Cyclotron resonance in two interacting electron systems with application to Si inversion layers

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We have investigated the cyclotron resonance for two degenerate electron systems interacting with one another through the electron-electron collision time $\tau_e(T)$ defined as the relaxation time of the relative momentum. The coupled kinetic equations for the total and the relative momentum are solved in the presence of a static magnetic field and a frequency-dependent electric field. The solution for the power absorption is discussed in terms of the concentration ratio of the electrons, n_1/n_2, and the parameters τ_1/τ_e, τ_2/τ_e, where $\tau_{1,2}$ are ordinary scattering times. We find that our results are consistent with the observed temperature dependence of the cyclotron resonance of inversion layer electrons in Si, for $T \gtrsim 25$°K. At lower temperatures also, the frequency dependence of τ_e becomes important and affects the cyclotron resonance in a two-electron system, even as $T \to 0$.

I. INTRODUCTION

The recent observations[1-4] of cyclotron resonance in electron inversion layers on Si have led to the following dilemma. At the Si (100) surface, the energy levels in the electrostatic potential wall can be grouped into two different sets of overlapping subbands.[5] The first set is characterized by the discrete energy levels, $E_0, E_1, \ldots$, arising from the orbital quantization perpendicular to the surface, and by the constant-energy circles associated with the itinerant motion parallel to the surface. The effective mass is $m_1 = 0.19m$. This set is twofold degenerate if the intervalley interaction caused by the surface potential is neglected. The neglect appears justified because of the smallness of the relevant valley-splitting matrix element.[6] The second set of subbands is characterized by the discrete levels, $E_0', E_1', \ldots$. It has constant-energy ellipses associated with the band motion. The principal effective masses are $m_{2x} = 0.91m$ and $m_{2y} = 0.19m$ and the corresponding cyclotron resonance mass m_c is given by $m_2 = (m_{2x}m_{2y})^{1/2}$. This set of subbands is fourfold degenerate, corresponding to the four energy ellipsoids in the bulk, the axes of which are oriented along k_x and k_y. The known details of the subband structure are discussed in the literature.[7] Of particular interest here is the finding that $E_0 < E_0'$, although the energy $E_0' - E_0$ is not well known as a function of the temperature T, the surface-electron concentration $N_s(\mathrm{cm}^{-2})$, and an applied uniaxial stress.

For $E_0 < E_0'$, one expects at sufficiently low temperatures only the 0 subband to be partially occupied. The cyclotron-resonance dilemma is that, when there is thermally induced partial occupation of the 0′ subband as evidenced from optical inter-subband transitions $0' \to 1'$,[8] only a single resonance line is observed. Its mass value is intermediate between m_1 and m_2. A partial occupation of the 0′ subband induced by uniaxial stress also results in a cyclotron resonance characteristic of one type of electrons.[9] Kelly and Falicov[10] have addressed themselves to this cyclotron-resonance behavior taking into account the electron-electron interaction. In particular their intervalley exchange interaction is assumed to be attractive by virtue of phonon-exchange. They find on the Si (100) surface a charge-density-wave ground state for a restricted range of pressures. In this range m_c varies between m_1 and m_2 in a continuous manner. A second attempt to understand intermediate values of m_c is made by Ando[11] using Landau's theory of a Fermi liquid. His result for the high-frequency conductivity $\sigma_+(\omega)$ depends on Landau's interaction function only via the p-wave components of this function. These particular components are relevant for the mass enhancement[12] whereas the electron-electron (e-e) collision rate is primarily due to the s-wave component of Landau's function. This fact becomes immediately clear if we make the δ function test, that is, if we assume the e-e interactions in coordinate space to be replaced by a δ function. Then, the mass renormalizations become zero. According to Ando's result, $\sigma_+(\omega)$, also the effect of e-e scattering on the cyclotron-resonance behavior vanishes (a wrong result). On the basis of the ω-dependent current-current correlation function Ganguly and Ting[13] and Ting, Ying, and Quinn[13a,b] have recently calculated the dynamic conductivity in the Si inversion layer. The complex memory function, that replaces i/τ, depends on the electron-impurity interaction screened through a T- and ω-dependent density-density correlation function. They obtain a mass shift $\Delta m/m$ as a function of T and ω which is of

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the order of 10%.

In this brief paper, we calculate the cyclotron resonance for a system composed of two different types of degenerate electrons, characterized by their respective masses m_1 and m_2. As the pertinent el-el interaction we take into account the scattering between electrons of systems 1 and 2. The coupled transport equations are solved in Sec. II using the proper relaxation time τ_e that describes the relaxation of the total relative momentum towards zero. The results for the high-frequency conductivity $\sigma(\omega)$ are discussed in Sec. III in terms of the ratio between the concentrations of electrons 1 and 2, n_1/n_2 and the parameters $\tau_e/\tau_{1,2}$ where $\tau_{1,2}$ are the relaxation times due to impurity and phonon scattering. A discussion of the question: can the theoretical results account for the experimental cyclotron resonance behavior, concludes the paper.

The evaluation of $\tau_e(T)$ is given in Appendix A and that of $\tau_e(T)$ in Appendix B.

II. POWER ABSORPTION FOR THE TWO INTERACTING ELECTRON SYSTEMS

We calculate the power P absorbed by the electrons of system 1 and system 2 from the high-frequency electric field. To that end we assume that the static magnetic field $\vec{H}$ is oriented in the z direction and that the electric field $\vec{E}=\vec{E}_0e^{i\omega t}$ lies in the $(x\text{-}y)$ plane. Decomposing the linearly polarized field $\vec{E}$ into two circular counter-rotating components E_+, E_-, we have

$$\frac{P}{P_0}=\frac{P_++P_-}{P_0}=\frac{1}{4P_0}\,\mathrm{Re}(J_+E_+^*+J_-E_-^*), \tag{1}$$

or, in terms of the corresponding conductivities,[14]

$$P/P_0=\tfrac{1}{2}(\mathrm{Re}\sigma_+/\sigma_0+\mathrm{Re}\sigma_-/\sigma_0). \tag{2}$$

Here P_0 is the total power for the linear wave and σ_0 is the dc conductivity of the interacting electrons at $H=0$.

The conductivities $\sigma_\pm$ can be defined in terms of the total and the relative momentum of the two electrons systems,[15]

$$\vec{P}=\vec{P}_1+\vec{P}_2;\quad \vec{\Pi}=\mu(\vec{P}_1/n_1m_1-\vec{P}_2/n_2m_2), \tag{3}$$

where $1/\mu=1/n_1m_1+1/n_2m_2$. Here $\vec{P}$ and $\vec{\Pi}$ are vectors in the $x\text{-}y$ plane so that the current density $\vec{J}$ is given by

$$J_\pm=-\frac{e}{V}\left[\left(\frac{n_1+n_2}{n_1m_1+n_2m_2}\right)P_\pm+\left(\frac{1}{m_1}-\frac{1}{m_2}\right)\Pi_\pm\right], \tag{4}$$

where V is the volume under consideration.

To get the conductivities $\sigma_\pm$ from Eq. (4), we determine the linear relations between the momenta $\vec{P},\vec{\Pi}$ and the electric field $\vec{E}$. To that end we start from the kinetic equations for $\vec{P}$ and $\vec{\Pi}$. For the electrons 1 and 2 these coupled equations are, respectively, given by

$$\dot{\vec{\Pi}}+\frac{\mu}{n_2m_2}\dot{\vec{P}}=-eVn_1\vec{E}-\frac{e}{cm_1}\left(\vec{\Pi}-\frac{\mu}{n_2m_2}\vec{P}\right)\times\vec{H}-\frac{\vec{\Pi}}{\tau_e}-\frac{1}{\tau_1}\left(\vec{\Pi}+\frac{\mu}{n_2m_2}\vec{P}\right), \tag{5}$$

$$-\dot{\vec{\Pi}}+\frac{\mu}{n_1m_1}\dot{\vec{P}}=-eVn_2\vec{E}-\frac{e}{cm_2}\left(-\vec{\Pi}+\frac{\mu}{n_1m_1}\vec{P}\right)\times\vec{H}+\frac{\vec{\Pi}}{\tau_e}-\frac{1}{\tau_2}\left(-\vec{\Pi}+\frac{\mu}{n_1m_1}\vec{P}\right). \tag{6}$$

The relaxation time τ_e relaxes the relative momentum $\vec{\Pi}$ to zero,

$$\left.\frac{d\vec{\Pi}}{dt}\right|_{e\text{-}e}=-\frac{\vec{\Pi}}{\tau_e} \tag{7}$$

and it relaxes the momenta $\vec{P}_1$ and $\vec{P}_2$ towards the respective nonequilibrium momenta for strong e-e scattering,

$$\left.\frac{d\vec{P}_1}{dt}\right|_{e\text{-}e}=-\frac{\vec{P}_1-\vec{P}_1^0}{\tau_e} \tag{8}$$

and

$$\left.\frac{d\vec{P}_2}{dt}\right|_{e\text{-}e}=-\frac{\vec{P}_2-\vec{P}_2^0}{\tau_e}=-\left.\frac{d\vec{P}_1}{dt}\right|_{e\text{-}e}. \tag{9}$$

Equations (7)–(9) are in complete analogy to the case of electron-hole scattering first discussed in this manner by Kukkonen and Maldague.[15] The effect of the scattering between two types of carriers of the same or of opposite sign on the electric transport coefficients in zero magnetic field has been discussed using the variation principle.[16] A simple illustration of the nonequilibrium distributions of electrons 1 and electrons 2 for the two limiting cases $\tau_e=\infty$ and $\tau_e=0$ is shown in Fig. 1.

To solve Eqs. (5) and (6), let us first rewrite these equations by taking $\vec{P}(t)=\vec{P}e^{i\omega t}$, $\vec{\Pi}(t)=\vec{\Pi}e^{i\omega t}$, and by introducing the momentum components, $P_\pm=P_x\pm iP_y$ and $\Pi_\pm=\Pi_x\pm i\Pi_y$.

The two coupled equations for P_+ and Π_+ are given by

$$[1+\tau_1/\tau_e+i\tau_1(\omega-\omega_1)]\Pi_+-(\mu/n_2m_2)[1+i\tau_1(\omega-\omega_1)]P_+=-eVn_1\tau_1E_+, \tag{10}$$

$$-[1+\tau_2/\tau_e+i\tau_2(\omega-\omega_2)]\Pi_+-(\mu/n_1m_1)[1+i\tau_2(\omega-\omega_2)]P_+=-eVn_2\tau_2E_+, \tag{11}$$

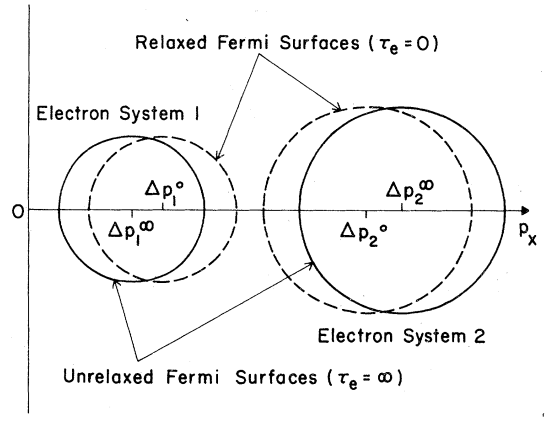

FIG. 1. Illustration of the nonequilibrium Fermi surfaces for electrons 1 and electrons 2 when an electric field $\vec{E}$ is applied in x direction. Here $\Delta p_{1,2}^{\infty} = -e\tau_{1,2}E$ are the momentum shifts of the Fermi spheres in the absence of e-e scattering, $\tau_e = \infty$. In the opposite case of strong e-e scattering $\tau_e = 0$ the momentum shifts $\Delta p_{1,2}^{0}$ of the relaxed Fermi spheres correspond to a complete relaxation of the relative momentum, $\vec{\Pi} = 0$.

where $\omega_{1,2} = eH/cm_{1,2}$. We get the two coupled equations for P_- and Π_- by replacing in Eqs. (10) and (11) E_+ by E_- and $\omega_{1,2}$ by $-\omega_{1,2}$. The solutions of Eqs. (10) and (11) are given by

$$P_+ = p_+ E_+ = \pi_+ E_+ , \tag{12}$$

where

$$p_+ = -\frac{eV}{D_+}\{n_1\tau_1[1+\tau_2/\tau_e + i\tau_2(\omega-\omega_2)] + n_2\tau_2[1+\tau_1/\tau_e + i\tau_1(\omega-\omega_1)]\} \tag{13}$$

and

$$\pi_+ = -(eV/D_+)\{\mu\tau_1/m_1[1+i\tau_2(\omega-\omega_2)] - (\mu\tau_2/m_2)[1+i\tau_1(\omega-\omega_1)]\}, \tag{14}$$

and

$$D_+ = 1 + \mu\left(\frac{1}{n_1 m_1}\frac{\tau_1}{\tau_e} + \frac{1}{n_2 m_2}\frac{\tau_2}{\tau_e}\right) - \tau_1(\omega-\omega_1)\tau_2(\omega-\omega_2) + i\tau_1(\omega-\omega_1)\left(1+\frac{\mu}{n_2 m_2}\frac{\tau_2}{\tau_e}\right) + i\tau_2(\omega-\omega_2)\left(1+\frac{\mu}{n_1 m_1}\frac{\tau_1}{\tau_e}\right). \tag{15}$$

The results for p_-, π_- are of the same form as Eqs. (13) and (14); however, $\omega_{1,2}$ is replaced by $-\omega_{1,2}$. In terms of the p's and π's, the conductivities are given by [Eq. (4)]

$$\sigma_\pm = -(e/V)[(n/M)p_\pm + (1/m_1 - 1/m_2)\pi_\pm], \tag{16}$$

where $n = n_1 + n_2$ and $M = M_1 + M_2 = n_1 m_1 + n_2 m_2$. The power absorption obtained from Eq. (16) is written down taking $\tau_1 = \tau_2 = \tau$ for the sake of simplicity, we get

$$\frac{P}{P_0} = -\frac{e}{2V\sigma_0}\left[\frac{n}{M}\mathrm{Re}(p_+ + p_-) + \left(\frac{1}{m_1} - \frac{1}{m_2}\right)\mathrm{Re}(\pi_+ + \pi_-)\right] = \frac{e^2 n\tau}{2\sigma_0 M(|D_+||D_-|)^2}\left[n(B_+|D_-|^2 + B_-|D_+|^2) + \left(\frac{m_2}{m_1} - \frac{m_1}{m_2}\right)(C_+|D_-|^2 + C_-|D_+|^2)\right], \tag{17}$$

where

$$B_\pm = \left(1+\frac{\tau}{\tau_e}\right)^2 - \frac{\tau}{\tau_e}\frac{n_1 M_1 + n_2 M_2}{nM}\tau^2(\omega\mp\omega_1)(\omega\mp\omega_2) + \left(1+\frac{\tau}{\tau_e}\frac{M_2}{M}\right)\frac{n_1}{n}\tau^2(\omega\mp\omega_2)^2 + \left(1+\frac{\tau}{\tau_e}\frac{M_1}{M}\right)\frac{n_2}{n}\tau^2(\omega\pm\omega_1)^2, \tag{18}$$

$$C_\pm = \left(1+\frac{\tau}{\tau_e}\right)\frac{n_1 M_2 - n_2 M_1}{nM} + \frac{\tau}{\tau_e}\frac{(n_1-n_2)M_1 M_2}{nM^2}\tau^2(\omega\mp\omega_1)(\omega\mp\omega_2) + \left(1+\frac{\tau}{\tau_e}\frac{M_2}{M}\right)\frac{n_1 M_2}{nM}\tau^2(\omega\mp\omega_2)^2 - \left(1+\frac{\tau}{\tau_e}\frac{M_1}{M}\right)\frac{n_2 M_1}{nM}\tau^2(\omega\mp\omega_1)^2, \tag{19}$$

and $\sigma_0 = \sigma_+ = \sigma_-$ for $H = \omega = 0$. It is readily seen that in the absence of e-e scattering ($\tau_e/\tau \to \infty$) and for a one-electron system ($m_1 = m_2$) the power absorption, Eq. (19), is given by the standard formula [cf. Ref. 14, Eq. I.5)].

III. CYCLOTRON RESONANCE ABSORPTION IN TERMS OF τ_{-e}

In Figs. 2 and 3, results for the absorption behavior are plotted taking $m_1 = 0.2\,m$ and $m_2 = 0.4\,m$. The parameter of the different absorption curves is τ/τ_e; τ_e is the e-e relaxation time for the relative momentum, Eq. (A23), and τ is the transport relaxation time due to thermal phonons and imperfections. Our choice of $\omega_1\tau = 3$ is characteristic of the experimental resonance conditions of Refs. 3 and 4.

The following general trends are seen from the

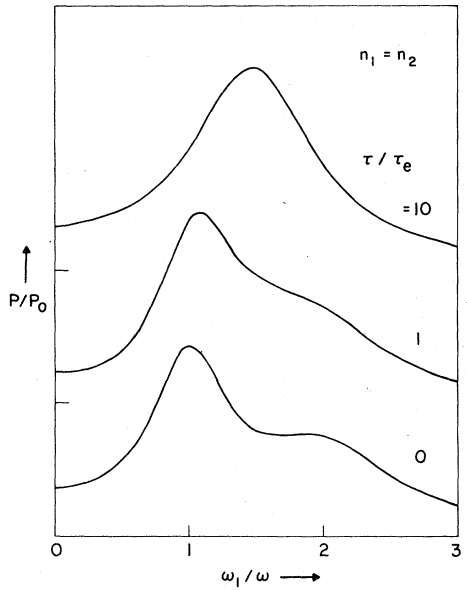

FIG. 2. Relative change in the power absorption P vs the magnetic field for $\omega_1\tau = 3$; $\omega_1 = eH/cm_1$ and τ is the relaxation time due to phonon and impurity scattering. The parameter on the absorption curves is τ/τ_e, where τ_e is the relative-momentum relaxation time, Eq. (7), due to scattering between electrons of the two different systems. n_1 and n_2 are the concentrations of the electrons is systems 1 and 2, respectively.

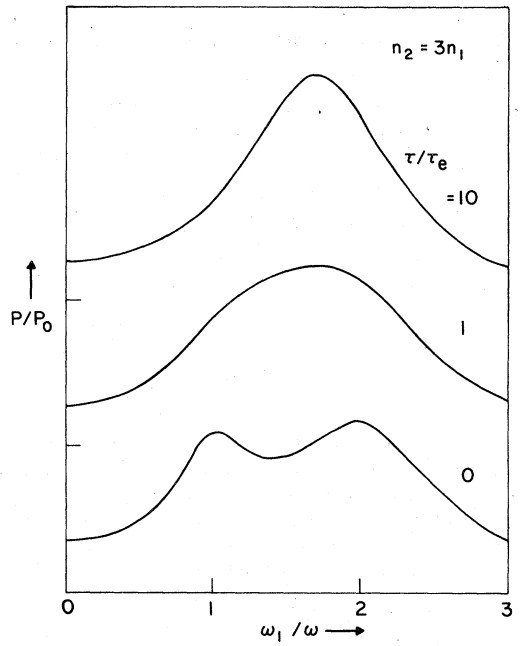

FIG. 3. Same caption as Fig. 2. The difference is the ratio n_2/n_1.

model results. For $\tau/\tau_e = 0$, we get the two peaks at ω_1 and ω_2 which are characteristic of noninteracting electrons. The relative peak heights depend on n_2/n_1. At an intermediate value $\tau/\tau_e = 1$, the two peaks begin to merge, although a shoulder remains for $n_1 = n_2$, Fig. 2. The resonance peaks have shifted towards intermediate values, between ω_1 and ω_2. Finally, for strong e-e interactions, $\tau/\tau_e = 10$, there is only a single resonance, the center of which is determined by n_2/n_1 and the width of which is governed by τ. The peak position depends on the concentration averaged mass, $m_c = (n_1m_1 + n_2m_2)/(n_1 + n_2)$, for $\tau/\tau_e \gg 1$.

A quantitative comparison between theory and experiment is not possible. One reason is that the distance between the 0 and 0′ subbands $E_0' - E_0$, is not known with sufficient accuracy as a function of temperature and applied uniaxial stress. This distance determines the occupation of the subbands, N_1 and N_2, by virtue of a self-consistent calculation of the surface potential. By comparing the qualitative features of the model results, Figs. 2 and 3, with the temperature dependence of the observed cyclotron resonance, Ref. 3, it appears possible that e-e scattering is responsible for the observation of a single resonance peak. A crucial role for the occurrence of a single peak is played the magnitude of the scattering time τ_e. We therefore do not merely consider τ_e to be an adjustable parameter but calculate it as the relaxation time for the relative momentum in Appendix A. The result is given by Eq. (A23). It is symmetric with respect to the parameters of system 1 and system 2 and it depends on the collision probability w between two electrons of 1 and 2, respectively. By taking w as constant, the integrations over the scattering angles were readily performed. For the sake of simplicity, we shall assume here that w is given by the $q = 0$ component of the Fourier transformed scattering potential.[17] In the Fermi-Thomas approximation, this potential is given by

$$V_{\mathrm{FT}}(r) = (e^2/\epsilon r)e^{-q_{\mathrm{FT}}r}, \tag{20}$$

where ϵ is the dielectric constant and q_{FT} is the screening parameter given by

$$q_{\mathrm{FT}}^2 = q_{\mathrm{FT},1}^2 + q_{\mathrm{FT},2}^2 = (4e^2/\pi\epsilon\hbar^4)(m_1p_{F1} + m_2p_{F2}). \tag{21}$$

Using $p_{F1,2} = \hbar^{-1}(3\pi^2 n_{1,2})^{1/3}$, we have

$$\omega = (2\pi/\hbar)|V_{\mathrm{FT}}(0)|^2 = 2\pi^3(\tfrac{1}{3}\pi)^{2/3} \times \hbar^3(m_1n_1^{1/3} + m_2n_2^{1/3})^{-2}, \tag{22}$$

where n_i is the volume concentration $(=N_i^{3/2})$. Inserting this expression in Eq. (A23) and, furthermore taking account of the degeneracies ν_1 and ν_2 of the electron systems 1 and 2, respectively, the relaxation time is given by

$$\frac{1}{\tau_e(T)} = \pi\left(\frac{\pi}{3}\right)^{2/3} \frac{(kT)^2}{\hbar^3}\, \nu_1\nu_2\, m_1^2\, m_2^2 \times\left(\frac{1}{n_1 m_1}+\frac{1}{n_2 m_2}\right) n_1 L[(n_1/n_2)^{1/3}] \times \frac{1}{(m_1 n_1^{1/3}+m_2 n_2^{1/3})^2}\,, \tag{23}$$

where the function L is of the order-of-magnitude 1 and is given by Eq. (A20) using $p_{F1}/p_{F2} = (n_1/n_2)^{1/3} = p$. In Fig. 4 the relaxation rate is plotted versus n_1/n_2 for a fixed set of parameter values for m_1, m_2, ν_1, ν_2, and n_1+n_2, and it is seen that the quantity $1/\tau_e T^2$ does not vary much in the range $10^{-2} \leq n_1/n_2 \leq 10^{+2}$, except for the cusp at $n_1 = n_2$. The main dependence of τ_e^{-1} on n_1, n_2 is given by the factor $(n_1+n_2)^{-2/3}$ arising from q_{FT}^{-2}. Assuming that the surface concentration $N_i = n_i^{2/3}$, the relaxation rate in the inversion layer of Si is of the order $\tau_e^{-1} \sim 3.10^{10}\, T^2[10^{11}/(N_1+N_2)]$ with T in °K and $N_{1,2}$ in cm^{-2}.

For a qualitative comparison with the experimental results for the T dependence of the cyclotron resonance in Ref. 3, we may assume that the parameter τ/τ_e in our formula for P/P_0, Eq. (17), depends on N_1, N_2 through the sum N_1+N_2, the magnitude of which is experimentally known from the gate voltage. The experimental values of τ determined in Ref. 3 from the width of the resonance lines at $T = 25.5$ and 65 °K are 6×10^{-13} and 3×10^{-13} sec, respectively. The corresponding values of τ/τ_e are 2 and 8, using $(N_1+N_2)_{expt} = 5\times10^{11}$ cm^{-2}. Hence at both temperatures, e–e scattering is so strong that only a single resonance line is to be expected (see, for example, Figs. 3 and 4). The position of the resonance peak depends on N_1/N_2, Eq. (17). The experimental power absorption at 25.5 °K yields a cyclotron resonance mass $m_c = 0.21\, m$. This value is larger than the 0-subband mass, $m_1 = 0.19\, m$, and therefore can be attributed to a small occupation of the 0′ subband $m_2 = 0.42\, m$. At 65 °K, $m_c = 0.27\, m$ indicating a partial occupation of the 0′ subband such that N_2 becomes comparable with N_1.

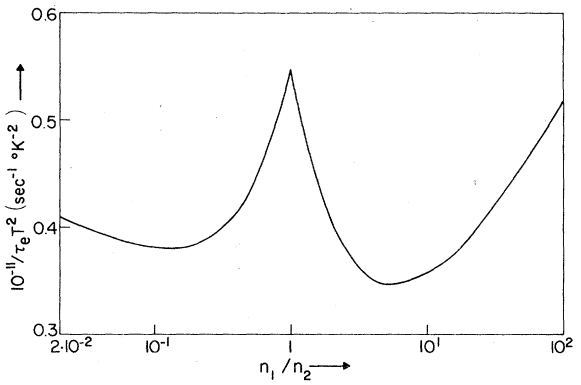

FIG. 4. Plot of the e-e scattering rate, Eq. (23), versus the concentration ratio n_1/n_2, where $n_{1,2}$ are the volume concentrations of the electrons in systems 1 and 2, respectively. The parameter values of the two systems as chosen as follows: $m_1 = 0.2m$, $\nu_1 = 2$ (degeneracy); $m_2 = 0.4m$, $\nu_2 = 4$; $n_1+n_2 = 3.16\times10^{16}$ cm^{-3}.

Hence, the T dependence of the cyclotron resonance mass of inversion-layer electrons in Si can be understood in a qualitative manner when we take into account the combined effects of both a partial thermal occupation of the 0′ subband and the e-e scattering between the carriers in different subbands. In particular, the e-e scattering rates are such that $\tau_e^{-1} \gtrsim \tau^{-1}$ for the experimental conditions. The observation that our results are consistent with the experimental findings rests on the assumption of a partial occupation of the 0′-subband. It appears from the results of the optical intersubband absorption[8] that this assumption is justified, although the exact dependence of N_1/N_2 is as yet unknown.

Finally, we would like to address ourselves to the question: What is the effect of the electron-electron collisions on the cyclotron resonance behavior at very low temperatures? As $T \to 0$, the collision rate $1/\tau_e$ vanishes as $(kT)^2$ (cf. Appendix A) if one neglects the effect of the oscillating electric field on $1/\tau_e$. This neglect, however, is no longer justified at frequencies ω where $\hbar\omega \gtrsim kT$. Even at $T = 0$, an electron-electron scattering process can be accompanied by the absorption of a photon and, thereby, lead to an $(\hbar\omega)^2$ contribution to $1/\tau_e$. The final result for the temperature *and* frequency-dependent collision rate, derived in Appendix B, is given by

$$1/\tau_e(T,\omega) = b[(kT)^2 + (\hbar\omega/2\pi)^2], \tag{24}$$

where b is the factor of $(kT)^2$ in Eq. (23). As for the experimental situation of Ref. 3, the frequency of the electric field is $\nu = 890.7$ GHz and corresponds to $T = 7$ °K, at which temperature both of the terms in the bracket of Eq. (24) have the same magnitude. The corresponding value of τ_e, Eq. (24), is $\sim 5\times10^{-12}$ sec and is comparable with the impurity scattering time τ at this temperature. Hence there is no reason to expect two distinct resonance lines at $T \sim 7$ °K if both the 0 and 0′ subbands are sufficiently occupied.

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APPENDIX A: SCATTERING TIME $\tau_{-e}(T)$

The time τ_e is defined by Eq. (7) as the relaxation time for the relative momentum $\vec{\Pi}$ between electrons 1 and 2; we can write

$$\left.\frac{d\vec{P}_1}{dt}\right|_{e-e} = -\frac{\vec{P}_1 - \vec{P}_1^0}{\tau_e} , \tag{A1}$$

where $\vec{P}_1^0$ is the total momentum of electrons 1 when $\vec{\Pi}=0$. In the presence of an electric field $\vec{E}$, we have

$$\vec{P}_1^0 = n_1 m_1 \vec{v}_{\text{c.m.}} , \tag{A2}$$

where n_1 is the concentration of electrons 1 and the center-of-mass velocity is

$$\vec{v}_{\text{c.m.}} = -[(n_1\tau_1 + n_2\tau_2)/(n_1 m_1 + n_2 m_2)]\, e\vec{E} . \tag{A3}$$

In general, $\vec{\Pi} \neq 0$, the momentum $\vec{P}_1$ is given by

$$\vec{P}_1 = \int f(\vec{p}_1)\vec{p}_1 \, d\tau_1 , \tag{A4}$$

where $d\tau_1 = [2/(2\pi\hbar)^3]d^3p_1$. The rate of change of $\vec{P}_1$ can be written in terms of the collision integral $I(\vec{p}_1)$

$$\left.\frac{d\vec{P}_1}{dt}\right|_{e-e} = \int \left.\frac{\partial f}{\partial t}\right|_{e-e} \vec{p}_1 d\tau_1 = -\int I(\vec{p}_1)\vec{p}_1 \, d\tau_1 , \tag{A5}$$

where

$$I(\vec{p}_1) = -\int d\tau_{\bar{1}} \int d\tau_{\bar{2}} \int d^3p_2 \, w(\vec{p}_1, \vec{p}_2; \vec{p}_{\bar{1}}\vec{p}_{\bar{2}})[f_1 f_2(1-f_{\bar{1}})(1-f_{\bar{2}}) - f_{\bar{1}} f_{\bar{2}}(1-f_1)(1-f_2)]$$
$$\times \delta(\epsilon_1 + \epsilon_2 - \epsilon_{\bar{1}} - \epsilon_{\bar{2}})\delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_{\bar{1}} - \vec{p}_{\bar{2}}) . \tag{A6}$$

Here w is the collision probability for two electrons, $f_1 \equiv f(\vec{p}_1)$ and $\epsilon_1 = p_1^2/2m_1$. The δ functions express energy and momentum conservation in a collision process. The collision integral vanishes, $I(\vec{p}_1)=0$, for $f_1 = f_1^0$, etc., where f_1^0 is the displaced Fermi distribution in Fig. 1, corresponding to $\vec{\Pi}=0$.

We proceed by expanding the distribution functions in Eq. (A6) around their displaced counter parts for $\vec{\Pi}=0$. For example,

$$f_1 = f_1^0 + \frac{\partial f_1^0}{\partial \epsilon_1}\Psi_1(\epsilon_1) . \tag{A7}$$

We may replace $\partial f_1^0/\partial\epsilon_1$ by

$$\frac{\partial f_{10}}{\partial \epsilon_1} = -\frac{1}{kT} f_{10}(1-f_{10}) , \tag{A8}$$

where $f_{10} = \{\exp[(\epsilon_1 - \zeta/kT] - 1\}^{-1}$ is the equilibrium distribution; ζ = Fermi energy. The perturbation Ψ_1 in the presence of an electric field $\vec{E}$ is given by

$$\Psi_1 = (\vec{\Delta}p_1 - \vec{\Delta}p_1^0)\vec{p}_1/m_1 . \tag{A9}$$

By substituting (A7) into Eq. (A6) we get

$$I(\vec{p}_1) = \frac{1}{kT}\int\int\int d\tau_{\bar{1}}\, d\tau_{\bar{2}}\, d^3p_2 \, w \left((\Delta\vec{p}_1 - \Delta\vec{p}_1^0)\frac{\vec{p}_1}{m_1} + (\Delta\vec{p}_2 - \Delta\vec{p}_2^0)\frac{\vec{p}_2}{m_2} - (\Delta\vec{p}_1 - \Delta\vec{p}_1^0)\frac{\vec{p}_1}{m_1} - (\Delta\vec{p}_2 - \Delta\vec{p}_2^0)\frac{\vec{p}_{\bar{2}}}{m_2} \right)$$
$$\times f_{10} f_{20}(1-f_{\bar{1}0})(1-f_{\bar{2}0})\delta(\epsilon_1 + \epsilon_2 - \epsilon_{\bar{1}} - \epsilon_{\bar{2}})\delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_{\bar{1}} - \vec{p}_{\bar{2}}) . \tag{A10}$$

The factor $\Delta\vec{p}_1 - \Delta\vec{p}_1^0$ can be taken out of the integral since

$$\Delta\vec{p}_2 - \Delta\vec{p}_2^0 = -(n_1/n_2)(\Delta\vec{p}_1 - \Delta\vec{p}_1^0),$$

by virtue of total-momentum conservation. Hence

$$I(\vec{p}_1) = n_1(\Delta\vec{p}_1 - \Delta\vec{p}_1^0)\vec{K}(\vec{p}_1), \tag{A11}$$

where

$$\vec{K}(\vec{p}_1) = \frac{1}{kT}\int\int\int d\tau_{\bar{1}}\, d\tau_{\bar{2}} d^3p_2 w \left(\frac{\vec{p}_1}{n_1 m_1} - \frac{\vec{p}_2}{n_2 m_2} - \frac{\vec{p}_{\bar{1}}}{n_1 m_1} + \frac{\vec{p}_{\bar{2}}}{n_2 m_2} \right) f_{10} f_{20}(1-f_{\bar{1}0})(1-f_{\bar{2}0})$$
$$\times \delta(\epsilon_1 + \epsilon_2 - \epsilon_{\bar{1}} - \epsilon_{\bar{2}})\ \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_{\bar{1}} - \vec{p}_{\bar{2}}) . \tag{A12}$$

By substituting (A11) into Eq. (A5) we get

$$\frac{1}{\tau_e} = \int \vec{K}(\vec{p}_1)\vec{p}_1 \, d\tau_1 . \tag{A13}$$

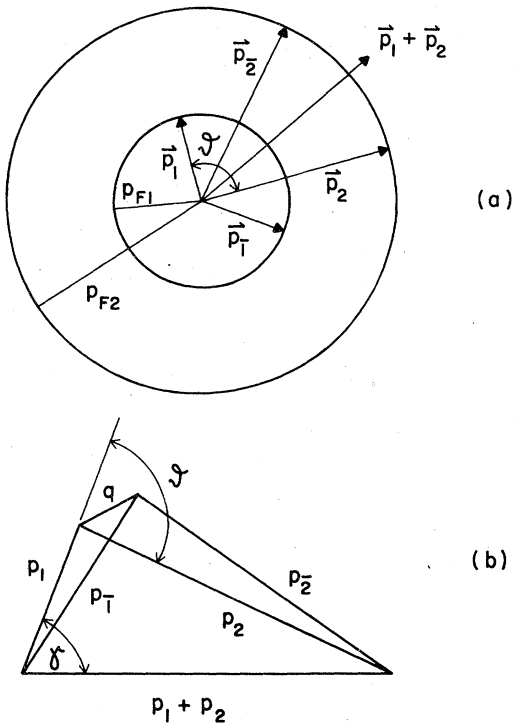

FIG. 5. Collision geometry. (a) The maximum angle between $\vec{p}_1$ and $\vec{p}_2$ is independent of the ratio between the Fermi momenta p_{F1} and p_{F2} for electron systems 1 and 2. (b) It is assumed that the plane spanned by $\vec{p}_{\bar{1}}$, $\vec{p}_{\bar{2}}$ is rotated by an angle φ into the plane of $\vec{p}_{\bar{1}}$ and $\vec{p}_{\bar{2}}$. Then $\sin\gamma = \sin\vartheta/[1+(p_{1F}/p_{2F})^2+2(p_{1F}/p_{2F})\cos\vartheta]^{1/2}$. For $p_{1F}=p_{2F}$, one has $\gamma=\frac{1}{2}\vartheta$ (Ref. 19).

We proceed by rewriting $\vec{K}$ in terms of suitable integration variables before we finally carry out the integration over $\vec{p}_1$ in Eq. (A13).

In calculating τ_e we consider only intravalley transitions, $\vec{p}_1 \to \vec{p}_1 + \vec{q}$, $\vec{p}_2 \to \vec{p}_2 - \vec{q}$; we neglect intervalley transitions, $\vec{p}_1 \to \vec{p}_2 + \vec{q}$, $\vec{p}_2 \to \vec{p}_1 - \vec{q}$. The reason is that since the 1 and 2 electron surfaces are so far apart in momentum space, that the $|\vec{q}|$ which is involved in the intervalley scattering process is large. This leads to a very small $V_{FT}(q)$ [of Eq. (20)]. The processes we keep involve $|\vec{q}|$'s which are at most $2p_F$ for the individual subbands and $2p_F/\hbar$ is very small compared to the Brillouin zone size.

Fixing $\vec{p}_1$, the momentum $\vec{p}_2$ is defined by the angles ϑ and φ_2 with respect to $\vec{p}_1$ as the polar axis. According to the collision geometry of Fig. 5(a),

$$0 \leq \vartheta \leq \pi; \quad 0 \leq \varphi_2 \leq 2\pi. \tag{A14}$$

The integration over $\vec{p}_{\bar{1}}$ is written in terms of the angle φ between the planes $\vec{p}_1$, $\vec{p}_2$ and $\vec{p}_{\bar{1}}$, $\vec{p}_{\bar{2}}$ and the two variables $p_{\bar{1}}$ and $p_{\bar{2}}$. According to Fig. 5(b) we have

$$\begin{aligned} d\tau_{\bar{1}} &= \frac{2}{(2\pi\hbar)^3} p_{F1} \sin\gamma \, dq_r \, dq_z \, d\varphi , \\ p_{\bar{1}} &= p_1 + q_z \cos\gamma + q_r \sin\gamma , \\ p_{\bar{2}} &= p_2 - q_2 \cos(\vartheta - \gamma) + q_r \sin(\vartheta - \gamma) , \\ dp_{\bar{1}} dp_{\bar{2}} &= [\sin\gamma \cos(\vartheta - \gamma) \\ &\quad + \cos\gamma \sin(\vartheta - \gamma)] dq_r dq_z . \end{aligned} \tag{A15}$$

Using $dp_{\bar{1}} = (m_1/p_{F1}) d\epsilon_{\bar{1}}$ and $dp_{\bar{2}} = (m_2/p_{F2}) d\epsilon_{\bar{2}}$ and introducing the variables $x = (\epsilon_{\bar{1}} - \zeta)/kT$ and $y = (\epsilon_{\bar{2}} - \zeta)/kT$, we get

$$d\tau_{\bar{1}} = \frac{2}{(2\pi\hbar)^3} \frac{1}{(1+p^2+2\cos\vartheta)^{1/2}} \times \frac{m_1 m_2}{p_{F2}} (kT)^2 \, dx \, dy \, d\varphi , \tag{A16}$$

where $p \equiv p_{F1}/p_{F2}$.

The integration over $\vec{p}_{\bar{2}}$ removes the δ function responsible for momentum conservation.

The integration over $\vec{p}_2$ is carried out by writing

$$\begin{aligned} d^3 p_2 &= p_2^2 dp_2 \sin\vartheta d\vartheta d\varphi_2 \\ &= p_{F2} m_2 kT dz \sin\vartheta d\vartheta d\varphi_2 . \end{aligned} \tag{A17}$$

Substituting (A16) and (A17) into Eq. (A13) for τ_e, we get

$$\begin{aligned} \frac{1}{\tau_e} = \int d\tau_1 \int\int\int dx\,dy\,dz \int_0^{\pi} d\varphi \int_0^{2\pi} d\varphi_2 \int_0^{\pi} \sin\vartheta d\vartheta \frac{w(\vartheta,\varphi)}{kT} \frac{2}{(2\pi\hbar)^3} \frac{1}{(1+p^2+2p\cos\vartheta)^{1/2}} \frac{m_1 m_2}{p_{F2}^2} (kT)^2 \\ \times \frac{2}{(2\pi\hbar)^3} m_2 p_{F2} kT \left(\frac{1}{n_1 m_1} + \frac{1}{n_2 m_2} \right) (p_1^2 - \vec{p}_1 \cdot \vec{p}_{\bar{1}}) \\ \times \frac{\delta(t+z-x-y)}{kT} f_0(t) f_0(z) [1 - f_0(x)][1 - f_0(y)] , \end{aligned} \tag{A18}$$

where $t = (\epsilon_1 - \zeta)/kT$ and

$$p_1^2 - \vec{p}_1 \cdot \vec{p}_{\bar{1}} = \frac{\sin^2\vartheta(1+\cos\varphi)}{1+p^2+2p\cos\vartheta} p_1^2 . \tag{A19}$$

Replacing $w(\vartheta, \varphi)$ by a constant, we can perform the integration over ϑ,

$$\begin{aligned} L(p) &= \frac{1}{2} \int_0^{\pi} \frac{\sin^3\vartheta d\vartheta}{(1+p^2+2p\cos\vartheta)^{3/2}} \\ &= \tfrac{2}{3}, \quad p \leq 1 \\ &= \tfrac{2}{3} \cdot \frac{1}{p^3}, \quad p \geq 1 . \end{aligned} \tag{A20}$$

The x, y, z integral is according to Ref. (18) given by

$$M(t)=\int\int\int dx\,dy\,dz\,\delta(t+z-x-y) \times f_0(t)f_0(z)[1-f_0(x)][1-f_0(y)] = \tfrac{1}{2}f_0(t)[1-f_0(t)](\pi^2+t^2). \quad \text{(A21)}$$

The t integration is performed by writing

$$d\tau_1 = 2(2\pi\hbar)^{-3}4\pi m_1 p_{F_1} kT\,dt,$$

$$\int_{-\infty}^{+\infty} M(t)dt = -\int_{-\infty}^{+\infty} \frac{\partial f_0}{\partial t}\frac{\pi^2+t^2}{2}dt = \frac{2\pi^2}{3}+\tfrac{1}{10}\pi^4\left(\frac{kT}{\zeta}\right)^2 + \cdots (kT/\zeta)^2,$$

where the dots denote higher-order terms. The final result for τ_e is given by

$$\frac{1}{\tau_e} = \frac{1}{6\pi^4\hbar^9}\left(\frac{1}{n_1 m_1}+\frac{1}{n_2 m_2}\right) \times m_1^2 m_2^2 p_{F_1}^3 L\left(\frac{p_{F_1}}{p_{F_2}}\right) w(kT)^2, \quad \text{(A23)}$$

where w is the constant collision probability substituted for $w(\vartheta, \varphi)$. Eq. (A23) is *symmetric* with respect to the interchange of indices 1 and 2. Assuming that 1 and 2 refer to the electron and hole Fermi surfaces of a degenerate semimetal, Eq. (A23) agrees with the first term in Eq. (9) of Ref. 15.

APPENDIX B: SCATTERING TIME $\tau_e(T, \omega)$

To obtain the frequency dependence of τ_e, we proceed by formulating the power absorption due to electron-electron collisions in the presence of a time-dependent, spatially homogeneous electric field. The procedure consists in two steps. First the Hamiltonian for two noninteracting electrons is solved for the time-dependent wave function in the presence of an oscillating electric field. Then, using these wave functions, we calculate the power absorption due to the electron-electron interaction in the framework of time-dependent first-order perturbation theory.

The time-dependent field is given by

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \quad \vec{A} = \frac{c}{\omega}\vec{E}_0\cos\omega t. \quad \text{(B1)}$$

In the presence of this field, the two-electron Schrödinger equation has the form

$$(H_1+H_2)\psi = i\hbar\frac{\partial\psi}{\partial t}, \quad \text{(B2)}$$

where

$$H_i = \frac{1}{2m_i}\left(\vec{p}_i - \frac{e}{c}\vec{A}_i\right)^2 = \frac{p_i^2}{2m_i} - \frac{e\vec{E}_0\cdot\vec{p}_i}{m_i\omega}\cos\omega t + \frac{e^2E_0^2}{2m_i\omega^2}E_0^2\cos^2\omega t, \quad i=1,2. \quad \text{(B3)}$$

The solution, $\psi_{12}\equiv\psi_{\vec{k}_1\vec{k}_2}$, of Eq. (B2) is given by

$$\psi_{12}(\vec{r}_1,\vec{r}_2;t) = \varphi_{12}(\vec{r}_1,\vec{r}_2)\exp\left\{-\frac{i}{\hbar}\left[(\epsilon_1+\epsilon_2)t - \left(\frac{\vec{k}_1}{m_1}+\frac{\vec{k}_2}{m_2}\right)\frac{\vec{E}_0 e\hbar\sin\omega t}{\omega^2}\right]\right\}, \quad \text{(B4)}$$

where $\varphi_{12} = (1/V)\exp(i\vec{k}_1\cdot\vec{r}_1 + i\vec{k}_2\cdot\vec{r}_2)$ and $\epsilon_i = \hbar^2k_i^2/2m_i$. Assuming that the external field is small,

$$\hbar e\vec{E}_0\cdot\vec{k}_i/m_i\omega \equiv \hbar\omega\vec{e}_i\cdot\vec{k}_i \ll \epsilon_i, \quad \text{(B5)}$$

we can expand the exponential function, Eq. (B4), in terms of $\vec{e}_i = (e/m_i\omega^2)\vec{E}_0$ and ignore terms of second and higher order in this parameter.

Then, using standard time-dependent perturbation theory, we have the following ansatz for the wave function perturbed by electron-electron collisions:

$$\Psi = \sum_{\vec{k}_1,\vec{k}_2} a_{12}(t)\varphi_{12}(\vec{r}_1,\vec{r}_2)\exp\left[-\frac{i}{\hbar}(\epsilon_1+\epsilon_2)t\right] \times [1+i(\vec{k}_1\cdot\vec{e}_1+\vec{k}_2\cdot\vec{e}_2)\sin\omega t]. \quad \text{(B6)}$$

The result for the first-order coefficient $a_{12}^{(1)}$ is given by

$$|a_{12}^{(1)}(t)|^2 = \frac{4}{\hbar^2}|H_{12,\bar{1}\bar{2}}^{(\mathrm{int})}|^2\left[\frac{\sin^2\frac{1}{2}\omega_{12,\bar{1}\bar{2}}t}{\omega_{12,\bar{1}\bar{2}}^2} + 4[(\vec{k}_{\bar{1}}-\vec{k}_1)\cdot\vec{e}_1+(\vec{k}_{\bar{2}}-\vec{k}_2)\cdot\vec{e}_2] \times\left(\frac{\sin^2\frac{1}{2}(\omega_{12,\bar{1}\bar{2}}+\omega)t}{(\omega_{12,\bar{1}\bar{2}}+\omega)^2} + \frac{\sin^2\frac{1}{2}(\omega_{12,\bar{1}\bar{2}}-\omega)t}{(\omega_{12,\bar{1}\bar{2}}-\omega)^2}\right)\right], \quad \text{(B7)}$$

where $H_{12,\bar{1}\bar{2}}^{(\mathrm{int})}$ is the matrix element of the screened scattering potential between φ_{12} and $\varphi_{\bar{1}\bar{2}}$. In terms of $a_{12}^{(1)}$, we get the transition probability, $\vec{k}_{\bar{1}}\to\vec{k}_1$ and $\vec{k}_{\bar{2}}\to\vec{k}_2$, in the form

$$w = t^{-1} \sum_{1,2} |a_{12}^{(1)}(t)|^2$$

$$= \frac{2\pi}{\hbar}\left(\frac{2V}{(2\pi)^3}\right) \int\int_{\substack{\text{empty}\\ \text{states}}} d^3k_1\, d^3k_2\, |H_{12,\bar{1}\bar{2}}^{(\text{int})}|^2 \{\delta(\epsilon_1+\epsilon_2-\epsilon_{\bar{1}}-\epsilon_{\bar{2}}) + 4[(\vec{k}_{\bar{1}}-\vec{k}_1)\cdot\vec{e}_1 + (\vec{k}_{\bar{2}}-\vec{k}_2)\cdot\vec{e}_2]$$

$$\times[\delta(\epsilon_1+\epsilon_2-\epsilon_{\bar{1}}-\epsilon_{\bar{2}}+\hbar\omega) + \delta(\epsilon_1+\epsilon_2-\epsilon_{\bar{1}}-\epsilon_{\bar{2}}-\hbar\omega)]\}\,. \qquad \text{(B8)}$$

To account for momentum conservation, we write

$$|H_{12,\bar{1}\bar{2}}^{(\text{int})}|^2 = W\,\delta(\vec{k}_1+\vec{k}_2-\vec{k}_{\bar{1}}-\vec{k}_{\bar{2}})\,, \qquad \text{(B9)}$$

assuming that W is a constant (dimension: energy^2/cm^2).

To get the power absorption P due to electron-electron scattering we simply integrate w over the filled states $\vec{k}_1, \vec{k}_2$, thereby multiplying the last two energy δ functions by $+\hbar\omega$ and $-\hbar\omega$, respectively. At $T=0$, only the last δ function, corresponding to the absorption of a phonon during a scattering event, contributes to P.

At $T\neq 0$, either an absorption or an emission process may take place in an electron-electron collision. The formal result for the power absorption, using Eqs. (B8) and (B9) and the momentum variables introduced in Appendix A, is given by

$$P = \left(\frac{2V}{(2\pi\hbar)^3}\right)^4 \int_{\epsilon_1} 4\pi m_1 p_{F1}\, d\epsilon_1 \int_{\epsilon_2} 2\pi m_2 p_{F2}\, d\epsilon_2 \int_0^{\pi} \sin\vartheta\, d\vartheta \int_{\epsilon_{\bar{1}}} d\epsilon_{\bar{1}} \int_{\epsilon_{\bar{2}}} d\epsilon_{\bar{2}} \int_0^{\pi} d\varphi\, \frac{m_1 m_2}{(1+p^2+2p\cos\vartheta)^{1/2}}$$

$$\times 8\pi W[(\vec{p}_{\bar{1}}-\vec{p}_1)(\vec{e}_1-\vec{e}_2)]^2\hbar\omega[\delta(\epsilon_1+\epsilon_2-\epsilon_{\bar{1}}-\epsilon_{\bar{2}}-\hbar\omega) - \delta(\epsilon_1+\epsilon_2-\epsilon_{\bar{1}}-\epsilon_{\bar{2}}+\hbar\omega)]$$

$$\times f_0(\epsilon_{\bar{1}}) f_0(\epsilon_{\bar{2}})[1-f_0(\epsilon_1)][1-f_0(\epsilon_2)]\,. \qquad \text{(B10)}$$

Using the approximation

$$[(\vec{p}_{\bar{1}}-\vec{p}_1)\cdot(\vec{e}_1-\vec{e}_2)]^2 = \tfrac{1}{3}|\vec{p}_{\bar{1}}-\vec{p}_1|^2\,|\vec{e}_1-\vec{e}_2|^2\,,$$

the integrations are readily carried out. The final result is given by

$$\frac{P}{V} = \frac{4}{3\pi^2}\,\frac{(1/m_1-1/m_2)^2}{1/n_1m_1+1/n_2m_2}\,\frac{e^2E_0^2}{\omega^2}\,\frac{1}{\tau_e(T,\omega)}\,, \qquad \text{B11)}$$

where $\tau_e(T,\omega)$ is given by Eq. (24). The coefficient b in Eq. (24) is the factor of $(kT)^2$ in Eq. (A23). We mentioned also that W, defined by Eq. (B9), is related to the collsions probability w in Eq. (A6) by writing $W=(\hbar/V^3)w$.

An expression of the form of Eq. (24) for $\tau_e^{-1}(T,\omega)$ has been given by Gurzhi,[20] Hopfield,[21] and Quinn,[22] without derivation of b.

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$$M = \frac{1}{\Omega_0}\iiint_{\text{u.c.}} u_{\vec{k}_1}^*(\vec{r})\,u_{\vec{k}_2}(\vec{r})\,e^{i\vec{K}\cdot\vec{r}}\,d^3r\,,$$

where $\vec{k}_1$ and $\vec{k}_2$ define the minima of the two opposite valleys along the z direction of $\vec{k}$ space; $\vec{K}$ is oriented in z direction and has the value $2\pi/a$, where a is the lattice constant, $\Omega_0(=a^3)$ is the volume of the unit cell (u.c.). By evaluating the cell-periodic wave functions $u_{\vec{k}}$ around the twofold degenerate state X_1 with the $\vec{k}\cdot\vec{p}$ perturbation method, we get $M=0$. A more accurate calculation of M using the cell periodic functions of M. Cardona and F. Pollak [Phys. Rev. 142, 530 (1966)] will yield a finite M. We are indebted to J. J. Quinn for a correspondence on the valley splitting. L. J. Sham and M. Nakayama [Second International Conference on Electronic Properties of Two-Dimensional Systems, edited by J. F Koch and G. Landweher, Berechtesgaden, Germany, Sept. 1977 (unpublished) p. 576] have evaluated the valley splitting using the effective mass approximation based on a set of electron wave functions which satisfy the boundary conditions at the oxide-semiconductor interface. The split-

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