Electromagnetic wave propagation at the interface between two polar semiconductors

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The interface between two polar semiconductors can support three types of phonon-plasmon-polariton modes propagating in three well-defined frequency windows $\Delta\omega_1 \equiv [\min(\omega_1,\omega_3),\omega_{R1}]$, $\Delta\omega_2 \equiv [\max(\omega_2,\omega_4),\omega_{R2}]$, and $\Delta\omega_3 \equiv [\min(\omega_2,\omega_4), \omega_{R3}]$. The limiting frequencies $\omega_{1,2,3,4}$ are defined by $\epsilon_1(\omega) = 0$, $\epsilon_2(\omega) = 0$, and $\omega_{R1,2,3}$ by $\epsilon_1(\omega) + \epsilon_2(\omega) = 0$, where $\epsilon_i(\omega)$ are dielectric functions of the two media with $i = 1,2$. The dispersion, decay distances, and polarization of the three modes are discussed. The variation of the limiting frequencies with the interface plasma parameter $\eta \equiv \omega_{p2}^2/\omega_{p1}^2$ reveals an interesting feature in the dispersion characteristics of these modes. For the interfaces for which the bulk coupled phonon-plasmon frequencies of medium 1 are greater than the LO frequency or are less than the TO frequency of medium 2, there exist two values of $\eta = \eta_1$ and η_2 $(< \eta_1)$ for which $\Delta\omega_1$ and $\Delta\omega_3$ are zero, respectively. Hence, for these values of η, the two interface modes defined by $\Delta\omega_1$ and $\Delta\omega_3$ propagate with constant frequencies equal to the bulk coupled phonon-plasmon frequencies of medium 1, i.e., without showing any dispersion.

The phenomenon of surface propagation of polaritons and the coupling of the latter, in the appropriate frequency ranges, with optical phonons have been studied extensively both theoretically and experimentally. The effects of the magnetic fields on the coupled modes have also received considerable attention. For review the reader is directed to the article by Hartstein *et al.*[1] Although these earlier studies were confined to the semiconductor-dielectric interface, more recently the case of the interface between two semiconductors has been considered by Halevi[2] and Uberoi and Rao.[3] It has been shown that the interface plasmons propagating along the semiconductor-semiconductor (SS) interfaces show novel dispersion characteristic different from those of the surface waves propagating along the semiconductor-vacuum (SV) interface.

In this general context, we consider the surface waves along the interface between two polar semiconductors (PS-PS). The plasmon polaritons along the SS interface propagate in a window with spectral range $[\min(\omega_{p1}, \omega_{p2}), \omega_{sp}]$, where $\omega_{sp} \equiv [(\epsilon_{\infty 1}\omega_{p1}^2 + \epsilon_{\infty 2}\omega_{p2}^2)/(\epsilon_{\infty 1} + \epsilon_{\infty 2})]^{1/2}$ is the plasmon resonant frequency and $\omega_{p1,2} = (4\pi n_{01,2}e^2/m_{1,2}^*\epsilon_{\infty 1,2})^{1/2}$ are the bulk plasmon frequencies of the two media with $\epsilon_{\infty 1,2}$ being the high frequency dielectric constants. For PS-PS interface the discussion of infinities and zeroes of the dispersion relation written in the form $n = f(\epsilon_1, \epsilon_2)$, where n is the refractive index, shows that a PS-PS interface can support three optical-phonon-plasmon polariton modes in three well defined frequency windows. The spectral range of the propagating windows varies with the change in the ratio of plasma frequencies of the two media at the interface. An interesting possibility is that for certain critical values of $\eta \equiv \omega_{p2}^2/\omega_{p1}^2$, $0 < \eta < 1$, the spectral range of the propagation windows for two of the modes can be made zero, thus resulting in the nondispersive modes propagating with constant frequencies. We note that these constant frequencies correspond to the bulk coupled phonon-plasmon frequencies of medium 1. It is interesting to compare this result with that obtained in Ref. 2, where the spectral range of propagating window can be made very narrow by making $\eta \sim 1$; but the value never reaches zero.

Consider a plane interface $x = 0$ of two semi-infinite samples of polar semiconductors filling the half-spaces $x < 0$ (medium 1) and $x > 0$ (medium 2). Each is characterized by the frequency-dependent dielectric functions

$$\epsilon_i(\omega) = \epsilon_{\infty i}\left(\frac{\omega^2 - \omega_{Li}^2}{\omega^2 - \omega_{Ti}^2} - \frac{\omega_{pi}^2}{\omega^2}\right), \quad i = 1, 2 ,$$

where ω_{pi}, ω_{Li}, and ω_{Ti} are, respectively, the plasma frequencies, longitudinal- and transverse-optical (LO and TO) frequencies of two media and $\epsilon_{\infty i}$ are the high-frequency background dielectric constants.

Assuming the perturbed electromagnetic-field components of the form $\exp i(ky - \omega t)$, using Maxwell's equations and normal boundary conditions at $x = 0$, we arrive at the following dispersion relation for the surface modes:

$$\epsilon_1\left(k^2 - \frac{\omega^2}{c^2}\epsilon_2\right)^{1/2} + \epsilon_2\left(k^2 - \frac{\omega^2}{c^2}\epsilon_1\right)^{1/2} = 0 , \qquad (1)$$

where $[k^2 - (\omega^2/c^2)\epsilon_{1,2}]^{1/2}$ are the decay constants of wave fields in mediums 1 and 2. Imposing a surface wave condition of decay constant being a positive real quantity in both the media, thereby allowing the fields to decay exponentially away from the interface makes $k^2 > (\omega^2/c^2)\epsilon_{1,2}$, thus making the phase velocity ω/k of the surface modes less than that of bulk modes, given by $k^2 = (\omega^2/c^2)\epsilon_{1,2}$ in two media.

The dispersion relation (1) can be written in the form

$$n^2 \equiv c^2k^2/\omega^2 = \epsilon_1\epsilon_2/(\epsilon_1+\epsilon_2) . \tag{2}$$

The solution of Eq. (2) [as seen also from Eq. (1)] exists only when ϵ_1 and ϵ_2 are of different signs such that $\epsilon_1+\epsilon_2<0$.

The values of ω for which $n^2=\infty$, i.e., the resonant or asymptotic frequencies of the surface modes are given by the roots of the equation $\epsilon_1+\epsilon_2=0$ which after normalizing all the frequencies with ω_{T1}, can be written

$$\begin{aligned}(1+\lambda)\omega^6 &- [(1+\lambda\alpha)\omega_{L1}^2+(1+\eta\lambda)\omega_{p1}^2+(\beta+\lambda)]\omega^4 \\ &+ [(\beta+\lambda\alpha)\omega_{L1}^2+(1+\beta)(1+\eta\lambda)\omega_{p1}^2]\omega^2 \\ &- (1+\eta\lambda)\omega_{p1}^2 = 0 ,\end{aligned} \tag{3}$$

where $\alpha=\omega_{L2}^2/\omega_{L1}^2$, $\beta=\omega_{T2}^2/\omega_{T1}^2$, $\lambda=\epsilon_{\infty 2}/\epsilon_{\infty 1}$, and $\eta=\omega_{p2}^2/\omega_{p1}^2$ are the interface parameters.

Equation (3) has always three positive real roots $\omega_{R3}<\omega_{R2}<\omega_{R1}$ for all values of α, β, η, and λ. The left-hand side of Eq. (3) changes sign at $\omega=0$, $\omega=1$, $\omega=\beta$, and $\omega=\infty$. Hence three roots of Eq. (3) are located in the following ranges:

$$\omega_{R3}^2<\min(1,\beta)<\omega_{R2}^2<\max(1,\beta)<\omega_{R1}^2 .$$

It is interesting to note that irrespective of the location of frequencies ω_{L1}, ω_{L2}, ω_{p1}, ω_{p2} with respect to ω_{T1}, ω_{T2}, the above inequality holds good for all PS-PS interfaces. For an interface with $\omega_{T2}>\omega_{T1}$, the asymptotic frequencies will satisfy the inequality $\omega_{R3}<\omega_{T1}<\omega_{R2}<\omega_{T2}<\omega_{R1}$.

For calculating the spectral range of propagation windows we find the zeros ($n^2=0$) of Eq. (2), which are given by the zeros of the functions ϵ_1 and ϵ_2, $\omega_{1,2}$ and $\omega_{3,4}$, defining the bulk coupled optical-phonon-plasmon frequencies:

$$\omega_{1,2}^2=\tfrac{1}{2}(\omega_{L1}^2+\omega_{p1}^2)\pm\{[\tfrac{1}{2}(\omega_{L1}^2+\omega_{p1}^2)]^2-\omega_{p1}^2\}^{1/2}$$

and

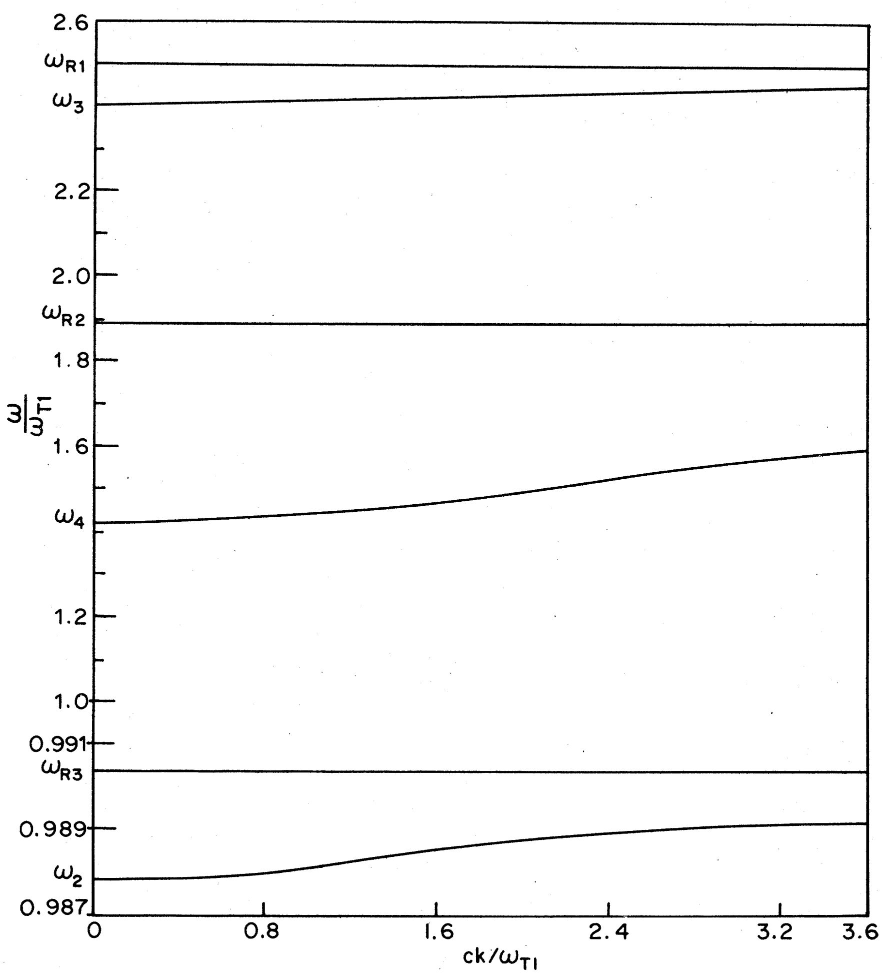

FIG. 1. Dispersion curves of optical-phonon-plasmon polaritons at the interface between two polar semiconductors. The frequencies (ω_2, ω_{R3}), (ω_4, ω_{R2}), and (ω_3, ω_{R1}) delineate the propagation windows. Numerical values correspond to the InSb-GaP interface as given in the text.

$$\omega_{3,4}^2=\tfrac{1}{2}(\alpha\omega_{L1}^2+\eta\omega_{p1}^2) \pm\{[\tfrac{1}{2}(\alpha\omega_{L1}^2+\eta\omega_{p1}^2)]^2-\eta\beta\omega_{p1}^2\}^{1/2},$$

such that $\omega_2^2<1<\omega_1^2$ and $\omega_4^2<\beta<\omega_3^2$. The frequencies are normalized with ω_{T1}. Since the frequency $\omega=\max(\omega_1,\omega_3)$ lies in the nonpropagating region $(\epsilon_1+\epsilon_2>0)$, the frequency range of the propagating windows is seen to be as follows: $\Delta\omega_1\equiv[\min(\omega_1,\omega_3),\omega_{R1}]$, $\Delta\omega_2\equiv[\max(\omega_2,\omega_4),\omega_{R2}]$, and $\Delta\omega_3\equiv[\min(\omega_2,\omega_4),\omega_{R3}]$. It is interesting to note that $\omega_3=\omega_1$ and $\omega_4=\omega_2$ when

$$\eta=\eta_{1,2}\equiv\omega_{1,2}^2(\omega_{1,2}^2-\alpha\omega_{L1}^2)/\omega_{p1}^2(\omega_{1,2}^2-\beta).$$

For these values of the interface parameter η a simple substitution in Eq. (3) shows that $\omega_{R1}=\omega_1$ when $\eta=\eta_1$ and $\omega_{R3}=\omega_2$ for $\eta=\eta_2$. Thus the spectral range of the windows denoted by $\Delta\omega_{1,3}$, becomes zero for $\eta=\eta_{1,2}$, respectively, resulting in corresponding surface waves propagating with constant frequencies, i.e., without any dispersion. However, we note that the critical values $\eta_{1,2}$ exist only for interfaces for which either $\omega_{1,2}^2>\omega_{L2}^2$ or $\omega_{1,2}^2<\omega_{T2}^2$, i.e., for the interfaces for which the bulk coupled phonon-plasmon frequencies of medium 1 are greater than the LO frequency or are less than the TO frequency of medium 2.

Figure 1 shows the dispersion characteristics of optical-phonon-plasmon polariton modes for parameters $\epsilon_{\infty1}=15.68$, $\omega_{p1}=0.873\times10^{14}$/sec, $\omega_{T1}=0.339\times10^{14}$/sec, $\omega_{L1}=0.362\times10^{14}$/sec, and $\epsilon_{\infty2}=8.46$, $\omega_{p2}=0.568\times10^{14}$/sec, $\omega_{T2}=0.69\times10^{14}$/sec, $\omega_{L2}=0.757\times10^{14}$/sec corresponding to InSb-GaP interface at room temperature. The corresponding interface parameters are $\alpha=4.372$, $\beta=4.141$, $\eta=0.424$, and $\lambda=0.54$. Figure 2 gives the variation of the limiting frequencies and the spectral range of the windows of these modes with the plasma parameter η. The limiting characteristic wavelengths (corresponding to ω_R's and ω_i's) are seen to be ($\lambda_{R1}=22$ μm, $\lambda_3=23$ μm), ($\lambda_{R2}=29.4$ μm, $\lambda_4=39$ μm), and ($\lambda_{R3}=56$ μm, $\lambda_2=56.27$ μm). The spectral range for the low-frequency modes becomes zero at $\eta=0.18$ and for the high-frequency mode at $\eta=0.7$. We note that these two modes propagating along InSb-GaP interface show very little dispersion for η greater than the critical value η_2 and η_1. The spectral range $\Delta\omega_2$ is also seen to decrease with increasing η, although it never reaches the value zero.

In Fig. 3, we plot (solid lines) the exponential decay distances into the two polar semiconductors defined by $\delta_{1,2}=(k^2-\omega^2/c^2\epsilon_{1,2})^{-1/2}$. Since we have neglected damping, we find that for the low-fre-

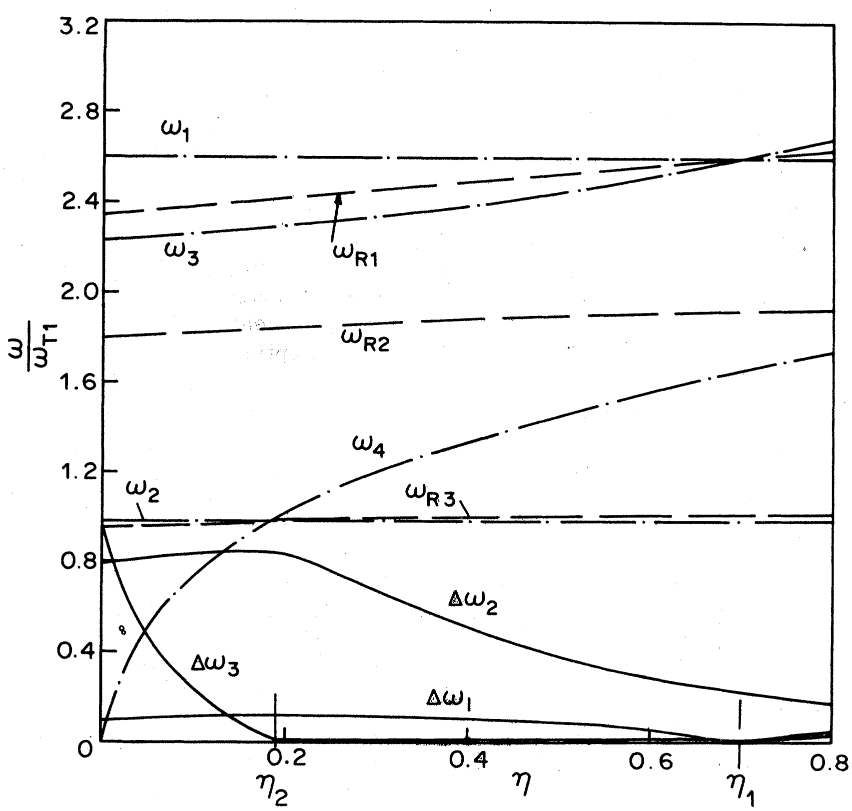

FIG. 2. Variation of the limiting frequencies ω_1, $\omega_2,\omega_3,\omega_4$ (dot-dashed lines) and $\omega_{R1},\omega_{R2},\omega_{R3}$ (dashed lines) corresponding to InSb-GaP interface when the ratio of plasma frequencies $\eta\equiv\omega_{p2}^2/\omega_{p1}^2$ is varied between 0 and 1. The variation of the spectral range of the windows $\Delta\omega_1,\Delta\omega_2$, and $\Delta\omega_3$ is also shown (solid lines). η_2 and η_1 are the critical values of η at which spectral range of two of the windows becomes zero. For InSb-GaP, $\eta_2=0.18$ and $\eta_1=0.7$.

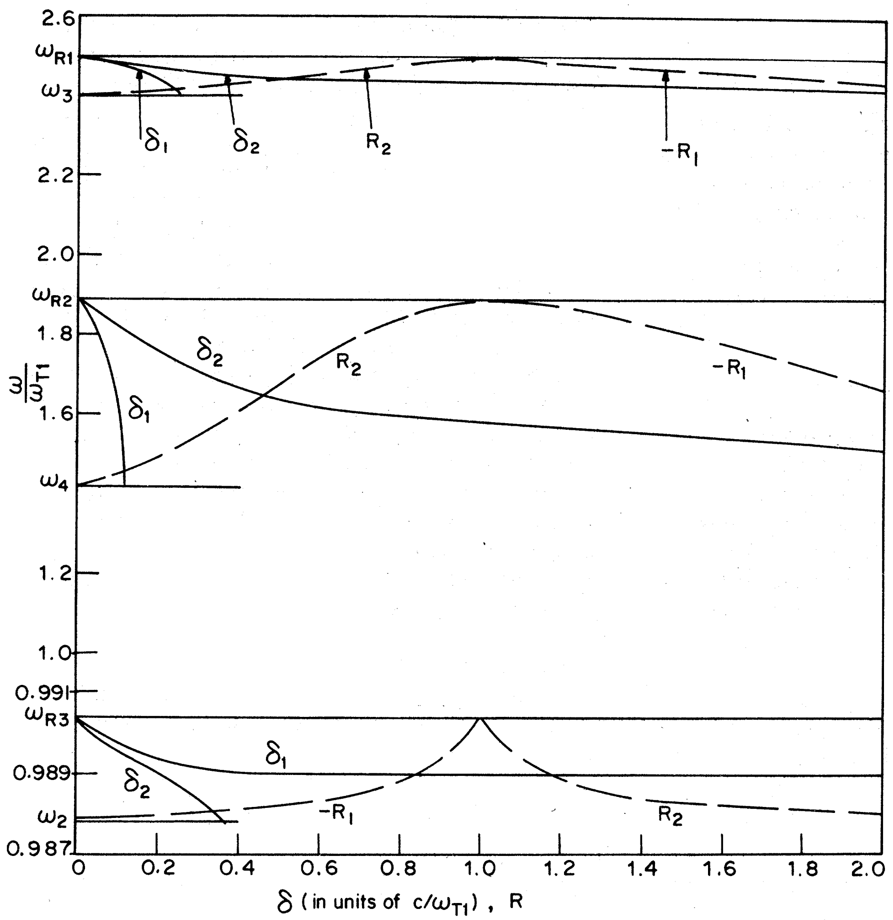

FIG. 3. Solid lines show the exponential decay distances (in units of c/ω_{T1}) into the two media and the broken lines the electric field ratio R at the interface $x=0$ in the two media.

quency mode, the decay distance is infinite at $\omega = \omega_2$ in medium 1, i.e., InSb and is infinite in medium 2, i.e., GaP for the other two modes at the corresponding limiting frequencies ω_4 and ω_3. We find penetration of 3.27×10^{-4} cm at $\omega=\omega_2$ of the low-frequency mode in medium 2 and 1.06×10^{-4} cm at $\omega=\omega_4$, 2.21×10^{-4} cm at $\omega=\omega_3$ in medium 1 for other two modes, respectively.

In Fig. 3, we also plot (broken lines) the electric field ratio, $R_{1,2}\equiv iE_y/E_{x1,2}=\mp(-\epsilon_{1,2}/\epsilon_{2,1})^{1/2}$ of wave fields at the interface. The behavior of electric field ratio for phonon polaritons at PS-PS interface is not very different from the plasmon polaritons at the SS interface (Ref. 1). At $\omega\gtrsim\omega_2$, the electric field for the low-frequency mode is approximately transverse ($E_y\sim0$) in medium 1 and it is approximately longitudinal ($E_x\sim0$) in medium 2. Similar situation arises at $\omega\gtrsim\omega_4$ and $\omega\gtrsim\omega_3$ for the other two modes where the electric field is approximately longitudinal in medium 1 and is approximately transverse in medium 2. In the limit $\omega\to\omega_{R1,2,3}$, we have $R=\pm i$, i.e., circular polarization in x-y plane.

The dispersion characteristics of surface modes along ionic crystal interfaces are being studied extensively in the experiments of surface-electromagnetic-wave spectroscopy by using the two-prism techniques developed by Schoenwald *et al.*[4] By employing the same technique, it should be possible to observe experimentally the dispersion of three types of coupled optical-phonon-plasmon surface polariton modes propagating in well defined windows along the interface between two polar semiconductor media. Even though the results reported in this investigation are based on considering an ideal situation of two semi-infinite media, the qualitative features of the discussion will persist in real experimental situation where thin polar semiconducting films of thickness d, which is greater than the decay distance δ of the modes, are deposited on other polar semiconductor substrates. For the considered example of InSb-GaP structure, δ is of the order of 2 μm for $\omega\lesssim\omega_R$. It is interesting to note that in actual experiments on thin films, film thickness usually varies from 1 to 10 μm. Further, in conclusion, we like to mention that the important feature of the frequency spectra of two of the coupled surface modes becoming zero at the specific values $\eta_{1,2}$ of the interface parameter η, the ratio of bulk plasmon frequencies of two media, can be used as a diagnostic tool to measure the carrier concentration of the films deposited on the polar semiconductors.

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