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# Impurity-induced free-carrier magnetoabsorption in semiconductors

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The theory of the intraband magnetoabsorption is derived in the quantum limit for semiconductors with ionized impurities. The $0\rightarrow 0$ and $0\rightarrow 1$ transitions are considered. For cyclotron-resonance-inactive polarization a logarithmic divergency of the absorption coefficient at cyclotron frequency is obtained. No such singularity exists for parallel polarization.

## I. INTRODUCTION

In the last few years, many observations of cyclotron-resonance harmonics in semiconductors were reported. Some of them were ascribed to the electron-impurity interaction (see, e.g., Ref. 1). Magnetoabsorption due to this interaction was studied theoretically by several authors.[2-5] Usually, electrons were treated as bound to impurities and the transition rates between their discrete states were considered.[2,3] The present authors have recently calculated the magnetoabsorption in the quantum limit for free electrons, i.e., assuming that the impurities are ionized.[4,5] This treatment is justified for high concentrations of impurities, and also if $kT$ is of the order of or higher than the binding energy in a magnetic field (but much lower than $\hbar\omega_c$). These conditions are usually fulfilled in narrow-gap semiconductors. Transitions $0\rightarrow n'$ for $n'\geqslant 2$ were studied, and it was shown that logarithmic divergencies of the absorption coefficient appear at all $\omega = n'\omega_c$ for both cyclotron-resonance-active and -inactive polarizations, but not for parallel polarization. The contribution given by the composition fluctuations in mixed semiconductors was also calculated.

A magnetoabsorption peak for cyclotron-resonance-inactive polarization was observed also for $\omega=\omega_c$.[6] The purpose of the present paper is to extend the results of Ref. 5 (hereafter referred to as I), to the case of impurity-induced transitions $0\rightarrow 0$ and $0\rightarrow 1$ for both cyclotron-resonance-inactive (CRI) and parallel ($\vec{E}\,||\,\vec{H}$) polarizations. Therefore, we consider the free electrons in the quantum limit, interacting with the radiation field and with some random time-independent potential. This potential will then be identified with the potential due to the ionized impurities distributed at random. The case of a mixed semiconductor with fluctuations of band edge due to composition fluctuations will also be considered. We assume a nondegenerate parabolic and spherical conduction band and we neglect spin effects. Plasma and screening effects will not be taken into account.

## II. MAGNETO-OPTICAL TRANSITION RATES

In the weak-radiation approximation, the effective-mass Hamiltonian for an electron of the charge $-e$ and the effective mass $m^*$ in the static magnetic field $\vec{H}$ along the $z$ axis, can be written in the form

$$\mathcal{H}=\mathcal{H}_0+U+\mathcal{H}_r\,, \tag{1}$$

where

$$\mathcal{H}_0=-\frac{\hbar^2}{2m^*}\left[\frac{\partial^2}{\partial x^2}-\left(\frac{x}{\lambda^2}-i\frac{\partial}{\partial y}\right)^2+\frac{\partial^2}{\partial z^2}\right]. \tag{2}$$

$$U=\sum_{\vec{q}}{}'\,v(\vec{q})\exp(i\vec{q}\cdot\vec{r})\,, \tag{3}$$

$$\mathcal{H}_r=-i\frac{\hbar e}{m^*\omega}\left[\mathrm{Im}\,(E_xe^{-i\omega t})\frac{\partial}{\partial x}+\mathrm{Im}\,(E_ye^{-i\omega t})\left(i\frac{x}{\lambda^2}+\frac{\partial}{\partial y}\right)+\mathrm{Im}(E_ze^{-i\omega t})\frac{\partial}{\partial z}\right]. \tag{4}$$

We have assumed here a uniform oscillating electric field of radiation of the form

$$\mathrm{Re}\,[\vec{E}\exp(-i\omega t)]\,,\quad \omega>0\,, \tag{5}$$

where $\vec{E}$ is a complex vector. The vector potential is chosen in the form

$$\vec{A}(\vec{r},t)=(0,Hx,0)+(c/\omega)\,\mathrm{Im}(\vec{E}e^{-i\omega t})\,, \tag{6}$$

and the scalar potential of the radiation vanishes. $\lambda=(c\hbar/eH)^{1/2}$ is the usual magnetic length. $v(\vec{q})$ are the Fourier components of the random potential due to crystal imperfections. In Eq. (3) the summation runs over all $\vec{q}\neq 0$ allowed by a periodicity box:

$$q_\alpha=(2\pi/L_\alpha)Q_\alpha\quad(\alpha=x,y,z)\,, \tag{7}$$

where $Q_\alpha$ are integers. To keep $U$ real we have to assume

$$v(-\vec{q})=v^*(\vec{q})\,. \tag{8}$$

The eigenstates of $\mathcal{H}_0$ are the Landau states $|k_y k_z n\rangle$ of the energies

$$\mathcal{E}_{k_z n}=\hbar\omega_c(n+\tfrac{1}{2})+\hbar^2k_z^2/2m^* , \tag{9}$$

where $n=0, 1, \ldots$ and $\omega_c=eH/cm^*$. $k_y$ and $k_z$ have the form

$$k_\alpha=(2\pi/L_\alpha)K_\alpha \quad (\alpha=y, z), \tag{10}$$

where $K_\alpha$ are integers. If the electron is in the periodicity box, its center of the orbit should be in a position between $-\frac{1}{2}L_x$ and $\frac{1}{2}L_x$. This yields a condition

$$-L_x/2\lambda^2<k_y\leq L_x/2\lambda^2 . \tag{11}$$

The perturbation given by Eqs. (3) and (4) can be written in the form

$$\mathcal{H}_1\cos\omega t+\mathcal{H}_2\sin\omega t+U . \tag{12}$$

Let us define

$$\mathcal{H}_\pm=\mathcal{H}_1\pm i\mathcal{H}_2 . \tag{13}$$

We will use the second-order perturbation calculus and neglect terms of higher than second order in $U$. We assume $\omega\neq\omega_c$ and consider only CRI or $\vec{E}\parallel\vec{H}$ polarizations, i.e., we put

$$E_x=iE_y . \tag{14}$$

Denoting

$$E_+=2^{-1/2}(E_x+iE_y)=2^{1/2}E_x, \tag{15}$$

we have

$$\langle k_y''' k_z''' n''' |\mathcal{H}_+|k_y'' k_z'' n''\rangle=-(\hbar e/m^*\omega)\delta_{k_y''' k_y''}\delta_{k_z''' k_z''}[iE_z k_z''\delta_{n''' n''}+(1/\lambda)E_+ n''^{1/2}\delta_{n''' n''-1}] \tag{16}$$

$$\langle k_y''' k_z''' n''' |\mathcal{H}_-|k_y'' k_z'' n''\rangle=-(\hbar e/m^*\omega)\delta_{k_y''' k_y''}\delta_{k_z''' k_z''}[-iE_z^* k_z''\delta_{n''' n''}+(1/\lambda)E_+^* n'''^{1/2}\delta_{n''' n''+1}] \tag{17}$$

With our assumptions, it follows from Eqs. (16) and (17) that for $\mathcal{E}_{k_z''' n'''}>\mathcal{E}_{k_z'' n''}$

$$\langle k_y''' k_z''' n''' |\mathcal{H}_+|k_y'' k_z'' n''\rangle=0 . \tag{18}$$

For $\mathcal{E}_{k_z''' n'''}<\mathcal{E}_{k_z'' n''}$

$$\langle k_y''' k_z''' n''' |\mathcal{H}_-|k_y'' k_z'' n''\rangle=0 . \tag{19}$$

For $\mathcal{E}_{k_z'' n''}=\mathcal{E}_{k_z n}$

$$\langle k_y' k_z' n' |\mathcal{H}_\pm|k_y'' k_z'' n''\rangle=0 . \tag{20}$$

For $\mathcal{E}_{k_z'' n''}=\mathcal{E}_{k_z' n'}$

$$\langle k_y'' k_z'' n'' |\mathcal{H}_\pm|k_y k_z n\rangle=0 . \tag{21}$$

Thus, the second-order perturbation calculus gives for $\mathcal{E}_{k_z' n'}>\mathcal{E}_{k_z n}$, i.e., for absorption process, the transition rate

$$\begin{aligned} W_{|k_y k_z n\rangle\to|k_y+q_y, k_z+q_z, n'\rangle}=\frac{\pi}{2\hbar^2}\delta\left(\omega-\frac{1}{\hbar}(\mathcal{E}_{k_z+q_z n'}-\mathcal{E}_{k_z n})\right)\Bigg| & -\sum_{\substack{k_y'' k_z'' n''\\ \mathcal{E}_{k_z'' n''}\neq\mathcal{E}_{k_z n}}}\langle k_y+q_y, k_z+q_z, n' |\mathcal{H}_+|k_y'' k_z'' n''\rangle \\ & \times\langle k_y'' k_z'' n'' |U|k_y k_z n\rangle(\mathcal{E}_{k_z'' n''}-\mathcal{E}_{k_z n})^{-1} \\ +\sum_{\substack{k_y'' k_z'' n''\\ \mathcal{E}_{k_z'' n''}\neq\mathcal{E}_{k_z+q_z, n'}}}\langle k_y+q_y, k_z+q_z, n' |U|k_y'' k_z'' n''\rangle & \\ \times\langle k_y'' k_z'' n'' |\mathcal{H}_+|k_y k_z n\rangle(\mathcal{E}_{k_z+q_z n'}-\mathcal{E}_{k_z'' n''})^{-1}\Bigg|^2 . & \end{aligned} \tag{22}$$

We consider here only the case of quantum limit at low temperatures. Therefore, only transitions $0\to n'$ with photon absorption are possible. As already mentioned, we restrict our calculations $0\to 0$ and $0\to 1$ transitions.

Contributions to the transition rates are visualized on Fig. 1.

The matrix elements of $U$ are (see paper I)

$$\langle k_y+q_y, k_z+q_z, n'' \,|\, U \,|\, k_y k_z 0\rangle = (\tfrac{1}{2}\lambda^2)^{n''/2}(n''!)^{-1/2} \sum_{q_x}{}' \exp(-i\lambda^2 k_y q_x) \times \exp(-\tfrac{1}{2}i\lambda^2 q_x q_y)(iq_x+q_y)^{n''} \exp(-\tfrac{1}{4}\lambda^2 q_\perp^2)\nu(\vec{q}) , \tag{23}$$

where summing over $q_x$ we omit $q_x=0$ if $q_y=q_z=0$. We have denoted

$$q_\perp^2 = q_x^2 + q_y^2 . \tag{24}$$

As we want to calculate the absorption, we need only the value of $W_{|k_y k_z 0\rangle \to |k_y+q_y, k_z+q_z, n'\rangle}$ averaged over all $k_y$. We will denote it by $\overline{W}(k_z, 0; q_y, q_z, n')$. To obtain $\overline{W}$, we use Fig. 1 and insert Eqs. (9), (16), and (23), into Eq. (22). Using the formula

$$\sum_{k_y} \exp[i\lambda^2(q_x' - q_x)k_y] = \frac{L_x L_y}{2\pi\lambda^2}\delta_{q_x' q_x} \tag{25}$$

strictly valid for $L_\alpha/2\pi^{1/2}\lambda$, $\alpha = x, y$, being integers), we find for $n'=0, 1$

$$\overline{W}(k_z, 0; q_y, q_z, n') = \pi \frac{e^2}{2\hbar^2 m^{*2}\omega^2}\, \delta\left(\omega - n'\omega_c - \frac{\hbar}{2m^*}(q_z^2 + 2k_z q_z)\right) \times \sum_{q_x}{}'' (\tfrac{1}{2}\lambda^2 q_\perp^2)^{n'} \exp(-\tfrac{1}{2}\lambda^2 q_\perp^2)\,|\nu(\vec{q})|^2 \left(|E_+|^2 \frac{q_\perp^2}{2(\omega+\omega_c)^2} + |E_z|^2 \frac{q_z^2}{\omega^2} + \frac{2^{1/2} q_z}{\omega(\omega+\omega_c)}[q_x \operatorname{Re}(E_+ E_z^*) + q_y \operatorname{Im}(E_+ E_z^*)]\right) . \tag{26}$$

## III. INTRABAND MAGNETOABSORPTION IN THE QUANTUM LIMIT

In the quantum limit, electron occupy only all the states with $n=0$ and $|k_z| \leq k_{zF}$, where

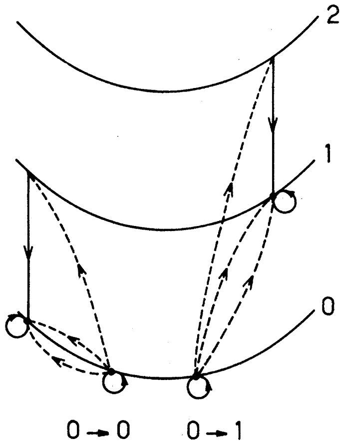

FIG. 1. Contributions to the transition rate [Eq. (22)] for transitions 0 → 0 and 0 → 1. Vertical transitions are due to radiation and the oblique ones to the potential of crystal imperfections.

$$k_{zF} = \pi^2\lambda^2 N_e \tag{27}$$

and $N_e$ is the electron concentration. The condition of the quantum limit reads

$$\hbar\omega_c > \hbar^2 k_{zF}^2/2m^* \tag{28}$$

or

$$\tfrac{1}{2}\lambda^2 k_{zF}^2 < 1 . \tag{29}$$

Power absorbed per unit volume because of transitions from all occupied states (of $n=0$) to all empty states of quantum number $n'$ ($n'=0,1$) will be denoted by $P_{n'}$. To calculate it, we have to multiply the expression (26) by $\hbar\omega$, to sum over all occupied states (accounting for spin degeneracy) and over all final empty states, and to divide by $L_x L_y L_z$. Then we can perform summation over $k_z$, and replace the random variable $|\nu(\vec{q})|^2$ by its value $\langle|\nu(\vec{q})|^2\rangle_{\text{av}}$.[7] Taking into account Eq. (8), using the notation

$$\langle|\nu(\vec{q})|^2\rangle = L_x L_y L_z \langle|\nu(\vec{q})|^2\rangle_{\text{av}} , \tag{30}$$

and replacing the summation over $\vec{q}$ by integration we obtain finally

$$P_0=\frac{e^2\omega_c}{16\pi^4\hbar^3\omega}\int d^2q_\perp \exp(-\tfrac{1}{2}\lambda^2q_\perp^2)\left[\int_{q_{z3}}^{q_{z4}} dq_z-\epsilon\left(\omega<\frac{\hbar k_{zF}^2}{2m^*}\right)\int_{q_{z5}}^{q_{z6}} dq_z\right]$$
$$\times q_z^{-1}\left(|E_+|^2\frac{q_\perp^2}{2(\omega+\omega_c)^2}+|E_z|^2\frac{q_z^2}{\omega^2}+\frac{2^{1/2}q_z}{\omega(\omega+\omega_c)}[q_x\operatorname{Re}(E_+E_z^*)+q_y\operatorname{Im}(E_+E_z^*)]\right)\langle|\nu(\vec{q})|^2\rangle, \tag{31}$$

$$P_1=\epsilon\left(\omega\geqslant\omega_c-\frac{\hbar k_{zF}^2}{2m^*}\right)\left((e^2/32\pi^4\hbar^2m^*\omega)\int d^2q_\perp q_\perp^2\exp(-\tfrac{1}{2}\lambda^2q_\perp^2)\int_{q_{z1}}^{q_{z2}} dq_z q_z^{-1}\right)$$
$$\times\left(|E_+|^2\frac{q_\perp^2}{2(\omega+\omega_c)^2}+|E_z|^2\frac{q_z^2}{\omega^2}+\frac{2^{1/2}q_z}{\omega(\omega+\omega_c)}[q_x\operatorname{Re}(E_+E_z^*)+q_y\operatorname{Im}(E_+E_z^*)]\right)\langle|\nu(\vec{q})|^2\rangle. \tag{32}$$

We have denoted here

$$q_{z1}=\left|k_{zF}-\left(k_{zF}^2+\frac{2m^*}{\hbar}(\omega-\omega_c)\right)^{1/2}\right|, \tag{33}$$

$$q_{z2}=k_{zF}+\left(k_{zF}^2+\frac{2m^*}{\hbar}(\omega-\omega_c)\right)^{1/2}, \tag{34}$$

$$q_{z3}=-k_{zF}+\left(k_{zF}^2+\frac{2m^*}{\hbar}\omega\right)^{1/2}, \tag{35}$$

$$q_{z4}=k_{zF}+\left(k_{zF}^2+\frac{2m^*}{\hbar}\omega\right)^{1/2}, \tag{36}$$

$$q_{z5}=k_{zF}-\left(k_{zF}^2-\frac{2m^*}{\hbar}\omega\right)^{1/2}, \tag{37}$$

$$q_{z6}=k_{zF}+\left(k_{zF}^2-\frac{2m^*}{\hbar}\omega\right)^{1/2}. \tag{38}$$

By definition $\epsilon=1$ if the indicated inequality is fulfilled, and equals zero otherwise.

For $\omega=\omega_c|P_1$ has a divergency. Close to $\omega=\omega_c$, i.e., for

$$|\omega-\omega_c|/\omega_c\ll\tfrac{1}{2}\lambda^2k_{zF}^2, \tag{39}$$

there is

$$P_1\approx|E_+|^2\frac{e^2}{256\pi^4\hbar^2m^*\omega_c^3}\ln\left(\frac{\frac{1}{2}\lambda^2k_{zF}^2\omega_c}{|\omega-\omega_c|}\right)$$
$$\times\int d^2q_\perp q_\perp^4\exp(-\tfrac{1}{2}\lambda^2q_\perp^2)\langle|\nu(\vec{q}_\perp)|^2\rangle, \tag{40}$$

where

$$\vec{q}_\perp=(q_x,q_y,0). \tag{41}$$

There is no divergency of $P_0$ for any $\omega>0$.

We will identify now the potential $U$ with the potential given by ionized impurities (point defects) present in the semiconductor. Suppose there are $S$ types of such ions. $Z_le$ and $N_l$ ($l=1,\ldots,S$) are the charge and concentration of the $l$-type ion, respectively, ($Z_l$ may be a positive or negative integer). Neutrality requirement yields the condition

$$\sum_{l=1}^{S}Z_lN_l=N_e. \tag{42}$$

Using the Poisson equation and assuming a perfectly random distribution of ions, we obtain

$$\langle|\nu(\vec{q})|^2\rangle=16\pi^2e^4DN_e/\kappa^2q^4, \tag{43}$$

where $\kappa$ is the dielectric constant of the crystal and

$$D=\sum_{l=1}^{S}Z_l^2\frac{N_l}{N_e}. \tag{44}$$

It is always $D\geqslant1$. $D=1$ only if all $Z_l=+1$. If all $Z_l=Z$ ($Z>0$) then $D=Z$. Suppose, finally, that there are both positive and negative ions of the charges $Ze$ ($Z>0$) and $-Ze$ and concentrations $N_+$ and $N_-$, respectively ($ZN_+-ZN_-=N_e$). Denoting

$$C=N_-/N_+ \tag{45}$$

($C$ is the compensation ratio if all donors and acceptors are ionized), we have

$$D=Z(1+C)/(1-C). \tag{46}$$

Inserting Eq. (43) into Eqs. (31) and (32), and multiplying by

$$8\pi/c\kappa^{1/2}(|E_+|^2+|E_z|^2), \tag{47}$$

we obtain the absorption coefficients $K_0^i$ and $K_1^i$ due to $0\to0$ and $0\to1$ transitions, respectively,

$$\begin{aligned} K_0^i = {} & \frac{4e^6 D N_e \omega_c}{c\hbar^3 \kappa^{5/2}\omega}\left(|E_+|^2+|E_z|^2\right)^{-1} \\ & \times \Bigg\{ \frac{|E_+|^2}{2(\omega+\omega_c)^2}\Big[A\left(\tfrac{1}{2}\lambda^2 q_{z3}^2\right) - A\left(\tfrac{1}{2}\lambda^2 q_{z4}^2\right) + \exp\left(\tfrac{1}{2}\lambda^2 q_{z3}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z3}^2\right) - \exp\left(\tfrac{1}{2}\lambda^2 q_{z4}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z4}^2\right) - \epsilon\left(\omega < \frac{\hbar k_{zF}^2}{2m^*}\right)\Big] \\ & \qquad \times \Big[A\left(\tfrac{1}{2}\lambda^2 q_{z5}^2\right) - A\left(\tfrac{1}{2}\lambda^2 q_{z6}^2\right) + \exp\left(\tfrac{1}{2}\lambda^2 q_{z5}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z5}^2\right) - \exp\left(\tfrac{1}{2}\lambda^2 q_{z6}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z6}^2\right)\Big] \\ & + \frac{|E_z|^2}{\omega^2}\Big[\exp\left(\tfrac{1}{2}\lambda^2 q_{z4}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z4}^2\right) - \exp\left(\tfrac{1}{2}\lambda^2 q_{z3}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z3}^2\right) - \epsilon\left(\omega < \frac{\hbar k_{zF}^2}{2m^*}\right) \\ & \qquad \times \left[\exp\left(\tfrac{1}{2}\lambda^2 q_{z6}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z6}^2\right) - \exp\left(\tfrac{1}{2}\lambda^2 q_{z5}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z5}^2\right)\right]\Big]\Bigg\}. \end{aligned} \tag{48}$$

$$\begin{aligned} K_1^i = {} & \epsilon\left(\omega \geqslant \omega_c - \frac{\hbar k_{zF}^2}{2m^*}\right)\frac{4e^6 D N_e \omega_c}{c\hbar^3\kappa^{5/2}\omega}\left(|E_+|^2+|E_z|^2\right)^{-1} \\ & \times \Bigg(\frac{|E_+|^2}{2(\omega+\omega_c)^2}\left[\left(1+\tfrac{1}{2}\lambda^2 q_{z2}^2\right)\exp\left(\tfrac{1}{2}\lambda^2 q_{z2}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z2}^2\right) - \left(1+\tfrac{1}{2}\lambda^2 q_{z1}^2\right)\exp\left(\tfrac{1}{2}\lambda^2 q_{z1}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z1}^2\right)\right] \\ & + \frac{|E_z|^2}{\omega^2}\left[\tfrac{1}{2}\lambda^2 q_{z1}^2\exp\left(\tfrac{1}{2}\lambda^2 q_{z1}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z1}^2\right) - \tfrac{1}{2}\lambda^2 q_{z2}^2\exp\left(\tfrac{1}{2}\lambda^2 q_{z2}^2\right)\mathrm{Ei}\left(-\tfrac{1}{2}\lambda^2 q_{z2}^2\right)\right]\Bigg). \end{aligned} \tag{49}$$

The exponential integral function is defined (for $x<0$)

$$\mathrm{Ei}(x) = -\int_{-x}^{\infty}\frac{1}{t}e^{-t}\,dt\,. \tag{50}$$

We have also denoted (for $x>0$)

$$A(x) = \int_0^{\infty}\ln(1+t)e^{-xt}\frac{dt}{t} \tag{51}$$

For $\omega$ close to $\omega_c$ there is

$$K_1^i \approx \frac{e^6 D N_e}{c\hbar^3\kappa^{5/2}\omega_c^2}|E_+|^2\left(|E_+|^2+|E_z|^2\right)^{-1} \times \ln\left(\frac{\frac{1}{2}\lambda^2 k_{zF}^2\omega_c}{|\omega-\omega_c|}\right). \tag{52}$$

If we identify the potential $U$ with the position-dependent band edge in a mixed semiconductor with composition fluctuations, then for a perfectly random alloy (see paper I)

$$\langle|\nu(\vec{q})|^2\rangle = \beta_0^2\frac{1}{N}v_0(1-v_0)\,. \tag{53}$$

$N$ is the number of unit cells per unit volume, $v_0$ is the average composition of the alloy of the type $A_{1-v}B_v$ (or $A_{1-v}B_vC$, etc.), and $\beta_0$ is a material parameter. For simplicity, we have assumed that another material parameter appearing in Eq. (53), $\gamma_0$, is equal to zero. Inserting Eq. (53) into Eqs. (31) and (32), and multiplying by expression (47), we obtain the absorption coefficients $K_0^c$ and $K_1^c$ due to 0 → 0 and 0 → 1 transitions, respectively,

$$\begin{aligned} K_0^c = {} & \left[\frac{e^2\beta_0^2 v_0(1-v_0)m^{*2}\omega_c^3}{\pi^2 c\hbar^5 N\kappa^{1/2}\omega}\right]\left(|E_+|^2+|E_z|^2\right)^{-1}\Bigg\{\frac{|E_+|^2}{(\omega+\omega_c)^2}\left[\ln\frac{q_{z4}}{q_{z3}} - \epsilon\left(\omega<\frac{\hbar k_{zF}^2}{2m^*}\right)\ln\frac{q_{z6}}{q_{z5}}\right] \\ & + \frac{|E_z|^2}{\omega^2}\left[\tfrac{1}{2}\lambda^2\left(q_{z4}^2-q_{z3}^2\right) - \epsilon\left(\omega<\frac{\hbar k_{zF}^2}{2m^*}\right)\tfrac{1}{2}\lambda^2\left(q_{z6}^2-q_{z5}^2\right)\right]\Bigg\}, \end{aligned} \tag{54}$$

$$\begin{aligned} K_1^c = {} & \epsilon\left(\omega \geqslant \omega_c - \frac{\hbar k_{zF}^2}{2m^*}\right)\left[e^2\beta_0^2 v_0(1-v_0)m^{*2}\omega_c^3/\pi^2 c\hbar^5 N\kappa^{1/2}\omega\right] \\ & \times\left(|E_+|^2+|E_z|^2\right)^{-1}\left[\frac{2|E_+|^2}{(\omega+\omega_c)^2}\ln\frac{q_{z2}}{q_{z1}} + \frac{|E_z|^2}{\omega^2}\tfrac{1}{2}\lambda^2\left(q_{z2}^2-q_{z1}^2\right)\right] \end{aligned} \tag{55}$$

For $\omega$ close to $\omega_c$ there is

$$K_1^c \approx [e^2\beta_0^2 v_0(1-v_0)m^{*2}/2\pi^2 c\hbar^5 N\kappa^{1/2}] \times |E_+|^2(|E_+|^2+|E_z|^2)^{-1}\ln\left(\frac{\frac{1}{2}\lambda^2 k_{zF}^2\omega_c}{|\omega-\omega_c|}\right) . \qquad (56)$$

## IV. DISCUSSION

Summarizing our results, for both CRI and $\vec{E}\parallel\vec{H}$ polarizations and for all frequencies the absorption is nonvanishing due to the presence of ionized impurities (point defects) or/and composition fluctuations. For CRI polarization there is a logarithmic divergency of the absorption coefficient at $\omega=\omega_c$.

To estimate the value of the absorption coefficient due to ionized impurities (point defects) let us consider the following numerical example corresponding roughly to $n$-type $Hg_{0.75}Cd_{0.25}Te$ at low temperatures: $\kappa=10$, $m^*=0.02m_0$, $N_e=6.8\times10^{16}$ cm$^{-3}$, $D=3$ (as for, e.g., $Z=1$ and $C=0.5$), and $H=50$ kG. Plasma effects should not be important for the normal incidence on a semiconductor plate, as $\hbar\omega_p=21$ meV ($\omega_p$ is the plasma frequency) and $\hbar\omega_c=29$ meV. The quantum-limit condition is fulfilled as $\frac{1}{2}\lambda^2k_{zF}^2=0.5$. The magnetic length $\lambda=115$ Å. $K_0^i+K_1^i$ for $E_z=0$ (CRI polarization) is plotted on Fig. 2. It seems that the divergence should be rather easily observable.

Using the above parameters, and taking $v_0=0.25$, $N=1.5\times10^{22}$ cm$^{-3}$, and $\beta_0=0.5$ eV, we obtain from Eqs. (52) and (56) that for $\omega$ close to $\omega_c$ there is $K_1^c/K_1^i\approx2.2\times10^{-3}$.

In the present paper we have not taken into account the scattering-induced broadening of the Landau levels. This broadening may seriously affect the shape and magnitude of the absorption at $\omega=\omega_c$. Because of that we cannot compare quantitatively our results with the experimental data of McCombe *et al.*[6] The other reason is that the sample presented in Ref. 6 is not degenerated enough (as $\hbar^2k_{zF}^2/2m^*\approx kT$). Nevertheless, if the parameters of this sample are inserted in our Eq. (52) (with $D=1$), the absorption coefficient is of roughly the same order as in Ref. 6.

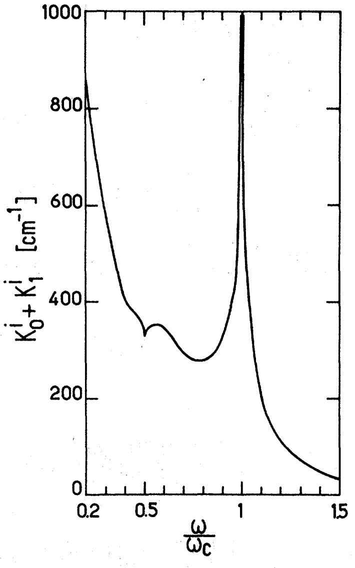

FIG. 2. CRI absorption coefficient due to ionized impurities (transitions 0 → 0 and 0 → 1) vs frequency for the numerical example discussed in the text.

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