $\vec{R}_{\vec{g}}$ is the $\vec{g}$th lattice displacement, $u_{\vec{k}}$ is the periodic part of Bloch-like wave function, and integrations are over a unit cell. In the present study we disregard the deformation-potential term and assume that the ion current, the current associated with the lattice motion, is fully represented by $\vec{J}=ne\times\partial\vec{u}/\partial t$ for a monovalent metal. We take the unperturbed energy band of cubic symmetry

$$E(\vec{k})=ak^2-bk^4-\eta k^4K_4 \tag{6}$$

in atomic units, and $b$ and $\eta$ are the small parameters. For the free electron model $a=1$ and $b=\eta=0$, and $a=m/m^*$ and $b=\eta=0$ for a band where $E=\hbar^2k^2/2m^*$.

In Sec. II, we give the constitutive equation. A general expression of the conductivity for the case of cubic relaxation time and an isotropic energy band based on Chamber's method is discussed. We found it more convenient to use the Jones-Zener method of iteration,[14] for a system of anisotropic energy band, when the geometry is $\vec{q}\parallel\vec{H}$. In Sec. III, the acoustical attenuation and dispersion are discussed.

## II. CONSTITUTIVE EQUATION

The constitutive equation is obtained by solving the linearized Boltzmann equation for the band and the relaxation time discussed in Sec. I.

In the presence of an acoustical excitation the Boltzmann equation

$$\frac{\partial f}{\partial t}+\frac{d\vec{r}}{dt}\cdot\nabla f+\frac{d\vec{k}}{dt}\cdot\nabla_{\vec{k}}f=\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} \tag{7}$$

should be modified in accordance with the collision-drag effect[15,16] in the relaxation-time approximation,

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}=-\frac{1}{\tau}\left[f-f^{(0)}\left(H-\vec{k}\cdot\frac{\partial\vec{u}}{\partial t}\right)\right], \tag{8}$$

where the effective Hamiltonian $H$ is given in Eq. (4). The perturbation term $H^{(s)}$ represents a real-metal effect of the sound wave and vanishes for the free-electron model. The electron conservation during collision is expressed by $\int d^3k(\partial f/\partial t)_{\text{coll}}=0$. The electron velocity is

$$\frac{d\vec{r}}{dt}=\nabla_{\vec{k}}H=\vec{v}+\nabla_{\vec{k}}H^{(s)}. \tag{9}$$

The "momentum" change contains an extra term

$$\frac{d\vec{k}}{dt}=-e\left(\vec{E}+\frac{d\vec{r}}{dt}\times\vec{H}\right)-\nabla H^{(s)}, \tag{10}$$

neglecting the magnetic field associated with the internal electric field. The extra term, $\nabla H^{(s)}$, is due to the presence of an acoustical wave.

We choose $\delta f$ to represent the deviation from the instantaneous equilibrium under the influence of the sound wave

$$\delta f=f^{(1)}=f-f^{(0)}(H)\,. \tag{11}$$

With this choice the linearized equation is

$$\left(\frac{1}{\tau}+i(\vec{q}\cdot\vec{v}-\omega)-\frac{e}{c}(\vec{v}\times\vec{H})\cdot\nabla_{\vec{k}}\right)f^{(1)} = \frac{\partial f^{(0)}}{\partial E}\left(e\vec{E}\cdot\vec{v}+i\omega H^{(s)}-\frac{1}{\tau}\vec{k}\cdot\frac{\partial\vec{u}}{\partial t}\right). \tag{12}$$

A different choice, such as $f=f^{(0)}(E)+f^{(1)}$, in place of Eq. (11) with corresponding change in the linearized equation, will equally well describe the problem. Alternatively one might have introduced a strain dependent momentum as $(\vec{I}+i\vec{q}\vec{u})\cdot[\vec{K}+(e/c)\vec{A}+(e/c)\vec{a}]$ from the onset with corresponding $H^{(s)}$ instead of the vector $\vec{k}$ defined as in the present paper.

The solution of the above Eq. (12) can compactly be written in Chamber's form as

$$f^{(1)}=\frac{\partial f^{(0)}}{\partial E}\int_{-\infty}^{t}dt'\left(e\vec{v}'\cdot\vec{E}-i(\vec{q}\cdot\vec{v}'-\omega)(\vec{k}'-m\vec{v}')\cdot\frac{\partial\vec{u}}{\partial t}-\frac{1}{\tau'}\vec{k}'\cdot\frac{\partial\vec{u}}{\partial t}\right)\exp\left[-\int_{t'}^{t}dt''\left(\frac{1}{\tau''}+i\vec{q}\cdot\vec{v}''-i\omega\right)\right] \tag{13}$$

because the spatial trajectory of the electron is unaffected by the electric field and by the acoustical field in the linear approximation.

Following standard procedures[10] one can write, in the absence of an acoustical wave, the conductivity

$$\vec{\sigma}=\frac{e^3H}{4\pi^3\hbar^2c}\int dk_z\int_0^T dt\,\vec{v}(t)\int_{-\infty}^{t}dt'\exp\left(-\int_{t'}^{t}dt''\,\phi(t'')\right)\vec{v}(t'), \tag{14}$$

where $\phi$ is introduced as $\phi = 1/\tau + i(\vec{q}\cdot\vec{v} - \omega)$ and the cyclotron period $T(E, k_z)$ is $T = 2\pi/\omega_c = (\hbar^2 c/eH)(\partial A/\partial E)_{k_z}$. By separating the aperiodic part from $\phi$, one can write $\phi = \langle\phi\rangle + (\phi - \langle\phi\rangle)$ where $\langle\phi\rangle = (1/T)\int_0^T dt\,\phi(t)$ is the average over a cyclotron period. Then the factor $\vec{v}(t)\vec{v}(t')\exp[-\int_t^{t'} dt''(\phi'' - \langle\phi\rangle)]$ is periodic in both $t$ and $t'$ with the common period $T$. Performing a double Fourier expansion of this factor, we obtain

$$\vec{\sigma} = \frac{e^2}{2\pi^2\hbar^2}\int dk_z\, m_c^* \sum_{l=-\infty}^{\infty} \frac{\vec{A}_l}{\langle\phi\rangle - il\omega_c}\,, \tag{15}$$

where

$$\vec{A}_l = \frac{1}{T^2}\int_0^T dt \int_0^T dt'\,\vec{v}(t)\vec{v}(t') \times \exp\left(-il\omega_c(t-t') - \int_{t'}^{t} dt''(\phi'' - \langle\phi\rangle)\right).$$

This expression is valid for arbitrary $\vec{k}$-dependent energy band and relaxation time. The explicit evaluation of $\vec{A}$ for the anisotropic relaxation time of Eq. (2), and an isotropic energy band has been made, with the result

$$(A_\pm)_l = \frac{v_\perp^2}{2}\sum_{\alpha=-\infty}^{\infty}\left\{J_\alpha\left[iw\left(\frac{k_\perp}{2k}\right)^4\frac{1}{\omega_c\tau_0}\right]\right\}^2 \delta_{l,4\alpha\pm1} \tag{16}$$

which gives

$$G_\pm = \frac{3}{4}\int_{-1}^{1} dx(1-x^2) \times \sum_{n=-\infty}^{\infty}\frac{\{J_n[i(w/16\omega_c\tau_0)(1-x^2)^2]\}^2}{1 + w\bar{K}_4 + iqv_F\tau_0 x - i\omega_c\tau_0 \mp i(4n+1)\omega_c\tau_0}\,, \tag{17}$$

where

$$A_\pm = A_{11} \mp iA_{12}\,,$$

$$\bar{K}_4 = \frac{1}{2\pi}\int_0^{2\pi} d\varphi K_4(\theta,\varphi) = \tfrac{4}{5}P_4(\cos\theta)\,,$$

with $\cos\theta = x$, and $J_n$ is the Bessel function of $n$th order. The dc conductivity $\sigma_0$ is independent of $w$. When $w = 0$, $J_{\pm n}(0) = \delta_{n,0}$, and the usual expression is obtained. In the high-field limit one may take only the $n = 0$ term, and it shows the relaxation time averaged over the cyclotron period enters into the conductivity. However, this may not be the case for a weak magnetic field. Besides the $w\bar{K}_4$ in the denominator of the integrand of Eq. (17), we note the first contributing term is proportional to $w^2$.

To treat the case of anisotropic relaxation time and anisotropic energy band, as given in Eqs. (2) and (6), we introduce the Fermi momentum and the Fermi velocity by

$$k_F = \left(\frac{1}{4\pi}\oint d\Omega\, k(E,\Omega)\right)_{E=E_F} \tag{18}$$

and

$$v_F = \left(\frac{1}{4\pi}\oint d\Omega\, v(E,\Omega)\right)_{E=E_F}\,, \tag{19}$$

where $E_F$ is the Fermi energy and $d\Omega$ is the element of solid angle in $\vec{k}$ space. Thus the electron density is $n = k_F^3/3\pi^2$ and $v_F/k_F = (a/m)[1 - (2b/a)k_F^2]$, for small anisotropy. The cyclotron frequency is

$$\omega_c = (eHv_F/ck_F)[1 + (3\eta/2a)k_F^2(\cos^2\theta - 1)] \tag{20}$$

and the cyclotron mass is

$$m_c^* = \frac{m}{a}\left(1 + \frac{2bE}{a^2} + \frac{3\eta E}{10a^2} - \frac{3\eta E}{2a}k_z^2\right). \tag{21}$$

When the band is anisotropic and $\vec{q}$ is parallel to $\vec{H}$, as in the present circumstance, we found that the direct calculation of the electron current using the solution given in Eq. (13) is less convenient than solving the linearized equation [Eq. (12)] by the Jones-Zener method. By writing $f^{(1)} = \frac{1}{2}(\partial f^{(0)}/\partial E)(\chi_+ + \chi_-)$, we found

$$\begin{aligned}\chi_\pm = {} & \left\{\frac{eE_\pm}{m^*} - \left[\frac{1}{\tau_0}(1 + w\bar{K}_4) + i(qv_z - \omega)\left(1 - \frac{m}{m_c^*}\right)\right]\frac{\partial u_\pm}{\partial t}\right\} \\ & \times\left\{\frac{k_\mp}{\langle\phi\rangle \mp i\omega_c} \mp i\frac{w}{32\omega_c\tau_0 k^4}\left[k_\pm^4 k_\mp\left(\frac{-1}{\langle\phi\rangle \mp i\omega_c} + \frac{1}{\langle\phi\rangle \pm 3i\omega_c}\right) + k_\mp^5\left(\frac{1}{\langle\phi\rangle \mp i\omega_c} - \frac{1}{\langle\phi\rangle \mp 5i\omega_c}\right)\right]\right\} \\ & - \eta\frac{eE_\pm k_\pm^3}{8m}\left(\frac{1}{\langle\phi\rangle \mp i\omega_c} + \frac{3}{\langle\phi\rangle \pm 3i\omega_c}\right) \\ & - \frac{w}{8\tau_0}\frac{\partial u_\pm}{\partial t}\frac{1}{k^4}\left(\frac{k_\pm^4 k_\mp}{\langle\phi\rangle \pm 3i\omega_c} + \frac{k_\mp^5}{\langle\phi\rangle \mp 5i\omega_c}\right) \\ & - \frac{i\eta}{8}\frac{\partial u_\pm}{\partial t}(qv_z - \omega)k_\pm^3\left(\frac{1 - m_c^*/m}{\langle\phi\rangle \mp i\omega_c} + \frac{3 + m_c^*/m}{\langle\phi\rangle \pm 3i\omega_c}\right).\end{aligned} \tag{22}$$

In this equation $\langle\phi\rangle = (1/\tau_0)(1+w\bar{K}_4)+i(qv_z-\omega)$ and $E_\pm = E_1 \pm iE_2$, etc. With this solution, we calculate the electron current $\vec{J} = -(e/4\pi^3)\int d^3k(d\vec{r}/dt)f$ where $d\vec{r}/dt$ is as given in Eq. (9). The nonvanishing contribution is $\vec{J} = -(e/4\pi^3)\int d^3k\,\vec{v}f^{(1)}$.

By writing

$$j_\pm = \sigma_\pm E_\pm - \Lambda_\pm J_\pm , \tag{23}$$

we obtain

$$\sigma_+ = ne^2\frac{3}{4}\int_0^\pi d\theta\sin\theta\frac{\langle(v_+v_-/v^2)(v_F/k)\rangle_{av}}{\langle 1/\tau\rangle_{av}+iq\bar{v}_z-i\omega-i\omega_c}$$

$$= \sigma_0\frac{3}{4}\int_0^\pi d\theta\sin^3\theta\frac{1+(\eta k/a)(3\cos^2\theta-\frac{3}{5}+\frac{7}{2}\bar{K}_4)}{1+w\bar{K}_4+i(q\bar{v}_z-\omega-\omega_c)\tau_0} . \tag{24}$$

In this equation, $\sigma_0 = ne^2\tau_0 v_F/k_F$ is the dc zero-field conductivity. The bars and $\langle\ \rangle_{av}$ indicate the azimuthal average at the Fermi surface, for instance

$$\bar{v}_z = \left(\frac{1}{2\pi}\int_0^{2\pi} d\varphi\, v_z(E,\Omega)\right)_{E=E_F}$$

$$= v_F\cos\theta\left(1+\frac{6\eta}{5a}k_F^2-\frac{2\eta}{a}k_F^2\cos^2\theta+\frac{\eta}{2a}k_F^2\bar{K}_4\right).$$

$\Lambda_+$ is given by

$$\Lambda_+ = 1-\frac{mv_F}{k_F}(1-L_+) + \frac{v_F}{k_F}\left[m+i\frac{eH}{c}\tau_0\left(1-\frac{mv_F}{k_F}\right)\right]G_+ , \tag{25}$$

where $G_+ = \sigma_+/\sigma_0$ and

$$L_+ = \frac{3}{4}\int_0^\pi d\theta\sin^3\theta \times\frac{w\bar{K}_4-i\eta(eH/c)(v_Fk_F/a\tau_0)(\frac{3}{2}\cos^2\theta-\frac{3}{10})}{1+w\bar{K}_4+i(q\bar{v}_z-\omega-\omega_c)\tau_0} . \tag{26}$$

$G_-(H) = G_+(-H)$, $\Lambda_-(H) = \Lambda_+(-H)$, and $L_-(H) = L_+(-H)$. The $L_\pm$ vanishes for a constant relaxation time and an isotropic energy band, and it depends on both $w$ and $\eta$.

## III. ULTRASONIC CYCLOTRON RESONANCE

With the constitutive equation, Eq. (23)–(26), one needs to consider the sound wave equation

$$nM\left(\frac{\partial^2\vec{u}}{\partial t^2}-c_0^2\frac{\partial^2\vec{u}}{\partial z^2}\right) = \vec{F}(\vec{E},\vec{j},\vec{J}) , \tag{27}$$

where $nM$ is the density of metal and $M$ is the mass of an ion. $\vec{F}$ is the force density acting on the ions, that is the force acting on the ions per unit volume of metal. This equation can be rewritten

$$(c_0^2q^2-\omega^2)J_\pm = -i(e\omega/M)F_\pm .$$

We take $\vec{F}$ as the difference between the total Lorentz force acting on the metal and the force acting on the conduction electrons[17]

$$\vec{F} = \frac{1}{c}(\vec{j}+\vec{J})\times\vec{H}-\frac{d}{dt}\left(\frac{1}{4\pi^3}\int d^3k\,m\frac{d\vec{r}}{dt}f\right) . \tag{28}$$

The second term contains the free-electron mass and represents the rate of change of "true" momentum of the electrons. The calculation of this term is

$$\frac{d}{dt}\int d^3k\frac{d\vec{r}}{dt}f = \int d^3k\frac{d^2\vec{r}}{dt^2}f+\int d^3k\frac{d\vec{r}}{dt}\left(\frac{\partial f}{\partial t}\right)_{coll} ,$$

where $d^2\vec{r}/dt^2$ is the acceleration of the electron which is[16]

$$\frac{d^2\vec{r}}{dt^2} = \left(\frac{\partial}{\partial t}+\frac{d\vec{r}}{dt}\cdot\nabla+\frac{d\vec{k}}{dt}\cdot\nabla_{\vec{k}}\right)\frac{d\vec{r}}{dt} , \tag{29}$$

and $f = f^{(0)}(H)+f^{(1)} = f^{(0)}(E)+H^{(s)}\partial f^{(0)}/\partial E+f^{(1)}$.

Explicit evaluation gives

$$F_+ = -\left(i\frac{H}{c}+\frac{m}{e\tau_0}\right)(J_++J_+)+\frac{mv_F}{k_F}\left(ne-neL_++i\frac{H}{c}\sigma_+\right)E_+ + \frac{mv_F}{k_F}\left\{-i\frac{H}{c}\Lambda_+ +\left[\frac{m}{e\tau_0}+i\frac{H}{c}\left(1-\frac{mv_F}{k_F}\right)\right]L_+\right\}J_+$$

$$+i\frac{m}{e\omega}\left\{\omega^2\left(1-\frac{mv_F}{k_F}\right)+\frac{1}{5}q^2v_F^2\left[1-a+\frac{2}{7}k_F^2\left(\frac{4b}{a}+3b-\frac{6\eta}{5}\right)\right]\right\}J_+ . \tag{30}$$

Inverse Laplace transforms are used in evaluating the terms that contain $f^{(0)}(E)$ in the integrand. When Eq. (30) is substituted in Eq. (27), one obtains the acoustical dispersion relation and attenuation $\alpha_\pm$. In the domain where the quasibalance of current holds

$$q_\pm^2 = \frac{\omega^2}{c_s^2}\left[1+\frac{m}{M}\left(1-\frac{mv_F}{k_F}\right)\mp\frac{eH}{Mc\omega}\left(\frac{mv_F}{k_F}\right)^2 + i\frac{m}{M\omega\tau_0}\left(\frac{mv_F}{k_F}\right)\left(\frac{1}{G_\pm}(1-2L_\pm)-1\right)\right] \tag{31}$$

and

$$\alpha_{\pm}=\frac{m}{M}\left(\frac{mv_F}{k_F}\right)\frac{1}{c_s\tau_0}\,\mathrm{Re}\left(\frac{1}{G_{\pm}}(1-2L_{\pm})-1\right), \quad (32)$$

where

$$c_s^2=c_0^2-\frac{m}{5M}v_F^2\left[1-a+\frac{2}{7}k_F^2\left(\frac{4b}{a}+3b-\frac{6\eta}{5}\right)\right]$$

is the modified sound velocity and $eH/Mc$ is the ion cyclotron frequency. The attenuation is dependent on the term $L_{\pm}/G_{\pm}$, which is a strictly anisotropic contribution, in addition to the generalized conductivity dependence, as in Eq. (1). Of course, $G_{\pm}$ is affected by the anisotropy. For the case of an isotropic energy band and a constant relaxation time, one can reduce Eq. (32) to the results given in Ref. 18. However, these results differ from the result given in Ref. 19 since no "diffraction" force is explicitly included.

From Eq. (31) we obtain the fractional change of the velocity of the acoustical wave

$$\left(\frac{\delta c_s}{c_s}\right)_{\pm}=\frac{1}{2M\omega}\left(\frac{mv_F}{k_F}\right)\left[\pm\frac{eH}{c}\left(\frac{mv_F}{k_F}\right)+\frac{m}{\tau_0}\,\mathrm{Im}\left(\frac{1}{G_{\pm}}(1-2L_{\pm})-1\right)\right] \quad (33)$$

after neglecting the term $(m/M)(1-mv_F/k_F)$ compared to unity.

From the constitutive equation, the Maxwell equations and the acoustical equation, we obtain the helicon-phonon dispersion relation

$$\left(q^2-i\frac{4\pi\omega}{c^2}\sigma_0 G_-\right)\left\{q^2-\frac{\omega^2}{c_s^2}\left[1+\frac{m}{M}\left(1-\frac{mv_F}{k_F}\right)\right]-\left(\frac{mv_F}{c_s k_F}\right)^2\frac{eH\omega}{Mc}-i\frac{mv_F}{k_F}\left(\frac{m\omega}{Mc_s^2\tau_0}\right)\left(\frac{1}{G_-}(1-2L_-)-1\right)\right\}$$
$$=\frac{mv_F\omega}{k_F}\left(\frac{q}{c_s}\right)^2\left[1-L_--\left(1-i\frac{eH}{mc}\tau_0\right)\left(1-\frac{mv_F}{k_F}\right)G_-\right]\left[\frac{eH}{Mc}\left(1-\frac{mv_F}{k_F}\right)+i\frac{m}{M\tau_0}\left(1-\frac{1}{G_-}(1-L_-)\right)\right]. \quad (34)$$

In the above, all calculations from Eq. (18) are performed to the first order in the anisotropic parameters $w$ and $\eta$.

One notes that the imaginary part of the denominator of the integrand of $G_+$, $\Lambda_+$, and $L_+$ vanishes at $q\bar{v}_z-\omega-\omega_c=0$. This equation can be rewritten $\Gamma(x)-\gamma=0$, where $\gamma(H)=eH/qk_Fc+\omega/qv_F$ and $\Gamma(x)=x\left[1+(\eta/2a)k_F^2(3-7x^2+\bar{K}_4)\right]$. When $\Gamma(x)$ is a monotonically increasing function of $x$, as in the case of the alkali metals, it has its maximum value at $x=1$. In the region of magnetic field for which the equation $q\bar{v}_z-\omega-\omega_c=0$ has a solution, the only solution is that of $\Gamma(x)=\gamma$. The denominator of the integrands has a strong minimum at this point for the case of $ql\gg1$, where $l=v_F\tau_0$ is the electron mean free path. The absorption edge occurs at $\gamma(H_A)=1-9\eta k_F^2/5a$. The absorption edge is related to the Gaussian curvature $K$ of the Fermi surface at the point where its normal is parallel to the magnetic field by $1/\sqrt{K}=-(1/2\pi)(\partial A/\partial k_z)_{\mathrm{ext}}$. In the region where the electron mean free path is large compared to the sound wavelength, the attenuation and the dispersion are insensitive to the relaxation-time anisotropy. The effect of relaxation-time anisotropy is significant when the mean free path is comparable to the wavelength.

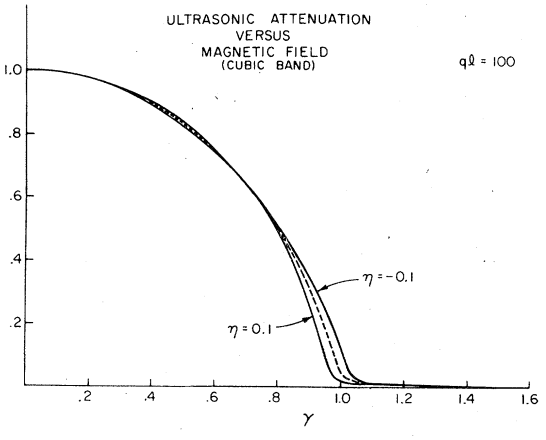


FIG. 1. Ultrasonic attenuation vs magnetic field (cubic band).

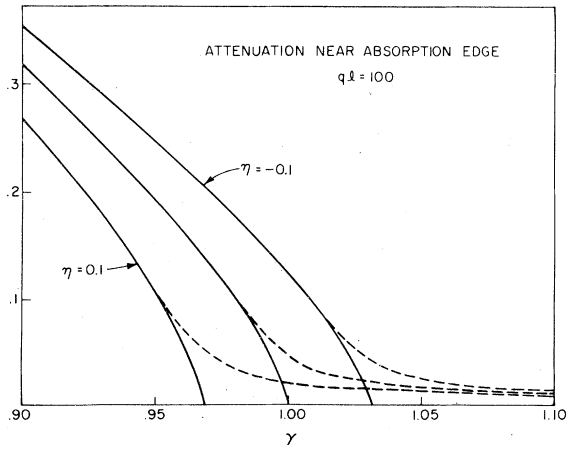


FIG. 2. Ultrasonic attenuation vs magnetic field near absorption edge (cubic band).

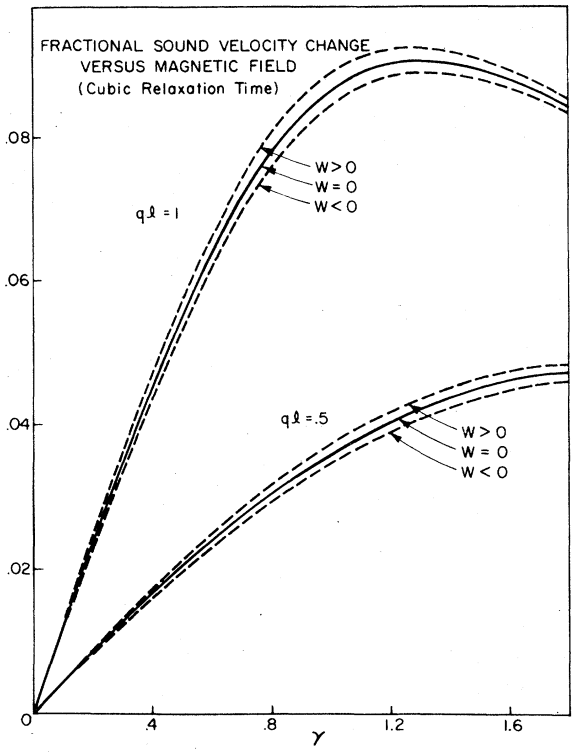


FIG. 3. Fractional sound velocity change vs magnetic field (cubic relaxation time).

By making measurements at very different values of frequency (i.e., $ql$) it should evidently be possible to separate the band and relaxation-time anisotropies.

The acoustical attenuation versus magnetic field below the absorption edge is graphed in Fig. 1 using potassium band parameters $a=1.16$, $b=0.85$, and $k_F=0.396$. The attenuation near the absorption edge is graphed in Fig. 2. The velocity change, [Eq. (33)], is shown in Fig. 3.

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