Polarization effects in the penetration of a barrier with a magnetic field applied; relativistic corrections

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The spin polarization of an electron moving through a potential barrier $V(z)$ with also a constant uniform magnetic field $(0,0,B)$ applied is calculated in such a way as to find the first relativistic corrections to the nonrelativistic result. The particle is described by the Dirac equation with Pauli anomalous-magnetic-moment term. The polarization is defined by the ratio of the first two components of the wave function. Approximations appropriate to a field-emission spin-polarization experiment are made and a WKB type of approximation for the z dependence of the wave function is used. The corrections are found to depend on a certain parameter β which is proportional to $(g-2)^{-1}$, the size of the electric field, and the square root of the area of the x-y projection of the orbit of the particle.

I. INTRODUCTION

It is established that the spin polarization of electrons field emitted from magnetic materials in the presence of an external magnetic field can be observed.[1-4] This raises the question: What happens to the polarization of the electron as it passes through the surface barrier in the presence of the externally applied magnetic field? This question was studied nonrelativistically by Schmit and Good[5]; they found that the electron jumps through the barrier with very little change of polarization.

The purpose of the present paper is to make a relativistic treatment of this problem. Although relativistic corrections are expected to be small it is worthwhile to have a quantitative estimate of them. Also the problem is perhaps of interest in itself, especially to see the effects of the anomalous moment and the strong field gradient at the surface.

As far as we know this problem has not been solved before. It was discussed by Regenfus[6] under the assumption that the Bargmann-Michel-Telegdi equations[7] apply even in the classically forbidden region inside the barrier. He was led to the conclusion that there would be a large change of polarization direction associated with the barrier penetration. We do not find confirmation of this assumption and our result is that the change of polarization is small, not much different than in the nonrelativistic case, and negligible under ordinary operating conditions in a field-emission experiment.

Since the calculations are lengthy they are described here only in sufficient detail to establish the notation and show the approximations made.[8]

II. HAMILTONIAN

The Dirac-Pauli Hamiltonian for the electron in potential V and magnetic field $\vec{B}$ is

$$H=\vec{\alpha}\cdot\vec{\pi}+\beta+V+\tfrac{1}{4}e\kappa\beta\vec{\sigma}\cdot\vec{B}-\tfrac{1}{4}ie\kappa\beta\vec{\alpha}\cdot\vec{E}\ , \qquad (1)$$

where $\vec{\pi}=\vec{p}+e\vec{A}$, $e\vec{E}=\vec{\nabla}V$, and $\kappa=g-2=2.32\times10^{-3}$. In terms of the 2×2 Pauli matrices $\vec{\sigma}$ the 4×4 matrices are

$$\vec{\alpha}=\begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix},\quad \beta=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \vec{\sigma}=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}\ .$$

The units are such that factors of m, c, and $\hbar$ do not appear. The symbol e indicates a positive number so the electron charge is $-e$. The particle eigenstates of H, with eigenvalues near unity, are of interest.

The potential is considered to be a function of z only, of the form indicated in Fig. 1. The magnetic field is taken to be of the form $(0,0,B)$ where B is a positive constant. (Results for negative B can be derived from those for positive B by a time-reversal consideration.) A convenient vector potential is $(-\tfrac{1}{2}By,\tfrac{1}{2}Bx,0)$. In this special case the Hamiltonian simplifies to

$$H=\begin{pmatrix}1+V+\tfrac{1}{4}e\kappa B\sigma_z & \vec{\sigma}\cdot\vec{\pi}-\tfrac{1}{4}ie\kappa E\sigma_z\\ \vec{\sigma}\cdot\vec{\pi}+\tfrac{1}{4}ie\kappa E\sigma_z & -1+V-\tfrac{1}{4}e\kappa B\sigma_z\end{pmatrix}\ . \qquad (2)$$

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III. INTEGRALS OF THE MOTION

The operators

$$x_0=\frac{1}{2}x-p_y/eB \ , \tag{3a}$$

$$y_0=\frac{1}{2}y+p_x/eB \ , \tag{3b}$$

$$J_z=xp_y-yp_x+\frac{1}{2}\sigma_z \tag{4}$$

commute with the Hamiltonian of Eq. (2). Classically x_0, y_0 are the coordinates of the center of the x-y projection of the orbit. The operator

$$R_0^2=x_0^2+y_0^2 \tag{5}$$

is introduced also in order to get to a separation of the variables in the problem. The point is that the operators H, R_0^2, J_z all commute with each other so one may consider functions that are simultaneously eigenstates of all three of them. In cylindrical coordinates (r,ϕ,z) the eigenvalue problems for R_0^2 and J_z determine the dependence of the wave function on r and ϕ. The problem of finding the eigenfunctions of H then reduces to a problem in ordinary differential equations for four functions of z only.

IV. EQUATIONS FOR z DEPENDENCE OF WAVE FUNCTION

The eigenfunctions of R_0^2 and J_z may be written

$$\psi=\begin{pmatrix} c_1\rho^{[m-(1/2)]/2} \ e^{-\rho/2} \ L_n^{[m-(1/2)]}(\rho) \ e^{i[m-(1/2)]\phi} \\ c_2\rho^{[m+(1/2)]/2} \ e^{-\rho/2} \ L_n^{[m+(1/2)]}(\rho) \ e^{i[m+(1/2)]\phi} \\ c_3\rho^{[m-(1/2)]/2} \ e^{-\rho/2} \ L_n^{[m-(1/2)]}(\rho) \ e^{i[m-(1/2)]\phi} \\ c_4\rho^{[m+(1/2)]/2} \ e^{-\rho/2} \ L_n^{[m+(1/2)]}(\rho) \ e^{i[m+(1/2)]\phi} \end{pmatrix} \tag{6}$$

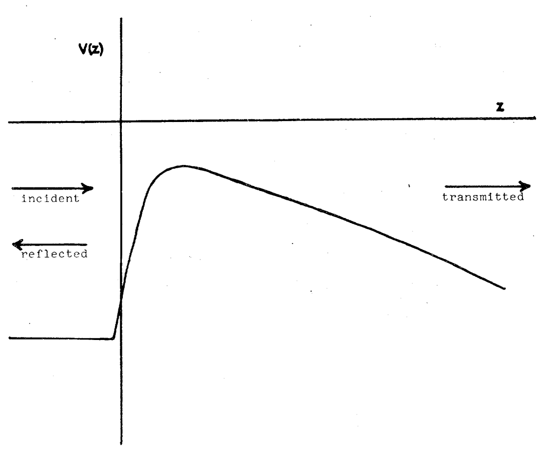

FIG. 1. Potential model for the surface of the material.

Here the c's are functions of z alone, ρ is $\frac{1}{2}eBr^2$, and $L_n^{(\alpha)}$ is the associated Laguerre polynomial in the notation of Magnus, Oberhettinger, and Soni.[9] As may be verified, these functions satisfy

$$R_0^2\psi=[(2n+1)/eB]\psi \ , \tag{7}$$

$$J_z\psi=m\psi \ , \tag{8}$$

where m and n are the quantum numbers for the states. The ranges of the quantum numbers are established from the requirement that the wave functions be integrable. If all the c's are not zero the allowed values are

$$n=0,1,2,\ldots, \tag{9a}$$

$$m=-n+\tfrac{1}{2},\ -n+\tfrac{3}{2},\ -n+\tfrac{5}{2},\ldots, \tag{9b}$$

and if c_1 and c_3 are zero but not c_2 and c_4 the allowed values are

$$n=0,1,2,\ldots, \tag{10a}$$

$$m=-n-\tfrac{1}{2} \ . \tag{10b}$$

The allowed values are displayed in Fig. 2.

With the r, ϕ dependence of the wave function thus established, the eigenvalue problem for the Hamiltonian

$$H\psi=W\psi \tag{11}$$

simplifies to a set of ordinary differential equations for the $c_i(z)$,

$$(1-W+V+\tfrac{1}{4}e\kappa B)c_1+\frac{1}{i}\frac{dc_3}{dz}-\tfrac{1}{4}ie\kappa Ec_3 -i(n+m+\tfrac{1}{2})(2eB)^{1/2}c_4=0 \ , \tag{12a}$$

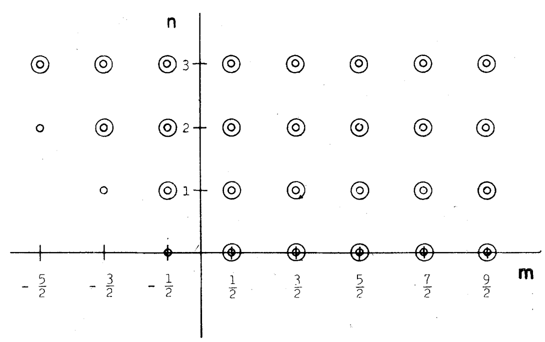

FIG. 2. Allowed values of the quantum numbers n, m for the eigenfunctions of R_0^2 and J_z, according to Eqs. (7) and (8). The double circles give the quantum numbers of eigenfunctions which have all four c_i in Eq. (6) not zero. The single circles are for functions which have c_1 and c_3 equal to zero, c_2 and c_4 not equal to zero.

$$(1-W+V-\tfrac{1}{4}e\kappa B)c_2+i(2eB)^{1/2}c_3-\frac{1}{i}\frac{dc_4}{dz}+\tfrac{1}{4}ie\kappa Ec_4=0\ , \quad (12b)$$

$$\frac{1}{i}\frac{dc_1}{dz}+\tfrac{1}{4}ie\kappa Ec_1-i(n+m+\tfrac{1}{2})(2eB)^{1/2}c_2+(-1-W+V-\tfrac{1}{4}e\kappa B)c_3=0\ , \quad (12c)$$

$$i(2eB)^{1/2}c_1-\frac{1}{i}\frac{dc_2}{dz}-\tfrac{1}{4}ie\kappa Ec_2+(-1-W+V+\tfrac{1}{4}e\kappa B)c_4=0\ . \quad (12d)$$

At this point the quantum numbers n and m only occur in the combination $(n+m+\frac{1}{2})$. This combination labels the eigenvalues of the operator

$$\tilde{A}=\pi[R_0^2+(2/eB)J_z] \quad (13)$$

according to

$$\tilde{A}\psi=(n+m+\tfrac{1}{2})(2\pi/eB)\psi\ . \quad (14)$$

The operator may also be expressed as

$$\tilde{A}=\pi[(x-x_0)^2+(y-y_0)^2]+(\pi/eB)\sigma_z \quad (15)$$

so it corresponds to the classical concept of the area of the x-y projection of the orbit with an additional spin contribution.

V. TWO-COMPONENT SYSTEM OF EQUATIONS

Further progress can be made by converting to a second-order system of equations. One can solve Eqs. (12c) and (12d) for c_3 and c_4 and substitute into Eqs. (12a) and (12b) to obtain equations involving only c_1 and c_2. Disregarding some of the smaller terms, as discussed in the next paragraph, one is led to this set of equations

$$\frac{d^2c_1}{dz^2}+\frac{eE}{1+W-V}\frac{dc_1}{dz}+\left[-1+(W-V)^2-\tfrac{1}{4}e\kappa\frac{dE}{dz}-(n+m+\tfrac{1}{2})2eB\right]c_1=\tfrac{1}{2}e\kappa Bc_1-\tfrac{1}{2}(\kappa+1)(n+m+\tfrac{1}{2})eE(2eB)^{1/2}c_2\ , \quad (16a)$$

$$\frac{d^2c_2}{dz^2}+\frac{eE}{1+W-V}\frac{dc_2}{dz}+\left[-1+(W-V)^2-\tfrac{1}{4}e\kappa\frac{dE}{dz}-(n+m+\tfrac{1}{2})2eB\right]c_2=-\tfrac{1}{2}(\kappa+1)eE(2eB)^{1/2}c_1-\tfrac{1}{2}e\kappa Bc_2\ . \quad (16b)$$

All quantities are expressed in terms of m, c, and $\hbar$. In a conceivable field-emission experiment: the kinetic energy would be about 5 eV so $1-W+V$ would be about 10^{-5}; the electric field, as estimated by the image field at the Fermi level, would be about 7×10^8 V/cm so eE would be about 5×10^{-8}; the magnetic field would be about 10^4 G so eB would be about 2×10^{-10}. To estimate the field gradient one considers the derivative of the image field $e/4x^2$ evaluated at the point where the work function ϕ equals the size of the image potential $e^2/4x$. This gives a field gradient of $32\phi^3/e^5$ and, for a work function of 5 eV, it makes $e\,dE/dz$ about 6×10^{-10}. The first approximation for the c's is that they vary as

$$\exp\left[i\int[2(1-W+V)]^{1/2}dz\right]$$

so d/dz applied to c_1 or c_2 is order of magnitude 4×10^{-3}.

In obtaining Eqs. (16) from Eqs. (12) terms 10^{-16} and larger were retained, terms 10^{-18} and smaller were discarded. The calculation is carried this far because then the effects of the field gradient and the coupling between the equations can be seen. In a lower degree of approximation, if the terms smaller than the $(n+m+\frac{1}{2})2eB$ terms (less than 4×10^{-10}) are disregarded, then the equations for c_1 and c_2 are uncoupled and each equation reduces to

$$\frac{d^2c}{dz^2}-2(1-W+V)c-(n+m+\tfrac{1}{2})2eBc=0\ .$$

This is the nonrelativistic limit of the problem, Eq. (12) of Ref. 5. The relativistic effects come in as the smaller terms are included in the calculation.

By changing the dependent variables according to

$$c_1=(1+W-V)^{1/2}(n+m+\tfrac{1}{2})^{1/2}f_1\ , \quad (17a)$$

$$c_2=(1+W-V)^{1/2}f_2\ , \quad (17b)$$

one obtains an equivalent pair of equations without first-derivative terms and with a symmetric coupling matrix on the right-hand side:

$$\frac{d^2f_1}{dz^2}+q^2(z)f_1=\alpha f_1+\alpha\beta(z)f_2\ , \quad (18a)$$

$$\frac{d^2f_2}{dz^2}+q^2(z)f_2=\alpha\beta(z)f_1-\alpha f_2\ , \quad (18b)$$

where q^2, α, and β are defined by

$$q^2(z) = -1 + (W-V)^2 - \tfrac{1}{4}e\kappa\frac{dE}{dz} - (n+m+\tfrac{1}{2})2eB - \frac{3}{4}\frac{e^2E^2}{(1+W-V)^2} - \frac{1}{2}\frac{e\,dE/dz}{1+W-V}\ , \tag{19}$$

$$\alpha = \tfrac{1}{2}e\kappa B\ , \tag{20}$$

$$\beta = -\frac{\kappa+1}{\kappa}(n+m+\tfrac{1}{2})^{1/2}eE\left(\frac{2}{eB}\right)^{1/2}\ . \tag{21}$$

The special case when $(n+m+\frac{1}{2})$ is zero is to be treated by discarding the f_1 term in Eq. (18b) and solving it for f_2 alone, then obtaining c_2 and c_4 from Eqs. (17b) and (12d) with $c_1=0$.

VI. SLOWLY-VARYING-WAVELENGTH APPROXIMATION

A WKB type of approximation can be developed for the $f(z)$ equations, along lines suggested by Pauli.[10]

The approximate solutions for waves propagating in the positive z direction are

$$f = p_\pm^{-1/2}\begin{pmatrix}\cos\theta_\pm \\ \sin\theta_\pm\end{pmatrix}\exp\left(i\int p_\pm\,dz\right)\ , \tag{22}$$

where

$$p_\pm = [q^2 \mp \alpha(1+\beta^2)^{1/2}]^{1/2}\ , \tag{23}$$

$$\cos\theta_\pm = [\tfrac{1}{2} \pm (1+\beta^2)^{-1/2}]^{1/2}\ , \tag{24a}$$

$$\sin\theta_\pm = \pm[\tfrac{1}{2} \mp \tfrac{1}{2}(1+\beta^2)^{-1/2}]^{1/2}\ . \tag{24b}$$

The two solutions correspond to the two possible spin states. For waves propagating in the negative z direction the opposite sign in the exponent is to be used, the rest of the solutions remaining the same. The solutions are written in a form appropriate for a classically allowed region, with $p_\pm$ having positive real values. In an unallowed region $p_\pm$ is a pure imaginary and the exponent will be $+\int |p_\pm|\,dz$ or $-\int |p_\pm|\,dz$.

A question is what to do about connecting the solutions across the zeroes of $p_\pm$. From a consideration of the uniform-electric-field case one concludes that the exponential factor connects in the same way as in the conventional WKB method, while the $\theta_\pm$ dependence remains the same on both sides of the zero.

One can now collect results and write complete formulas for a barrier transmission problem, as set up in Fig. 1. For definite values of the quantum numbers W, n, m and in terms of two arbitrary constants D_+ and D_-, the incident wave from the left is given by

$$c_1 = (1+W-V)^{1/2}\left(n+m+\tfrac{1}{2}\right)^{1/2}\left[D_+p_+^{-1/2}\cos\theta_+\exp\left(-i\int_z^{a_+}p_+(\zeta)\,d\zeta\right) + D_-p_-^{-1/2}\cos\theta_-\exp\left(-i\int_z^{a_-}p_-(\zeta)\,d\zeta\right)\right]\ , \tag{25a}$$

$$c_2 = (1+W-V)^{1/2}\left[D_+p_+^{-1/2}\sin\theta_+\exp\left(-i\int_z^{a_+}p_+(\zeta)\,d\zeta\right) + D_-p_-^{-1/2}\sin\theta_-\exp\left(-i\int_z^{a_-}p_-(\zeta)\,d\zeta\right)\right]\ , \tag{25b}$$

where $a_\pm$ are the smaller roots of $p_\pm$. Using the connection formulas on the exponential factors in the usual way, one finds that the transmitted wave on the right is correspondingly given by

$$c_1 = (1+W-V)^{1/2}\left(n+m+\tfrac{1}{2}\right)^{1/2}\left[D_+p_+^{-1/2}\cos\theta_+\exp\left(-\int_{a_+}^{b_+}|p_+|\,d\zeta\right)\exp\left(i\int_{b_+}^{z}p_+\,d\zeta\right) + D_-p_-^{-1/2}\cos\theta_- \times\exp\left(-\int_{a_-}^{b_-}|p_-|\,d\zeta\right)\exp\left(i\int_{b_-}^{z}p_-\,d\zeta\right)\right]\ , \tag{26a}$$

$$c_2 = (1+W-V)^{1/2}\left[D_+p_+^{-1/2}\sin\theta_+\exp\left(-\int_{a_+}^{b_+}|p_+|\,d\zeta\right)\exp\left(i\int_{b_+}^{z}p_+\,d\zeta\right) + D_-p_-^{-1/2}\sin\theta_- \times\exp\left(-\int_{a_-}^{b_-}|p_-|\,d\zeta\right)\exp\left(i\int_{b_-}^{z}p_-\,d\zeta\right)\right]\ , \tag{26b}$$

where $b_\pm$ are the larger roots of $p_\pm$. There will also be a reflected wave on the left which we disregard. The above results for the c's, along with Eqs. (12c) and (12d), are to be used in Eq. (6) to get the complete wave function.

VII. POLARIZATION EFFECTS

We define the polarization of the particle, in the presence of the external fields, to be given by the Pauli matrices $\vec{\sigma}$ acting within the space of the first two components of the wave function ψ_1 and ψ_2. This means that, at any point in space, the spherical polar angles (θ, ϕ) of the direction of polarization are given by

$$\tan\tfrac{1}{2}\theta\, e^{i\phi} = \psi_2/\psi_1 \ . \tag{27}$$

This definition agrees with the way polarization is defined for a free particle (this subject was reviewed by Fradkin and Good[11]). Also Sidhu and Good[12] have shown that the polarization so defined agrees, in the nonquantum limit, with the definition that leads to the Bargmann-Michel-Telegdi equations.

In principle Eqs. (25) and (26) solve the barrier transmission problem since they provide the connection across the barrier for a complete set of solutions. However to see what the solution implies about the polarization is not straightforward because the states with definite quantum numbers, W, n, m do not have a polarization which varies with z alone. Thus the ratio ψ_2/ψ_1 as given by Eq. (6) depends on ρ and ϕ as well as z. What is needed here is to get relativistic corrections to the nonrelativistic problem, in which the ρ, ϕ dependence is

$$\rho^{\bar{m}/2}e^{-\rho/2}L_n^{(\bar{m})}(\rho)\exp(i\bar{m}\phi)$$

for both components and there is a definite polarization depending on z alone. This nonrelativistic state is an eigenstate of the operator for the area of the x-y projection of the orbit,

$$\begin{aligned}\bar{A} &= \pi R_0^2 + (2\pi/eB)L_z \\ &= \pi[(x-x_0)^2+(y-y_0)^2] \ ,\end{aligned} \tag{28}$$

with eigenvalue $(n+\bar{m}+\frac{1}{2})(2\pi/eB)$.

For the incident wave in the region of constant potential β is zero and in that case one can build a relativistic wave with the same properties as the nonrelativistic wave. Thus when $\beta=0$, Eqs. (25) simplify to

$$c_1 = (1+W-V)^{1/2}(n+m+\tfrac{1}{2})^{1/2}D_+p_+^{-1/2} \times\exp\left[-i\int_z^{a_+}p_+\,d\zeta\right] \ ,$$

$$c_2 = -(1+W-V)^{1/2}D_-p_-^{-1/2}\exp\left[-i\int_z^{a_-}p_-\,d\zeta\right] \ .$$

If now one superposes a D_+ solution with quantum numbers, W, n, m and a D_- solution with quantum numbers, W, n, $m'=m-1$ then the wave function components in the $\beta=0$ region are

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = (1+W-V)^{1/2}\rho^{[m-(1/2)]/2}e^{-\rho/2}L_n^{[m-(1/2)]}e^{i[m-(1/2)]\phi}\begin{pmatrix}[n+m+\frac{1}{2}]^{1/2}D_+p_+^{-1/2}\exp\left[-i\int_z^{a_+}p_+\,d\zeta\right]\\ -D_-p_-'^{\,-1/2}\exp\left[-i\int_{z_-}^{a_-'}p_-'\,d\zeta\right]\end{pmatrix} \ , \tag{29}$$

where the prime on a symbol means it is to be evaluated at $m'=m-1$. These states, eigenstates of $\bar{A}$ with eigenvalue $(n+m)(2\pi/eB)$, form a complete set just as well as those that are eigenstates of $\bar{A}$ but these have a polarization that depends on z alone.

We will make expansions in the type of integral that occurs here in order to isolate the polarization effects and provide a ready comparison with the nonrelativistic problem. Let

$$\bar{p}^{\,2} = -1+(W-V)^2-2eB(n+m) \tag{30}$$

and let $\bar{a}$ and $\bar{b}$ be the roots of $\bar{p}^{\,2}$. In the integrals we expand for the case when the rest of the radicand is small compared to the $\bar{p}^{\,2}$ term. This leads to

$$\int_z^{a_+}p_+(\zeta)\,d\zeta = \int_z^{\bar{a}}\bar{p}(\zeta)\,d\zeta + \tfrac{1}{2}\int_z^{\bar{a}}\left[-\frac{1}{4}e\kappa\frac{dE}{d\zeta} - \frac{3}{4}\frac{e^2E^2}{(1+W-V)^2} - \frac{1}{2}\frac{2dE/d\zeta}{1+W-V}\right]\frac{d\zeta}{\bar{p}} - \tfrac{1}{2}\int_z^{\bar{a}}\frac{eB+\alpha(1+\beta^2)^{1/2}}{\bar{p}}\,d\zeta \ ,$$

$$\int_z^{a_-'}p_-'(\zeta)\,d\zeta = (\text{same first two terms}) + \tfrac{1}{2}\int_z^{\bar{a}}\frac{eB+\alpha(1+\beta'^2)^{1/2}}{\bar{p}}\,d\zeta \ .$$

We will disregard the difference between the factors $p_+^{-1/2}$ and $p_-'^{\,-1/2}$.

Applying these approximations to the incident wave of Eq. (29), one finds in the $\beta=0$ region

$$\frac{\psi_2}{\psi_1} = \frac{-D_-}{(n+m+\frac{1}{2})^{1/2}D_+}\times\exp\left[-i\tfrac{1}{2}\int_z^{\bar{a}}\frac{2eB+\alpha(1+\beta^2)^{1/2}+\alpha(1+\beta'^2)^{1/2}}{\bar{p}}\,d\zeta\right] \ . \tag{31}$$

We let

$$\frac{-D_-}{(n+m+\frac{1}{2})^{1/2}D_+}=\tan\tfrac{1}{2}\theta_0\,e^{i\phi_0}\ , \tag{32}$$

so then the polarization direction for the incident wave (θ_i, ϕ_i) is given by

$$\tan\tfrac{1}{2}\theta_i e^{i\phi_i}=\tan\tfrac{1}{2}\theta_0\exp i\left[\phi_0-eB\int_z^{\bar{a}}\frac{1+\frac{1}{4}\kappa(1+\beta^2)^{1/2}+\frac{1}{4}\kappa(1+\beta'^2)^{1/2}}{\bar{p}}\,d\zeta\right]\ . \tag{33}$$

The polarization precesses at constant angle $\theta_i=\theta_0$ to the z axis. The contributions to the integral from the $\beta\neq 0$ region near the turning point influence the phase of the precession but the exponent varies simply as $i\frac{1}{2}geB\int d\zeta/\bar{p}$ in the $\beta=0$ region.

For the transmitted wave there is not a $\beta=0$ region. However, away from the surface, β will be small in ordinary applications. Thus with a magnetic field of 10^4 G and an electric field of 5×10^7 V/cm, as might be found just outside a field-emission tip, the value of β is about $0.2(n+m+\frac{1}{2})^{1/2}$. The value decreases rapidly with distance from the tip. Accordingly we assume that, although β might be appreciable near the barrier, there is a region outside the tip where β is negligible and we evaluate the polarization there. Using the same ideas as for the incident wave, one finds for the polarization direction (θ_t, ϕ_t) of the transmitted wave

$$\tan\tfrac{1}{2}\theta_t e^{i\phi_t}=\tan\tfrac{1}{2}\theta_0\exp\left[eB\int_{\bar{a}}^{\bar{b}}\frac{1+\frac{1}{4}\kappa(1+\beta^2)^{1/2}+\frac{1}{4}\kappa(1+\beta'^2)^{1/2}}{|\bar{p}|}\,d\zeta\right]$$
$$\times\exp i\left[\phi_0+eB\int_{\bar{b}}^{z}\frac{1+\frac{1}{4}\kappa(1+\beta^2)^{1/2}+\frac{1}{4}\kappa(1+\beta'^2)^{1/2}}{\bar{p}}\,d\zeta\right]\ . \tag{34}$$

Equations (33) and (34) solve the problem of what happens to the polarization in the penetration of a barrier by a wave component. Equation (33) may be used to get the polarization incident far from the barrier and Eq. (34) to get the polarization transmitted far from the barrier.

VIII. DISCUSSION

It is interesting to compare the results with those of the nonrelativistic problem, Eqs. (23) and (24) of Ref. 5. If $\Delta\phi$ denotes the phase of the transmitted wave far from the barrier, $\phi_t(z\to\infty)$, minus the phase of the incident wave far from the barrier, $\phi_i(z\to-\infty)$, then

$$\Delta\phi_{\rm rel}-\Delta\phi_{\rm nonrel}=\tfrac{1}{4}\kappa eB\left(\int_{-\infty}^{\bar{a}}+\int_{\bar{b}}^{\infty}\right)\times\frac{-2+(1+\beta^2)^{1/2}+(1+\beta'^2)^{1/2}}{\bar{p}}\,d\zeta\ . \tag{35}$$

This formula provides a convenient way to test whether a relativistic treatment of the phase of the polarization is necessary. As is seen, it depends on the value of β in the classically allowed region. There is a similar relativistic correction in the relation between θ_i and θ_t, depending on β in the unallowed region.

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