Theory of Larmor waves

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A theory of Larmor waves (recently discovered by Janossy and Monod) is presented, which takes account of Fermi-liquid interactions (that can be measured by measuring the frequency of the Larmor-wave oscillations), Fermi-surface anisotropy, and of g-tensor anisotropy. Exact results are presented for cylindrically symmetric Fermi surfaces, as well as approximate results for general Fermi surfaces (including effects due to open orbits). A prediction of the Larmor-wave spectrum for potassium is made.

I. INTRODUCTION

Recently, Janossy and Monod[1] discovered a new type of spin-density oscillation in metals, which they called a Larmor wave, and which they studied in microwave transmission experiments. By studying a model of noninteracting electrons having a constant scalar g factor and spherical Fermi surface, they showed theoretically how Larmor waves arise. They also pointed out that, unlike conduction-electron spin resonance, the Larmor-wave transmission is associated with electrons on particular parts of the Fermi surface. Furthermore, they noted that the Larmor-wave phenomenon could in principle be observed in metals with large g-factor anisotropy.

The purpose of this paper is to develop a theory which takes account of some of the effects of the Fermi-liquid interactions, the momentum dependence of the conduction-electron g tensor, and Fermi-surface anisotropy, on the Larmor-wave phenomenon.

The mechanism by which microwave power is transmitted through thin metallic slabs via Larmor waves is very similar to the mechanism of microwave power transmission in the cyclotron phase-resonance phenomenon.[2] The microwave skin depth must be much smaller than the slab thickness, whereas the electron mean free path should be comparable to or greater than the slab thickness. To be specific, consider the case of a static magnetic field normal to the slab , and an incident microwave field of frequency ω, circularly polarized so that it rotates in the same sense as the free precessional motion of the electron spin. An electron spin enters the skin depth and has its spin orientation tipped away from its equilibrium direction by the microwave field. It then crosses the slab, precessing at its free precessional frequency, and radiates power into the receiving cavity upon reaching the far surface of the metal. If the free precessional frequency of the electron spin is $g_{sp}\mu_B H/\hbar$ (which may depend on the electron momentum $\vec{p}$), the phase lag suffered by the electron spin relative to the microwave field during its transit across the metal is

$$\Phi = \int_{t_0}^{t_1} [\omega - \hbar^{-1} g_{sp}(\vec{p}(t'))\mu_B H]\, dt' \ , \qquad (1.1)$$

where t_0 and t_1 are the times at which the electron leaves the skin depth, and strikes the far surface, respectively. Different electrons suffer different phase lags depending on their momentum-space (or real-space) trajectories, and their contributions to the total transmitted field interfere. If the average phase lag for a given electron trajectory has an extremum, then neighboring trajectories will have nearly the same average phase lag, and the constructive interference which results will give rise to a particularly large signal. An experiment measuring the component of the transmitted field having a particular phase relative to the incident field will measure a signal proportional to $\cos(\Phi_{ex} + \alpha)$, where α is a constant the same for all electron spins, and Φ_{ex} is the extremal value of the average phase lag just referred to. The phase lag varies linearly with magnetic field [see Eq. (1.1)] giving rise to oscillations in the signal amplitude as the magnetic field strength is varied.

The frequency $\Omega = g_{sp}(\vec{p})\mu_B H/\hbar$ which appears in Eq. (1.1) is the precessional frequency of a spin with momentum $\vec{p}$, in an external field of magnitude H, when all other spins are frozen in their equilibrium positions. Thus, this precessional frequency is determined not only by the external magnetic field, but also by the effective exchange field due to neighboring spins. Furthermore, if spin-orbit coupling effects are important, the tensor interactions[3] between a given spin and its neighbors should be considered. All of these effects play a role in determining the single-particle g factor, $g_{sp}(\vec{p})$, which occurs in Eq. (1.1), and which is defined more precisely below in terms of the interaction functions occurring in the Landau theory of Fermi liquids. Thus, the single-particle g factor is quite different from the g factor determining the position of the conduction-electron spin-resonance line, and is in fact, closely related to the g factor measured in the de Haas–van Alphen effect. Finally, it should be noted that although most of our discussion here and below is in terms of a scalar g factor $g_{sp}(\vec{p})$, a complete theory requires the use of a g tensor, and some formal results for this more general case will be given.

There is a relatively simple case of a cylindrically symmetric Fermi surface, and a constant scalar g factor and exchange interaction for which an exact solution is possible (this is studied in Sec. III). The effects of exchange, and of different types of extrema resulting from different types of Fermi surfaces, are studied in detail. Of particular interest is a prediction of the Larmor-wave spectrum in potassium, which is known to have a relatively large exchange interaction.

Brief discussions are also given of the effects of more general Fermi-surface anisotropy, including some effects of open orbits.

In the experiments of Janossy and Monod,[1] the metallic slab through which the Larmor waves propagate is coated on both surfaces with a ferromagnetic layer which enhances the amplitude of the spin-dependent transmission. The role played by the ferromagnetic layers in the excitation and radiation process is not examined in this paper.

II. KINETIC EQUATION FOR SPIN-DEPENDENT OSCILLATIONS

The spin-dependent oscillations of a metallic Fermi liquid were originally studied by Silin[4] within the framework of the Landau theory of Fermi liquids. Because Larmor waves can be observed in metals with large g-factor anisotropy,[1] the tensor nature[3] of g, as well as tensor interactions[3] between the quasiparticles should be taken into account, in addition to the exchange interactions considered by Silin. Therefore, we begin by defining a single particle g tensor $\overleftrightarrow{g}_{sp}(\vec{p})$ which will be an essential element of the ultimate theory of Larmor waves.

The spin-dependent contribution to the quasiparticle energy will be written

$$\epsilon_{op} = \tfrac{1}{2}\mu_B \vec{\sigma}_{op}\cdot\overleftrightarrow{g}(\vec{p})\cdot\vec{H} + \vec{\sigma}_{op}\cdot\sum_{\vec{p}'}\overleftrightarrow{\Psi}(\vec{p},\vec{p}')\cdot\vec{\sigma}(\vec{p}') \ , \tag{2.1}$$

where an op subscript indicates an operator in spin space, and the components of $\vec{\sigma}_{op}$ are the Pauli matrices. The quantity $\vec{\epsilon}(\vec{p})$ is defined by the equation

$$\epsilon_{op} = \vec{\sigma}_{op}\cdot\vec{\epsilon}(\vec{p}) \ . \tag{2.2}$$

The equilibrium value (designated by a subscript zero) of the spin distribution function $\vec{\sigma}_0(\vec{p})$ in the static magnetic field $\vec{H}_0$, can be found from

$$\vec{\sigma}_0(\vec{p}) = 2\vec{\epsilon}_0(\vec{p})\frac{\partial n}{\partial \epsilon} \ , \tag{2.3}$$

where n is the Fermi-Dirac distribution function. This gives an integral equation for $\vec{\epsilon}_0(\vec{p})$, namely,

$$\vec{\epsilon}_0(\vec{p}) = \tfrac{1}{2}\mu_B\overleftrightarrow{g}(\vec{p})\cdot\vec{H}_0 + 2\sum_{\vec{p}'}\overleftrightarrow{\Psi}(\vec{p},\vec{p}')\cdot\vec{\epsilon}_0(\vec{p}')\frac{\partial n}{\partial \epsilon'} \ . \tag{2.4}$$

An explicit solution of this equation can be obtained only if the Fermi surface and interactions are explicitly given. The solution will be linear in $\vec{H}_0$ and thus has the form

$$\vec{\epsilon}_0(\vec{p}) = \tfrac{1}{2}\mu_B\overleftrightarrow{g}_{sp}(\vec{p})\cdot\vec{H}_0 \ , \tag{2.5}$$

which defines $\overleftrightarrow{g}_{sp}(\vec{p})$.

To study the time-dependent part of the spin-distribution function $\delta\vec{\sigma}(\vec{p},\vec{r},t)$ in the presence of a microwave magnetic field $\delta\vec{H}(\vec{r},t)$, let

$$\begin{aligned} \vec{\sigma}(\vec{p},\vec{r},t) &= \vec{\sigma}_0(\vec{p}) + \delta\vec{\sigma}(\vec{p},\vec{r},t) \ , \\ \vec{\epsilon}(\vec{p},\vec{r},t) &= \vec{\epsilon}_0(\vec{p}) + \delta\vec{\epsilon}(\vec{p},\vec{r},t) \ , \\ \vec{H}(\vec{r},t) &= \vec{H}_0 + \delta\vec{H}(\vec{r},t) \ . \end{aligned} \tag{2.6}$$

$\delta\vec{\sigma}$ can be expressed in terms of a function $\vec{G}$ defined only on the Fermi surface by

$$\delta\vec{\sigma}(\vec{p},\vec{r},t) = -2\frac{\partial n}{\partial\epsilon}\vec{G}(\vec{p},\vec{r},t) \tag{2.7}$$

and a quantity $\vec{G}^*$ is defined by

$$\vec{G}^*(\vec{p},\vec{r},t) = \vec{G}(\vec{p},\vec{r},t) + \delta\vec{\epsilon}(\vec{p},\vec{r},t) \ . \tag{2.8}$$

Assuming a time dependence $\exp(-i\omega t)$, the kinetic equation which determines $\vec{G}$ (and hence $\delta\vec{\sigma}$) is

$$-i\omega\vec{G} + \gamma\vec{G}^*\times\vec{H}_{\rm eff} + \left(\vec{v}\cdot\frac{\partial}{\partial\vec{r}} + \frac{\partial}{\partial t_1}\right)\vec{G}^* = -\frac{\vec{G}^* - \langle\vec{G}^*\rangle}{\tau_0} - \frac{\langle\vec{G}^*\rangle}{\tau_s} \ , \tag{2.9}$$

with

$$\gamma = 2\mu_B/\hbar, \quad \vec{H}_{\rm eff} = \tfrac{1}{2}\overleftrightarrow{g}_{sp}(\vec{p})\cdot\vec{H}_0 \ . \tag{2.10}$$

In Eq. (2.9), instead of the variables p_x, p_y, and p_z, it is convenient to use the variables ϵ, p_z, t_1 to denote the position of an electron in momentum space (t_1 is the arrival time of an electron at a given point on its trajectory as measured from some fixed reference point). The times τ_0' and τ_s' are momentum and spin reorientation times, and the angular brackets denote a Fermi-surface average.

A quantity of physical interest is the departure of the magnetization density from its equilibrium value, which is given in terms of the solution of Eq. (2.9) by

$$\delta \vec{M}(\vec{r},t) = -\frac{\mu_B}{(2\pi\hbar)^3}\frac{eH}{c} \times \int dp_z\, dt_1\, \vec{G}(p_z,t_1;\vec{r},t)\cdot \vec{g}(p_z,t_1) \quad . \tag{2.11}$$

III. LARMOR WAVES AND SPIN WAVES — EXACT SOLUTION

Here the spin-dependent oscillations of a Fermi liquid with a momentum-independent scalar g factor and a momentum-independent exchange interaction, in the thin slab and magnetic field normal geometry, with a cylindrically symmetric Fermi surface (the axis of symmetry being normal to the slab), and with a specular scattering boundary condition, will be studied in detail. For this case, an exact solution can be found.

Thus, by assumption, the g tensor of Eq. (2.1) is written $\vec{g}(\vec{p}) = \vec{1}g$, where g is independent of momentum. Also, the quasiparticle interaction is assumed to have the form $\rho\vec{\Psi} = \vec{1}B_0$, B_0 being independent of momentum, and ρ being the density of states. Equation (2.4) can now be solved to yield

$$g_{sp} = g/(1+B_0) \quad . \tag{3.1}$$

The slab has thickness L and a static magnetic field is assumed normal to it in the z direction. Microwaves are incident on the surface $z=0$, and excite spin-density oscillations, which result in a transmitted microwave field emanating from the surface $z=L$. The distribution function $\vec{G}$ is independent of t_1 and determined by Eq. (2.9), which can be rewritten

$$-i\omega G_\alpha + \left(v_z \frac{\partial}{\partial z} - i\alpha\Omega_0 \right) G_\alpha{}^* = -\frac{G_\alpha{}^* - \langle G_\alpha{}^* \rangle}{\tau_0'} - \frac{\langle G_\alpha{}^*\rangle}{\tau_s'} \quad , \tag{3.2}$$

where the G_α's, $\alpha = 0, \pm 1$, are the circularly polarized components of $\vec{G}$ [e.g., $G_{\pm1} = 2^{-1/2}(G_x \pm iG_y)$], and $\hbar\Omega_0 = g_{sp}\mu_B H_0$. The boundary conditions on $G_\alpha{}^*(p_z,z)$ at the surfaces $z=0$ and $z=L$ are specular-scattering boundary conditions which, by definition, are

$$G_\alpha{}^*(p_z,0) = G_\alpha{}^*(-p_z,0) \quad ,$$
$$G_\alpha{}^*(-p_z,L) = G_\alpha{}^*(p_z,L) \quad . \tag{3.3}$$

Equation (3.2) with the boundary condition (3.3) can be solved exactly using a method outlined in a paper by Azbel *et al.*,[5] the result being

$$\langle G_\alpha{}^* \rangle = \frac{-\frac{1}{2} i\omega g\mu_B}{(1+B_0)\tau_0^{-1} - i\omega B_0} \times \sum_{n=-\infty}^{\infty} \frac{R_{\alpha n} H_{\alpha n} \exp(ik_n z)}{F_\alpha - (R_{\alpha n} - 1)} \quad , \tag{3.4}$$

where we have

$$k_n = n\pi/L \quad ,$$
$$H_{\alpha n} = L^{-1}\int_0^L \cos(k_n z)\,\delta H_\alpha(z)\, dz \quad ,$$
$$R_{\alpha n} = \langle [1 + (k_n v_z t_\alpha)^2]^{-1} \rangle \quad ,$$
$$F_\alpha = [\tau_s^{-1} - i(\omega + \alpha\omega_0)]/[(1+B_0)\tau_0^{-1} - iB_0\omega] \quad , \tag{3.5}$$
$$\hbar\omega_0 = g\mu_B H_0, \quad \hbar\Omega_0 = g_{sp}\mu_B H_0 \quad ,$$
$$t_\alpha^{-1} = \tau_0^{-1} - i(\omega + \alpha\Omega_0) \quad ,$$
$$\frac{1}{\tau_0} = \frac{1}{\tau_0'} - \frac{1}{\tau_s'}, \quad \frac{1}{\tau_s} = \frac{1+B_0}{\tau_s'} \quad .$$

For the observation of Larmor waves, the skin depth must be much smaller than the slab thickness or the electron mean free path, and it is therefore a good approximation when evaluating Eq. (3.4) to write

$$H_{\alpha n} \approx H_{\alpha 0} = L^{-1}\int_0^L \delta H_\alpha(z)\, dz \quad . \tag{3.6}$$

The simplest way to proceed to obtain an accurate evaluation of Eq. (3.4) valid over the entire range of magnetic fields of interest, is to evaluate it numerically. First, however, some analytic results will be presented.

Note that close to the condition for conduction-electron spin resonance, i.e. $\omega \approx \omega_0$, one has $|F_-| \ll 1$. Also, at small k_n, one has

$$R_{-1n} - 1 \approx -k_n^2 \langle v_z^2 \rangle t_{-1}^2 \quad . \tag{3.7}$$

A major contribution to the sum in Eq. (3.4) is expected to come from the region where the denominator is small, and where the approximation

$$\langle G_-{}^*(z) \rangle = \left(i\omega g\mu_B \int_0^L dz\, \delta H_-(z)/DL \right) \times \sum_n \frac{e^{ik_n z}}{k_n^2 - k_s^2} \tag{3.8}$$

is valid. Here we have

$$k_s^2 = (1/D\tau_s)[i(\omega - \omega_0)\tau_s - 1] \tag{3.9}$$

and

$$D = (1+B_0)\langle v_z^2\rangle t_- \quad . \tag{3.10}$$

The amplitude of the transmitted field is proportional to the amplitude of the precessing component of the magnetization density at the transmitting surface $z = L$, which in turn is proportional to $\langle G_-^*(L)\rangle$ [see Eq. (2.11)]. The evaluation of Eq. (3.8) by contour integration gives

$$\langle G_-{}^*(L)\rangle = \tfrac{1}{2} i\omega g \mu_B \int_0^L dz\, \delta H_-(z)/Dk_s \sin(k_s L) \quad . \tag{3.11}$$

This well-known result has been used extensively in the interpretation of microwave transmission due to long-wavelength spin waves.[6,7]

Under the condtions $(|D\tau_s|)^{1/2} << L$, i.e., $|k_s L| << 1$, one finds that transmission via the conduction-electron spin resonance occurs with a signal amplitude proportional to

$$\delta M_-(L) = \chi \frac{-\omega_0}{\omega-\omega_0+i/\tau_s} \frac{1}{L}\int_0^L \delta H_-(z)\, dz \quad , \tag{3.12}$$

where χ is the static spin susceptibility.

The Larmor waves occur at frequencies far from the conduction-electron spin-resonance frequency where we have $|F_\alpha| >> 1$, i.e.,when

$$|\omega + \alpha\omega_0| >> |(1+B_0)/\tau_0 - iB_0\omega| \quad . \tag{3.13}$$

A detailed study of the case of a spherical Fermi surface showed that we have $|R_{\alpha n} - 1| < |F_\alpha|$ under these conditions and we assume that this is true in general. Thus, to evaluate Eq. (3.4), the terms $(R_{\alpha n} - 1)$ in the denominator will be put equal to zero. The result

$$\sum_n R_{\alpha n} e^{ik_n L} = \left\langle \frac{L/v_z t_\alpha}{\sinh(L/v_z t_\alpha)} \right\rangle \tag{3.14}$$

can now be used to show that Eq. (3.4) gives

$$\langle G_\alpha{}^*(L)\rangle = ig\mu_B\omega \times \int_0^\infty \delta H_\alpha(z)\, dz \left\langle \frac{1}{v_z}\sum_{n=0}^{\infty} \exp\left[-\frac{(2n+1)L}{v_z t_\alpha}\right] \right\rangle_+ , \tag{3.15}$$

where the average is over those parts of the Fermi surface for which $v_z > 0$ and $v_z(-p_z) = -v_z(p_z)$ is sumed.

Consider electrons with velocity v_z normal to the slab, so that the time taken to cross the slab once is L/v_z. Electrons arriving at the surface $z = L$ at time $t = 0$ were subjected to exciting fields at times $t = -L/v_z$, $-3L/v_z$,..., and $-(2n+1)L/v_z$,.... Hence the response at a given time is the sum of the infinite series displayed in Eq. (3.15). A reasonable guess at the solution in the case of a diffuse-scattering boundary condition (as opposed to the specular-scattering boundary condition used here) would be the solution (3.15) with the terms $n \geq 1$ eliminated (see also Sec. IV).

Combining the $n = 0$ term of Eq. (3.15) with Eq. (2.11), one finds

$$\delta M_\alpha(L) = \int_+ dp_z\, F_\alpha(p_z) e^{i\Phi_\alpha(p_z)} \quad , \tag{3.16}$$

where

$$F_\alpha(p_z) = i\omega g^2 \mu_B^2 m^* e^{-L/v_z\tau_0'} \times \int_0^L \delta H_\alpha(z)\, dz\, [2\pi^2\hbar^3(1+B_0)v_z]^{-1} \tag{3.17}$$

and

$$\Phi_\alpha(p_z) = (\omega + \alpha\Omega_0)L/v_z \quad . \tag{3.18}$$

Assuming that the phase lag $\Phi_\alpha(p_z)$ varies sufficiently rapidly in the range of integration, the integration can be performed by the method of stationary phase; the principal contribution to the integral then comes from those points where the phase lag Φ_α has an extremum. The condition that the phase lag Φ_α has an extremum is equivalent to the condition that v_z has an extremum, in the present case of a momentum-independent g factor and exchange interaction.

Suppose that the extremal value of v_z occurs at a limiting point (e.g. for a spherical Fermi surface this occurs when the electron velocity $\vec{v}$ is parallel to the external field $\vec{H}_0$). Then the evaluation of the limiting point contribution to the integral (3.16) gives

$$\delta M_\alpha(L) = \chi_1 \frac{2\omega}{\omega+\alpha\Omega_0} e^{-L/v_z\tau_0'} \left[\int_0^L \delta H_\alpha(z)\, dz/L\right] \times \exp\left[\frac{i(\omega+\alpha\Omega_0)L}{v_z}\right] \quad , \tag{3.19}$$

where

$$\chi_1 = g^2\mu_B m^{*2} v_z/4\pi^2\hbar^3(1+B_0) \tag{3.20}$$

is a quantity which, for a spherical Fermi surface, is equal to the spin susceptibility; the quantities v_z and m^* appearing in Eqs. (3.19) and (3.20) are to be evaluated at the limiting point.

If v_z has an extremum in the middle of the range of integration in Eq. (3.16), say at $p_z = p_e$, write

$$\Phi(p_z) \approx \Phi(p_e)[1+\beta(p_z-p_e)^2] \quad , \tag{3.21}$$

where β is a constant. We shall call this an ordinary extremum. The stationary phase method applied to the integral (3.16) now yields

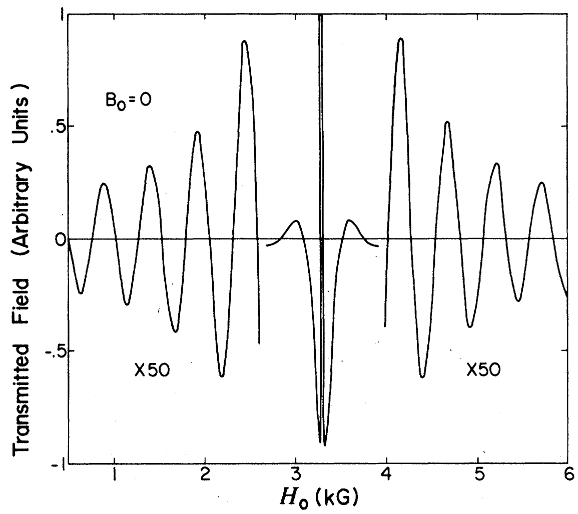

FIG. 1. Transmitted field as a function of H_0 in the case of a spherical Fermi surface, and zero exchange interaction. In the regions far from the CESR frequency, the signal has been scaled by the factors shown.

$$\delta M_\alpha(L) = \chi_2 \left(\frac{\omega}{\omega + \alpha\Omega_0} \frac{\omega L}{v_z} \right)^{1/2} e^{-L/v_z\tau_0'} \times \left(\int_0^L \delta H_\alpha(z)\, dz/L \right) \times \exp i \left(\frac{\pi}{4} + \frac{(\omega+\alpha\Omega_0)L}{v_z} \right) , \quad (3.22)$$

where

$$\chi_2 = \frac{g^2 \mu_B^2 m^*}{2\pi^2 \hbar^3 (1+B_0)} \left(\frac{\pi}{\beta} \right)^{1/2} \quad (3.23)$$

has the order of magnitude of a spin susceptibility, and m^* and v_z are evaluated at $p_z = p_e$.

Finally, it may happen that v_z, and hence the phase lag, are relatively constant over an interval Δp of the range of integration in Eq. (3.16). In this case one finds

$$\delta M_\alpha(L) = i\chi_3 \left(\frac{\omega L}{v_z} \right) e^{-L/v_z\tau_0'} \times \left(\int_0^L \delta H_\alpha(z)\, dz/L \right) \times \exp\left(\frac{i(\omega+\alpha\Omega_0)L}{v_z} \right) , \quad (3.24)$$

where

$$\chi_3 = g^2 \mu_B^2 m^* \Delta p / 2\pi^2 \hbar^3 (1+B_0) \ . \quad (3.25)$$

Typical experimental conditions have $\omega L/v_z >> 1$. In fact the data in Fig. 2 of the paper by Janossy and Monod[1] was obtained for $\omega \sim 6\times10^{10}$ sec^{-1}, $L = 0.07$ cm, and $v_F = 10^8$ cm/sec giving $\omega L/v_F = 42$. Under these conditions, the transmission due to Larmor waves associated with limiting point orbits is the least intense [Eq. (3.19)], the transmission associated with ordinary extrema is somewhat more intense [Eq. (3.22)], and the transmission associated with a finite p_z interval of nearly constant phase lag is the most intense [Eq. (3.24)].

The Larmor-wave observations of Janossy and Monod[1] on copper (shown in their Fig. 2) were carried out with the external magnetic field in the [100] direction and perpendicular to the surface of the slab. The component of the electron velocity along the field direction v_z has been calculated, as a function of p_z, by Phillips, Baraff, and Schmidt[2] and is shown in their Fig. 4. Their calculation shows that v_z is approximately constant over a relatively large p_z interval, one of whose end points is the limiting point. Thus Eq. (3.24) gives a closer approximate description of the observations in copper than Eqs. (3.22) and (3.19). (The fact that this p_z interval contains a limiting point is incidental, and Eq. (3.19) will not be a good approximation in this case.) The data of Ref. 11 shows that v_z has other extrema, but in view of the remarks of the preceding paragraph, the contribution of the orbits of the p_z interval for which v_z is approximately constant, should dominate.

With a particular model for the Fermi surface, $R_{\alpha n}$ may be calculated explicitly [see Eq. (3.5)] and an ac-

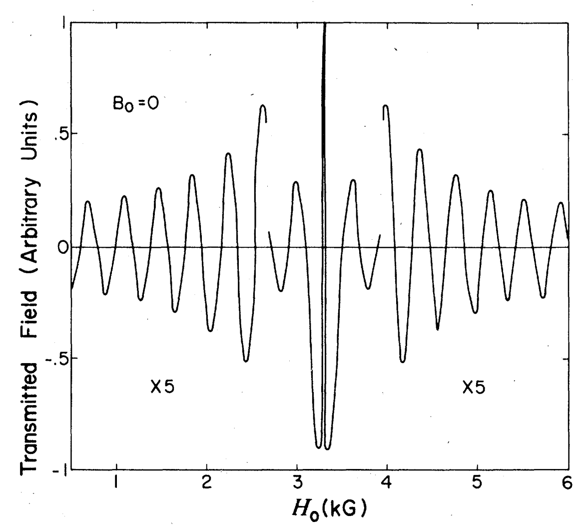

FIG. 2. Transmitted microwave field as a function of H_0 for a Fermi surface with an ordinary extremum of v_z, and zero quasiparticle interaction.

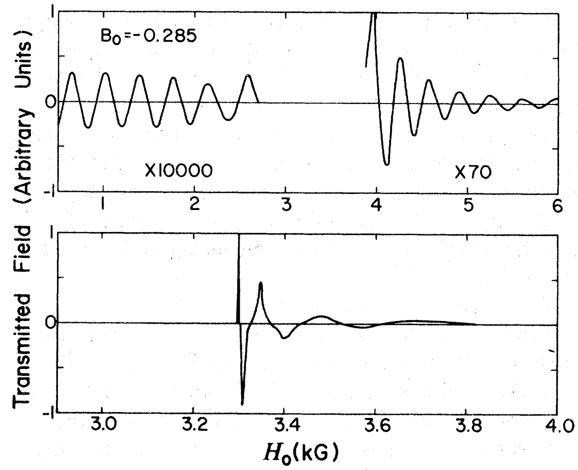

FIG. 3. Transmission spectrum for a metal having a spherical Fermi surface, and strong exchange interaction. The scale factors are taken relative to the lower plot.

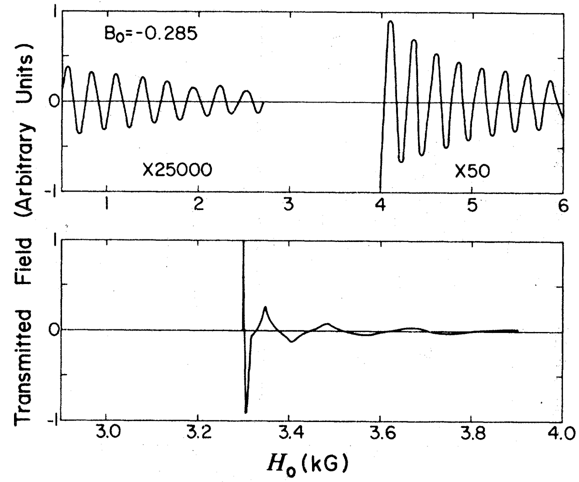

FIG. 4. Transmission spectrum for a metal having strong quasiparticle interaction, and a Fermi surface in the form of a corrugated cylinder. Notice that the signal is roughly 500 times greater on the high-field side of the CESR.

curate numerical evaluation of $\delta M_\alpha(L)$ can be obtained from Eq. (3.4). Figs. 1–4 show $\delta M_\alpha(L)$ for a spherical Fermi surface, where the limiting point contribution should be dominant, and for a surface in the form of a corrugated cylinder, having an ordinary extremum of v_z. Figures 3 and 4 display the transmitted field for $B_0=-0.285$, which is typical of potassium.[7]

In the case of the spherical Fermi surface, characterized by its Fermi velocity v_F, $R_{\alpha n}$ is a function of $z_\alpha = i/k_n v_F t_\alpha$, and is given by

$$R_{\alpha n} = \tfrac{1}{2} z_\alpha [\ln(1+z_\alpha) - \ln(1-z_\alpha) - i\pi\, \mathrm{sgn}(n)] \ , \tag{3.26}$$

where the argument of the natural logarithm is an angle θ, with $-\pi < \theta \leq \pi$.

For a model corrugated cylinder defined by

$$v_z^2(p_z) = 4 v_e^2 \left[\left| \frac{p_z}{p_0} \right| - \left(\frac{p_z}{p_0} \right)^2 \right] \ , \qquad -p_0 \leq p_z \leq p_0 \tag{3.27}$$

and with the cyclotron frequency chosen to be independent of p_z, the integral in Eq. (3.5) can be performed. For such a Fermi surface, $R_{\alpha n}$ is a function of $\lambda = (1+K^2)^{1/2}/K$ and $K = k_n v_e t_\alpha$, and has the form

$$R_{\alpha n} = (1/2K^2\lambda)[\ln(1+\lambda) - \ln(1-\lambda) - i\pi\, \mathrm{sgn}(\mathrm{Im}\lambda)] \ . \tag{3.28}$$

Here, the imaginary part of the logarithm ranges over $(-\pi, \pi)$.

The figures show the transmission through a sample 0.7 mm thick, with the microwave field penetrating to a depth of $\delta = 4\ \mu$m. For purposes of comparison, the mean free times for scattering and for spin relaxation are taken to be $\tau_0 = 3\times 10^{-10}$ sec and $\tau_s = 2\times 10^{-7}$ sec for all graphs. These values are typical of some alkali metals at low temperature.[7] For the spherical Fermi surface $v_F = 10^8\ \mathrm{cm\,sec^{-1}}$ and for the cylindrical surface $v_e = 0.75\times 10^8\ \mathrm{cm\,sec^{-1}}$; with these values we have $\omega L/v_F \gg 1$, $\omega\tau_0 \gg 1$, and $L/v_F\tau_0 \approx 2$, so that multiple reflections do not play a role.

In the absence of exchange interaction, it is expected that $\delta M_\alpha(L)$ will be symmetric around the CESR, from Eqs. (3.19) and (3.22). This symmetry is evident in Figs. 1 and 2.

Close to the CESR, the transmission is dominated by diffusing electrons and when $B_0 = 0$ and $(2D\tau_s)^{1/2} \lesssim L$, narrow lobes appear.[8] This is in accordance with Eq. (3.11). The Larmor oscillations are found farther from the resonance frequency, where Eq. (3.13) is valid. In this region, Eqs. (3.1), (3.5), (3.19), and (3.22) predict that $\delta M_\alpha(L)$ will oscillate with a period ΔH_0, given by

$$1 + B_0 = g\mu_B(\Delta H_0)L/2\pi\hbar v_z \ . \tag{3.29}$$

Here, v_z is evaluated at the limiting point of a spherical surface, or the ordinary extremum of a corrugated cylinder. By measuring ΔH_0, it is possible to determine B_0. Taking ΔH_0 from the graphs of Figs. 1 and 2 gives $B_0 = 0$ for both cases.

The envelopes of the curves in Figs. 1–4 give the amplitude of the transmitted field. Although for $B=0$ the magnitude is close to that given by the analytic formula, the amplitude falls off somewhat more rapidly than $(\omega + \alpha\Omega_0)^{-1}$ in the case of the spherical Fermi surface, and faster than

$(\omega+\alpha\Omega_0)^{-1/2}$ for the corrugated cylinder. This discrepancy arises when the terms $(R_{\alpha n}-1)$ in the denominator of Eq. (3.4) are put equal to zero.

As the figures show, expressions (3.19) and (3.22) yield very poor approximations to the intensity when there is exchange interaction. If we have $|B_0\omega\tau_0(1+B_0)^{-1}| \geq 1$, then inequality (3.13) reduces to $|(\omega-\omega_0)/B_0\omega| >> 1$, and is not satisfied within the domain of H_0 shown on Figs. 3 and 4. There is pronounced assymmetry around the CESR, the intensity on the high-field side being greater than on the low-field side for $B_0<0$, even far from the spin resonance.

Nevertheless, the period of the Larmor oscillations is given very well by Eq. (3.29). With ΔH_0 measured from Figs. 3 and 4, that equation gives $B_0=-0.28\pm0.03$ for both Fermi surfaces.

IV. LARMOR WAVES IN THE CASE OF ANISOTROPIC FERMI SURFACES AND A SCALAR g

In discussing the case of a general anisotropic Fermi surface, a scalar g will be assumed at first for simplicity. Thus, from Eq. (2.10) we have

$$\vec{H}_{\text{eff}}=\tfrac{1}{2}g_{\text{sp}}(p_z,t_1)\,\vec{H}_0 \ .$$

Equation (2.9) can then be written

$$-i\omega_\alpha G_\alpha{}^*+v_z\frac{\partial G_\alpha{}^*}{\partial z}+\frac{\partial G_\alpha{}^*}{\partial t_1}=-i\omega\delta\epsilon_\alpha +\left(\frac{1}{\tau_0'}-\frac{1}{\tau_s'}\right)\langle G_\alpha{}^*\rangle \ , \tag{4.1}$$

where $\alpha=0,\ \pm1$, and

$$\omega_\alpha=\omega+\alpha\Omega(p_z,t_1)+i\tau_0'^{-1} \ ,$$
$$\hbar\Omega(p_z,t_1)=g_{\text{sp}}(p_z,t_1)\mu_B H_0 \ . \tag{4.2}$$

As discussed in Sec. I, Larmor-wave transmission occurs only when a substantial number of electrons all undergo nearly the same phase lag relative to the microwave field during their transit through the metal, from the exciting surface $z=0$ to the transmitting surface $z=L$; the signals due to these electrons then add coherently to give an observable transmission. The term $\tau_0'^{-1}G_\alpha{}^*$ on the left-hand side of Eq. (4.1) describes the rate of decrease of $G_\alpha{}^*(\vec{p})$ at point $\vec{p}$ due to the scattering of electrons from point $\vec{p}$ to other momentum states on the Fermi surface; the term proportional to $\langle G_\alpha{}^*\rangle$ on the right-hand side of Eq. (4.1), on the other hand, describes the rate of increase of $G_\alpha{}^*(\vec{p})$ due to electrons scattered into state $\vec{p}$ from other states. Electrons which have been scattered into state $\vec{p}$ from other states arrive with random phases and therefore will not contribute to the Larmor-wave signals. Therefore, the term proportional to $\langle G_\alpha{}^*\rangle$ on the right-hand side of Eq. (4.1) will be dropped.

Also, the contribution of the quasiparticle interaction to $\delta\epsilon_\alpha$ will be dropped since it is responsible for the collective spin-wave modes, but is expected to have little effect on the Larmor-wave frequency.

Thus the equation describing Larmor waves is

$$-i\omega_\alpha G_\alpha{}^*+v_z\frac{\partial G_\alpha{}^*}{\partial z}+\frac{\partial G_\alpha{}^*}{\partial t_1}=h_\alpha \ , \tag{4.3}$$

$$h_\alpha(z,t_1)=-\tfrac{1}{2}i\omega g(t_1)\mu_B\delta H_\alpha(z) \ . \tag{4.4}$$

Both g and h_α depend on p_z also, but this dependence is not shown explicitly in Eq. (4.4).

Recall again that Larmor-wave transmission is due to the coherent superposition of signals from groups of electrons which undergo approximately the same phase lag relative to the microwave field during their transit across the sample. In view of this, if the scattering of electrons at the metallic surface is diffuse, the past history of an electron, which has just been scattered at the surface $z=0$ and is starting towards the surface $z=L$, is unimportant. Thus, it is appropriate to study the solution of Eq. (4.3) for electrons with $v_z>0$ for their entire cyclotron-orbit, subject to the boundary condition

$$G_\alpha{}^*|_{z=0}=0 \ . \tag{4.5}$$

The solution of Eq. (4.3) subject to the boundary condition (4.5) can be found by the method of characteristics, and is

$$G_\alpha{}^*(z,t_1)=\int_{t_0}^{t_1}dt'\exp\left(i\int_{t'}^{t_1}\omega_\alpha(t_1')\,dt_1'\right)\times h_\alpha\left(\int_{t_0}^{t'}v_z(t_1')\,dt_1',t'\right) \ , \tag{4.6}$$

where $t_0=t_0(z,t_1)$ is determined by the equation

$$z=\int_{t_0}^{t_1}v_z(t_1')\,dt_1' \ . \tag{4.7}$$

For cases of interest, the skin depth δ is sufficiently small that we have $|\omega_\alpha\delta/v_z| << 1$, and thus Eq. (4.6) can be written

$$G_\alpha{}^*(z,t_1)=\exp\left(i\int_{t_0}^{t_1}\omega_\alpha(t_1')\,dt_1'\right)\times\int_{t_0}^{\infty}dt'\,h_\alpha\left(\int_{t_0}^{t'}v_z(t_1')\,dt_1',\ t'\right). \tag{4.8}$$

The situation where the Fermi surface has cylindrical symmetry about the z axis, so that v_z, Ω_0, and g are independent of t_1 (but dependent on p_z) is particularly simple. In this case, the combination of Eq. (4.8) with Eq. (2.11) gives the result (3.16), except that in the present case $F(p_z)$ is smaller by a factor of 4 than the result (3.17). Because of the boundary condition (4.5), an electron striking the surface $z=0$ is only excited by the microwave field as it leaves the surface, whereas for the specular-reflection boundary condition of Sec. III, the electron spin is excited both as it approaches and leaves the surface $z=0$; this accounts for one factor of 2. Furthermore, $G_\alpha{}^*(L)$ as found from Eq. (4.8), includes only contributions from those electrons with $v_z>0$; electron spins which have just been scattered from the surface $z=L$, and thus have $v_z<0$ should also be counted. This accounts for a second factor of 2.

It is clear that the approximate method of this section will suffer from the same limitations as does Eq. (3.16) of Sec. III. Thus, although it is expected that the frequency of the Larmor-wave oscillations will be predicted correctly by Eq. (4.8), the amplitude of the oscillations will not be predicted very well, particularly in the case of large exchange interactions.

The detailed evaluation of the integral (3.16) for the case of a cylindrically symmetric Fermi surface will not be repeated here. The main extension of the work in Sec. III is that now g, and hence Ω_0, depend on p_z. This dependence on momentum must be taken into account in determining those values of p_z at which the phase lag $\Phi_\alpha(p_z)$ is stationary (the condition that $\Phi_\alpha(p_z)$ is stationary is no longer equivalent to the condition that v_z is stationary). This can lead to Φ_+ and Φ_- having different extremal values.

The contribution to the Larmor-wave transmission of electrons following closed orbits will now be considered for the case where the Fermi surface does not have cylindrical symmetry. As above, the static magnetic field is assumed normal to the slab and only contributions of electrons for which we have $v_z>0$ for the entire orbit are considered. The distribution function is again given by Eq. (4.8). Now write

$$i\int_{t_0}^{t_1}\omega_\alpha(t_1')\,dt_1' = i\Phi_\alpha(t_1)-\int_{t_0}^{t_1}\tau_0'^{-1}\,dt_1' \;, \tag{4.9}$$

where the electron collision time τ_0' has been allowed to depend on momentum, $t_0(z=L,t_1)$ is determined by Eq. (4.7) with $z=L$, and

$$\Phi_\alpha=\int_{t_0}^{t_1}[\omega+\alpha\Omega(t_1')]\,dt_1' \tag{4.10}$$

is the phase lag acquired by the electron during its transit through the metal. The phase lag is a periodic function of t_1 (the period being the cyclotron period) and can be written as a sum of its average value $\overline{\Phi}_\alpha$ (averaged over a period) and a correction term, i.e.,

$$\Phi_\alpha=\overline{\Phi}_\alpha+\delta\Phi_\alpha \;. \tag{4.11}$$

Whereas for thick slabs $\overline{\Phi}_\alpha$ grows in proportion to L, $\delta\Phi_\alpha$ will have a magnitude of the order of 2π or less. Furthermore, $\delta\Phi_\alpha$ will depend on magnetic field in such a way that, at those magnetic fields at which electrons for a given p_z complete an integral number of cyclotron orbits while crossing the slab, $\delta\Phi_\alpha$ will be zero. This introduces an additional periodicity in magnetic field strength which was not present in the case of cylindrical symmetry. Combining Eqs. (2.11), (4.8), (4.9), and (4.11) yields

$$\delta M_\alpha(L)=\int dp_z\, e^{i\overline{\Phi}_\alpha}F_\alpha \;, \tag{4.12}$$

where

$$F(p_z)=-\frac{\mu_B}{(2\pi\hbar)^3}\frac{eH}{c}\int dt_1\, g(p_z,t_1) \times\exp\left[i\delta\Phi_\alpha-\int_{t_0}^{t_1}\tau_0'^{-1}\,dt_1\right] \times\int_{t_0}^{t'}h_\alpha\left[\int_{t_0}^{t'}v_z(t'')\,dt''\right]dt' \;. \tag{4.13}$$

Since the portion $\overline{\Phi}_\alpha$ of the phase lag Φ_α which is large for large L has been separated out and appears explicitly in Eq. (4.11), the stationary phase method can be applied to Eq. (4.12) in exactly the same way as it was applied to Eq. (3.16). The new feature which will affect experimental results and which comes from the lack of cylindrical symmetry relative to the z axis, is that there will be an additional magnetic field dependence of the amplitude of the oscillations; this arises because $F(p_z)$ is field dependent through the field dependence of $\delta\Phi_\alpha$, for example, as well as through field dependence of the interval t_1-t_0.

The above formalism is also applicable to the case of periodic open orbits when the applied static magnetic field is normal to the slab.

V. FIELD-PARALLEL GEOMETRY — EFFECTS OF OPEN ORBITS

One of the more important effects of open orbits, will be to allow Larmor-wave transmission in the case where the magnetic field is parallel to the surface of the slab. Consider the geometry show in Fig. 5, in which the magnetic field is in the z direction parallel to the surface of the slab. The electrons are excited at the surface $x=0$, and travel across to the surface $x=L$, where they radiate power into the receiving cavity. A Fermi surface in the form of a corrugated cylinder is shown in Fig. 5 and several open-orbit trajectories (labeled a, b, c, and d) are also shown. It is evident that the electrons which follow trajectory d will cross the slab in the shortest time; the average phase lag of these electrons will thus be at a

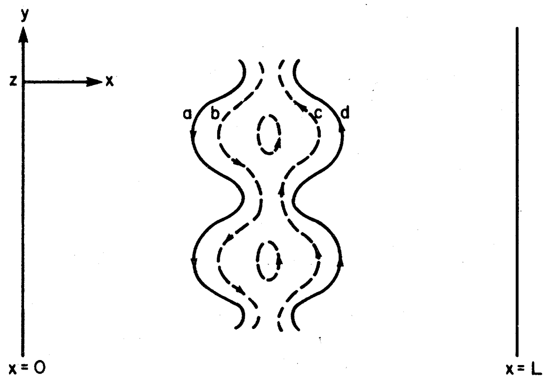

FIG. 5. Part of a Fermi surface in the form of a corrugated cylinder, with some of the open orbits shown.

minimum and will determine the frequency of the associated Larmor-wave oscillations. A formal theory of Larmor-wave oscillations due to periodic open orbits for the geometry shown in Fig. 5, is given by Eqs. (4.3)–(4.13) ; in all these equations, however, let $z \rightarrow x$, $v_z \rightarrow v_x$ (but keep p_z unchanged). The situation just described is similar to that which occurs in copper when a [111] direction coincides with the y direction in Fig. 5.

VI. THEORY FOR A g TENSOR

If spin-orbit coupling effects are important, it is essential to allow for the tensor nature of g in a general theory. The starting point is Eq. (2.9) and, in studying the transmission in the Larmor-wave region only [defined roughly by Eq. (3.13)], approximations are made analogous to those made in arriving at Eq. (4.3) from Eq. (4.1). Also, the field normal geometry is assumed as in Sec. IV. Thus, the starting equation is

$$\left(-i\omega + \frac{1}{\tau_0'}\right) G_i^* + \sum_j \Omega_{ij} G_j^* + \left(v_z \frac{\partial}{\partial z} + \frac{\partial}{\partial t_1}\right) G_i^* = h_i \ , \tag{6.1}$$

where Cartesian components have been used ($i = x,\ y,\ z$ as distinct from $\alpha = 0,\ \pm 1$) and we have

$$\Omega_{ij} = \gamma \sum_k \epsilon_{ijk} H_{\mathrm{eff},k} \ , \tag{6.2}$$

with ϵ_{ijk} being the Levi-Civita density. Define the matrix ω_+ by

$$\omega_+ = \omega 1 + i\Omega \ , \tag{6.3}$$

where 1 is the unit matrix, and the matrix elements of Ω are given by Eq. (6.2); note that ω_+ is Hermitian. The components G_i^* and h_i define the column vectors G^*, h of a vector space, and ω_+ can be viewed as an operator in this vector space. Thus, Eq. (6.1) is written

$$\left(-i\omega_+ + \frac{1}{\tau_0'}\right) G^* + \left(v_z \frac{\partial}{\partial z} + \frac{\partial}{\partial t_1}\right) G^* = h \ . \tag{6.4}$$

Considering only electron orbits for which $v_z > 0$, the solution of Eq. (6.4) subject to the boundary condition $G^* = 0$ at $z = 0$, is

$$G^* = \int_{t_0}^{t_1} dt'\, T_{t_1'}\left[\exp\left(i \int_{t'}^{t_1} \omega_+(t_1')\, dt_1'\right)\right] \times \exp\left(-\int_{t_0}^{t_1} \frac{dt'}{\tau_0'(t')}\right) \times h\left(\int_{t_0}^{t'} v_z(t'')\, dt''\right) \ , \tag{6.5}$$

where $T_{t_1'}$ is the time ordering operator with respect to the time variable t_1' which is the argument of the operator $\omega_+(t_1')$.

The formal solution (6.5) is as far as we have proceeded with the analysis for the case of a g tensor. Nevertheless, it was thought to be useful to include it in order to emphasize some of the shortcomings of the discussion for a g scalar by way of contrast.

VII. CONCLUSIONS

A quantity of principal importance in the theory is the phase lag suffered by an electron spin during its transit across the slab. The electron orbits for which this phase lag is an extremum give the dominant contributions to the transmission signal. There are three types of extrema: (i) the limiting point, (ii) the ordinary extremum and (iii) an extremum associated with a phase lag constant over a finite p_z interval. In general (i) gives the least intense signal, whereas (iii) will give the most intense signal. The value of the phase lag at the extremum determines the frequency of the Larmor-wave oscillations; since this phase lag depends upon the Fermi-liquid interactions, it can be used to measure these interactions experimentally. The amplitude of the oscillations also contains information, but it is shown above that although our approximate theory gives the frequency of the oscillations accurately, it does not predict the amplitude well in the presence of moderately strong exchange interactions. In the case of potassium, the amplitude of the transmission signal can be determined exactly. Thus, it is possible to find the field dependence of the excitation, for a potassium foil plated with ferromagnetic layers. Finally, it should be mentioned that although most of our paper studies the case where the magnetic field is oriented normal to the slab, it is predicted that the presence of open orbits will give rise to Larmor-wave transmission in the case where the magnetic field is parallel to the slab.

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