Energy distribution of photoexcited hot electrons in a quantizing magnetic field

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We calculate the distribution function of the electrons in the conduction band of a semiconductor that are photoexcited by a laser. We assume extreme-quantum-limit conditions and we describe the recombination processes by a relaxation time. This distribution function is the solution of a Pauli master equation that we transform by means of a Fokker-Planck analysis. We determine the change in shape of this distribution function for different values of the mean energy lost by an electron during its lifetime and for different laser excitation energies ϵ_P. The influence of optical phonons is studied qualitatively. Using this distribution function we are able to calculate the mean energy of the electron gas and are able to obtain its dependence on ϵ_P, on the recombination time, and on the magnetic field.

I. INTRODUCTION

Recent experiments on hot electrons photoexcited in the conduction band of semiconductors allow the measurement of the electron distribution function (EDF) in the conduction band. This is in contrast with transport results, where the EDF is averaged over all the carriers and hence can only be obtained indirectly.

Photoexcitation experiments were performed both in the absence and in the presence of a magnetic field B.[1-3] Without magnetic field, a theoretical expression of the EDF was predicted by several authors.[4-8] In this case electron-electron collisions are in general important and it is quite difficult to take these collisions properly into account, even approximately. In a preceding paper[9] we have already examined the influence of a very high magnetic field. In the extreme quantum limit (EQL) we made the simplest assumption, i.e., we took for the EDF a Maxwellian distribution, characterized by an electron temperature T_e. By usual arguments, based on the energy-balance equation, we then calculated the dependence of T_e on the laser excitation frequency, on the phonon temperature T, and on the strength of the magnetic field B.

The aim of the present work is to eliminate this Maxwellian assumption. The exact EDF is obtained as the solution of a general master equation. We will deduce from this distribution function the mean energy of the system and compare this new result with that obtained in Ref. 9. It is only possible to accomplish this task if the following three conditions are realized:

(i) We assume an EQL situation, that is, the electrons occupy only the first Landau level; consequently, two-body electron-electron collisions cannot exist. Furthermore, we neglect three-body collisions,[10,11] like electron-electron-impurity or electron-electron-phonon interactions.

(ii) We are able to transform the electron–acoustical-phonon collision term of the master equation by means of a Fokker-Planck analysis.

(iii) Finally we choose the simplest model to describe the electron recombination, that is, we use an electron recombination time with a convenient energy dependence.

The paper is divided as follows: Section II is devoted to the description of the model (including broadening effects on the energy levels). In Sec. III we transform the master equation into a Fokker-Planck equation, the solution of which is given in Sec. IV, where the shape of the EDF obtained is discussed. In Sec. V the mean energy is derived and compared to our previous results, and finally, in Sec. VI the qualitative influence of optical phonons on the EDF is examined. Our conclusions are summarized in Sec. VII.

II. DESCRIPTION OF THE MODEL

It is well known that the electron states in a magnetic field B, parallel to the z axis, can be specified by three quantum numbers: n, k_y, and k_z summarized by ν. If we neglect all phenomena related to the electronic spin, the electron energies reduce to

$$\epsilon_\nu = (n+\tfrac{1}{2})\hbar\omega_c + \hbar^2 k_z^2/2m, \tag{1}$$

where m is the effective mass of the parabolic conduction band. Using the same model as in Ref. 9, we may write the stationary master equation, obeyed by the EDF ρ_ν, as follows:

$$\frac{\partial \rho_\nu}{\partial t} = 0 = \frac{\alpha I}{g(\epsilon_\nu)}\,\delta\left(\epsilon_\nu - \frac{\hbar\omega_c}{2} - \epsilon_P\right) - \frac{\rho_\nu}{\tau_\nu} + \sum_{\nu'} \rho_{\nu'} W_{\nu'\nu} - \rho_\nu W_{\nu\nu'}. \tag{2}$$

The first term of Eq. (2) describes the creation of electrons by laser excitation to an energy ϵ_P above the first Landau level of the conduction band. As in Ref. 9, we assume that the electrons are created by photoionization of impurities; α is the absorption coefficient, I the photon flux density, and $g(\epsilon_\nu)$ the density of states of the electrons.

The second term corresponds to the annihilation of electrons by means of various recombination mechanisms; we will assume that these mechanisms can be described by a recombination time; more precisely, let τ_ν be the recombination time of an electron in state ν. We will assume that this recombination time is energy dependent,

$$\tau_\nu = \tau(\epsilon_\nu). \tag{3}$$

Finally we have introduced in Eq. (2) the variation of ρ_ν due to the collisions. In our EQL case, the elastic collisions on impurities do not contribute to $\partial\rho_\nu/\partial t$. Thus $W_{\nu\nu'}$ is the transition probability per unit time to transfer an electron from state ν to state ν' due to an electron-phonon collision. We can write $W_{\nu\nu'}$ in the simplest form as

$$W_{\nu\nu'} = \frac{2\pi}{\hbar}\sum_{\vec{q}} |\gamma_{\nu\nu'}(\vec{q})|^2 [N_q \delta(\epsilon_\nu - \epsilon_{\nu'} + \hbar\omega_q) + (1+N_q)\delta(\epsilon_\nu - \epsilon_{\nu'} - \hbar\omega_q)], \tag{4}$$

where N_q is the thermal equilibrium phonon distribution function at temperature T and where

$$|\gamma_{\nu\nu'}(\vec{q})|^2 = c^2(q)|\langle \nu | e^{i\vec{q}\cdot\vec{r}} | \nu' \rangle|^2. \tag{5}$$

In Eq. (5) $c(q)$ describes the electron-phonon coupling, its strength and its character. The two $\delta(\epsilon)$ terms in Eq. (4) describe the energy conservation when absorption or emission of a phonon occurs during an electron-phonon collision.

We should be more careful in the description of the electronic states: Eq. (1) does not take into account any collision broadening effects. As is well known,[12] both multiple scattering of electrons on impurities and the fluctuations in the potential (produced by the random distribution of the impurities) lead to a broadening Γ, as well as a shift Δ, of each Landau level. Consequently, Eq. (4) must be modified. We will replace each $\delta(\epsilon)$ term by a spectral density $A(\epsilon)$. If Δ and Γ are assumed to be energy independent, $A(\epsilon)$ has a Lorentzian shape and the density of states

$$g(\epsilon) = \sum_\nu A(\epsilon - \epsilon_\nu) = \sum_n g_n(\epsilon) \tag{6}$$

is simply given by[13]

$$g_n(\epsilon) = \frac{2\Omega}{(2\pi\delta)^2}\left(\frac{2m}{\hbar^2}\right)^{1/2} \mathrm{Re}[\epsilon - (n+\tfrac{1}{2})\hbar\omega_c - \Delta - i\Gamma]^{-1/2}, \tag{7}$$

where $\Omega = L^3$ is the volume of the sample and $\delta = (\hbar/m\omega_c)^{1/2}$ the cyclotron radius. Actually, in order to obtain tractable calculations, we will neglect the shift Δ and replace $A(\epsilon)$ by a Gaussian[14]:

$$A(\epsilon) = (2\pi\Gamma^2)^{-1/2} e^{-\epsilon^2/2\Gamma^2}, \tag{8}$$

and we will replace the density of states by

$$g_n(\epsilon) = \begin{cases} [2\Omega/(2\pi\delta)^2](2m/\hbar^2)^{1/2}[\epsilon - (n+\tfrac{1}{2})\hbar\omega_c + \Gamma]^{-1/2} & \text{if } \epsilon > (n+\tfrac{1}{2})\hbar\omega_c \\ 0 \text{ if } \epsilon \leq (n+\tfrac{1}{2})\hbar\omega_c. \end{cases} \tag{9}$$

One can easily check that we regain the undamped density of states when $\Gamma = 0$. Although the approximations (8) and (9) are not entirely consistent, they are necessary to avoid lengthy calculations. We assume that they do not change the main features of the results.

III. DERIVATION OF A FOKKER-PLANCK EQUATION

From now on we will assume that only the first Landau level is populated (EQL). This implies that $\hbar\omega_c \gg kT$ and that the energy ϵ_P, to which the electrons are photoexcited, is much less than $\hbar\omega_c$ (low-energy excitation). Under these conditions the set of states ν and ν' of Eqs. (2) and (4) are reduced to ν_0 and ν_0' such that $n = n' = 0$. The energies and the density of states in the EQL case become

$$\epsilon_{\nu_0} = \tfrac{1}{2}\hbar\omega_c + \hbar^2 k_z^2/2m = \tfrac{1}{2}\hbar\omega_c + \epsilon$$

and

$$g(\epsilon) = [2\Omega/(2\pi\delta)^2](2m/\hbar^2)^{1/2}(\epsilon+\Gamma)^{-1/2}, \tag{10}$$

while

$$|\langle \nu_0 | e^{i\vec{q}\cdot\vec{r}} | \nu_0' \rangle|^2 = e^{-q_\perp^2\delta^2/2}\delta_{k_y', k_y+q_y}\delta_{k_z', k_z+q_z}. \tag{11}$$

Let us focus our attention on the collision term of Eq. (2),

$$\mathcal{C}(\epsilon) = \sum_{\nu_0'} W_{\nu_0'\nu_0}\rho_{\nu_0'} - W_{\nu_0\nu_0'}\rho_{\nu_0}. \tag{12}$$

Since $W_{\nu_0\nu_0'}$ takes into account interactions with both acoustic and optical phonons we will write

$$\mathcal{C}(\epsilon) = \mathcal{C}^{\mathrm{ac}}(\epsilon) + \mathcal{C}^{\mathrm{op}}(\epsilon). \tag{13}$$

In this section we will deal only with $\mathcal{C}^{\mathrm{ac}}(\epsilon)$; the treatment of $\mathcal{C}^{\mathrm{op}}(\epsilon)$ will be developed in Sec. VI.

In order to transform $\mathcal{C}^{\mathrm{ac}}(\epsilon)$ into a differential

equation we follow the same line as in Ref. 15. We define the new transition probability as

$$P(\epsilon, \epsilon') = \frac{2\pi\delta^2}{L^2} g(\epsilon)g(\epsilon')\sum_{k_y'} W_{\nu_0\nu_0'} . \tag{14}$$

As long as the average energy exchanged during a collision is small compared to the thermal energy, that is

$$\hbar s\delta^{-1} \ll kT , \tag{15}$$

where s is the sound velocity, we show in Appendix A that the probability $P(\epsilon, \epsilon')$ may be written

$$P(\epsilon, \epsilon') = g(\epsilon)g(\epsilon')\mathcal{P}(\epsilon - \epsilon') , \tag{16}$$

where $\mathcal{P}(\epsilon - \epsilon')$ is given by Eqs. (A4) and (A6).

If we define the energy distribution function by

$$f(\epsilon) = g(\epsilon)\rho(\epsilon) , \tag{17}$$

then (i) for energies large enough, typically, $\epsilon \gg \hbar s\delta^{-1}$, and (ii) for functions $f(\epsilon)$ varying slowly enough over the range of $\mathcal{P}(\epsilon - \epsilon')$, the acoustical collision term $\mathcal{C}^{ac}(\epsilon)$ may be approximated by

$$\mathcal{C}^{ac}(\epsilon) = M_1 \frac{df}{d\epsilon} + M_2 \frac{d^2f}{d\epsilon^2} . \tag{18}$$

M_n is the nth-order moment of $\mathcal{P}(\xi)$ and is energy independent.

The exact criteria of validity of Eq. (18) will be obtained by using condition (ii) once the energy dependence of $f(\epsilon)$ is known.

The calculation of the moments is also done in Appendix A. It is shown that the zero-order moment is simply related to the acoustic collision time $\tau_{ac}(\epsilon)$ [Eq. (A13)].

The expression of first and second moments are given for two different kinds of electron-phonon interactions: the deformation potential and the piezoelectric coupling by Eq. (A19).

Finally, as long as we do not take into account optical phonons effect, the master equation (2) can be replaced by a differential equation satisfied by $f(\epsilon)$. From Eqs. (17) and (18) we obtain

$$M_1 \frac{df}{d\epsilon} + M_2 \frac{d^2f}{d\epsilon^2} - \frac{f(\epsilon)}{\tau(\epsilon)g(\epsilon)} + \frac{\alpha I}{g(\epsilon)} \delta(\epsilon - \epsilon_P) = 0 . \tag{19}$$

The transformation of Eq. (2) into Eq. (19) is the usual treatment of Fokker-Planck. We recall that this is valid for $\epsilon \gg \hbar s\delta^{-1}$, $\hbar s\delta^{-1} \ll kT$, and when the optical phonons effects are neglected. This last restriction will be dropped in Sec. VI.

For the present let us simply note that a Fokker-Planck treatment cannot be valid for optical phonons because the energy transferred between an optical phonon and an electron during a collision is far too large compared to the mean electron energy. Nevertheless, under the EQL conditions and when $\epsilon_P < \hbar\omega_c$, none of the electrons have an energy ϵ greater than $\hbar\omega_c$. For convenient magnetic fields and for a semiconductor like GaAs on which many of photoexcitation experiments have been performed, $\hbar\omega_c$ is of the same order of magnitude as the optical-phonon energy $\hbar\omega_0$. Thus in the EQL there is no electron available to emit an optical phonon and the temperature is sufficiently low that one can neglect the absorption of an optical phonon. This is the main reason that we can use the solution of Eq. (19) to obtain a reasonable approximation for $f(\epsilon)$.

IV. SHAPE OF THE DISTRIBUTION FUNCTION

In order to solve Eq. (19), it is necessary to discuss a last physical point of our model: the energy dependence of the recombination time $\tau(\epsilon)$. We know very little about this function when a magnetic field is present. In the absence of magnetic field the Lax model[5,16] of a cascade capture of the carriers by shallow donors predicts that $\tau(\epsilon)$ increases as a power of ϵ. However, when ϵ goes to zero it gives a divergent recombination rate, a result that is obviously wrong. To avoid this spurious result, a cut-off ϵ_0 was introduced. In the presence of B we will still assume an increasing dependence of $\tau(\epsilon)$ with ϵ as well as the existence of a cutoff. Moreover, for the convenience of the calculations we take $\tau(\epsilon)$ proportional to $(\epsilon + \epsilon_0)^{1/2}$ and we will assume that ϵ_0 is of the same order as the level broadening Γ. Under such an assumption the product $\tau(\epsilon)g(\epsilon)$ is constant,

$$\tau(\epsilon)g(\epsilon) = a^{-1} \tag{20}$$

The equation obeyed by $f(\epsilon)$ is now reduced to a second-order differential equation with constant coefficients,

$$0 = M_1 \frac{df}{d\epsilon} + M_2 \frac{d^2f}{d\epsilon^2} - af(\epsilon) + \frac{\alpha I}{g(\epsilon)} \delta(\epsilon - \epsilon_P) . \tag{21}$$

We look for a solution of (21) which satisfies the two following conditions:

$$\text{(i)}\quad f(\epsilon) \to 0 \quad \text{as} \quad \epsilon \to \infty , \tag{22}$$

$$\text{(ii)}\quad \int_0^\infty g(\epsilon)f(\epsilon)\,d\epsilon = \frac{\alpha I}{a} . \tag{23}$$

This last equality results from the conservation of the number of particles deduced from Eq. (2),

$$\sum_\nu \frac{\rho_\nu}{\tau_\nu} = \sum_\nu \frac{\alpha I}{g(\epsilon_\nu)} \delta\left(\epsilon_\nu - \frac{\hbar\omega_c}{2} - \epsilon_P\right) \tag{24}$$

and from the assumption (20).

Let us define the three energy-independent pa-

rameters:

$$\eta = \frac{4M_2 a}{(M_1)^2} = 4\left(\frac{kT}{\hbar s\delta^{-1}}\right)^2 \frac{\tau_{ac}(\epsilon)}{\tau(\epsilon)}\,\alpha\left[1+\frac{\alpha}{2}\left(\frac{\Gamma}{\hbar s\delta^{-1}}\right)^2\right], \tag{25}$$

$$r = (M_1/2M_2)[(1+\eta)^{1/2}+1], \tag{26}$$

and

$$t = (M_1/2M_2)[(1+\eta)^{1/2}-1]. \tag{27}$$

Equation (A1 9) of Appendix A leads to

$$M_2/M_1 = kT[1+\tfrac{1}{2}\alpha(\Gamma/\hbar s\delta^{-1})^2] = kT_{\text{eff}}, \tag{28}$$

with

$$T_{\text{eff}} = T[1+\tfrac{1}{2}\alpha(\Gamma/\hbar s\delta^{-1})^2]. \tag{29}$$

Note that for $\Gamma = 0$, $M_2/M_1 = kT$, which is the temperature of the thermodynamic equilibrium distribution function. We shall see later that T_{eff} is also the temperature of the thermodynamic equilibrium EDF in the presence of collision broadening.

Moreover, η may be easily understood in terms of average energy lost during a collision. In each collision an electron looses an energy of order $\hbar s\delta^{-1}$ with the probability

$$[1+N(\hbar s\delta^{-1})]/[1+2N(\hbar s\delta^{-1})]$$

and receives the same energy with the probability

$$N(\hbar s\delta^{-1})/[1+2N(\hbar s\delta^{-1})];$$

hence the net energy lost during a collision is[15]

$$\bar{\epsilon} = \hbar s\delta^{-1}\tanh\frac{\hbar s\delta^{-1}}{kT} \sim \frac{(\hbar s\delta^{-1})^2}{kT} \quad \text{if} \quad \frac{\hbar s\delta^{-1}}{kT} \ll 1.$$

But the number of collisions that an electron undergoes during its lifetime is τ/τ_{ac}; so the total energy lost by an electron during τ is simply

$$\bar{\epsilon}_{\text{tot}} = \bar{\epsilon}\tau/\tau_{ac} = [(\hbar s\delta^{-1})^2/kT]\tau/\tau_{ac}$$

and

$$\eta^{-1} \simeq \bar{\epsilon}_{\text{tot}}/kT_{\text{eff}}.$$

Finally, we note that for given B and T we have the following two limiting cases: (i) $\eta \to 0$ corresponds to an infinite recombination time and we expect the electrons to have a thermal distribution; and (ii) $\eta \to \infty$ corresponds to an instantaneous recombination, and we expect in this case a distribution peaked around $\epsilon = \epsilon_P$.

We now come back to the solution of Eq. (21). A straightforward calculation gives:

$$f(\epsilon) = Ae^{-r\epsilon} + [\alpha I a^{-1} rt/g(\epsilon_P)(r+t)] \times[\Theta(\epsilon_P-\epsilon)e^{t(\epsilon-\epsilon_P)} + \Theta(\epsilon-\epsilon_P)e^{-r(\epsilon-\epsilon_P)}], \tag{30}$$

with

$$A = \frac{\alpha I a^{-1} rt}{g(\epsilon_P)(r+t)}\,\frac{1}{1-\text{erf}[(r\Gamma)^{1/2}]}\left(\frac{r^{1/2}}{\pi^{1/2}(\epsilon_P+\Gamma)^{1/2}}\,\frac{r+t}{rt}\,e^{-r\Gamma} - e^{r\epsilon_P}\,\text{erfc}\{[r(\epsilon_P+\Gamma)]^{1/2}\} - e^{-t\epsilon_P-(r+t)\Gamma}\left(\frac{r}{t}\right)^{1/2}\frac{1}{i}\left(\text{erf}\{i[t(\epsilon_P+\Gamma)]^{1/2}\} - \text{erf}(i\sqrt{t\Gamma})\right)\right), \tag{31}$$

where $\Theta(x)$ is the usual step function and $\text{erf}(x) = 1-\text{erfc}(x)$ is the error function.[17]

The final expression of $f(\epsilon)$ varies essentially in an exponential way characterized by the two parameters r and t. We need to remember that we have obtained the Fokker-Planck equation (21) only if $f(\epsilon)$ is a slowly varying function in the range of $\mathcal{P}(\epsilon-\epsilon')$ defined by (16). A typical energy range of $f(\epsilon)$ is r^{-1} or t^{-1}, whereas the energy range of $\mathcal{P}(\epsilon-\epsilon')$ is of order $(2M_2/M_0)^{1/2}$. Then the smallest of r^{-1} and t^{-1}, i.e., r^{-1}, must satisfy the condition

$$r^{-1} \gg (2M_2/M_0)^{1/2}. \tag{32}$$

Using Eqs. (A19) and (26) the above inequality is equivalent to

$$(1+\eta)^{1/2}+1 \ll (2\alpha)^{1/2}kT_{\text{eff}}/\hbar s\delta^{-1}. \tag{33}$$

For given values of $kT/\hbar s\delta^{-1}$ and Γ/kT the inequality (33) leads to a minimum value of the ratio τ/τ_{ac}, below which our results are no longer valid.

Let us now examine the change in the shape of $f(\epsilon)$ with the parameters η, ϵ_P/kT, and Γ/kT.

In Figs. 1(a)–1(c) we have plotted the dependence of the reduced distribution function

$$\varphi(x) = f(\epsilon)/f(\epsilon_P)$$

on the relative energy $x = \epsilon/\epsilon_P$ in the case of GaAs at $B = 5$ T, $T = 10$ K.

With regard to the influence of the collision broadening, Figs. 1(b) and 1(c) show that for the two values of Γ/kT choosen, the general features of the curves are quite similar. However, as expected, the curves at the larger value of Γ are flatter than at the lower value of Γ. Let us now discuss the change of $\varphi(x)$ at fixed values of ϵ_P/kT for different η. On one hand, as η and τ/τ_{ac} are increased the distribution function departs more and more from the Maxwellian because the number of collisions suffered by an electron dur-

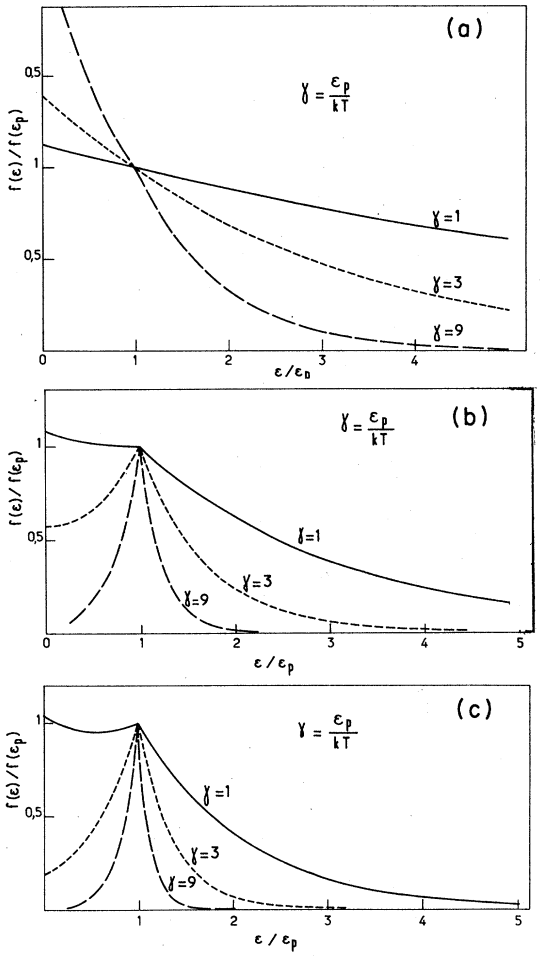

FIG. 1. Dependence of the reduced distribution function $f(\epsilon)/f(\epsilon_p)$ on the reduced energy ϵ/ϵ_p for different values of the ratio $\gamma=\epsilon_p/kT$. Magnetic field $B=5$ T, temperature $T=10$ K, sound velocity $s=3.8\times10^3$ m sec^{-1}. (a) $\Gamma/kT=1$, $\tau/\tau_{ac}=10^3$, $\eta=0.576$; (b) $\Gamma/kT=1$, $\tau/\tau_{ac}=10$, $\eta=57.6$; (c) $\Gamma/kT=\frac{1}{2}$, $\tau/\tau_{ac}=10$, $\eta=19.2$.

ing its lifetime is lowered, and hence the thermalization of the electrons is diminished. On the other hand, when η decreases the memory of the initial photoexcitation at ϵ_P disappears and a Maxwellian shape emerges.

In particular, when $\eta=0$, $f(\epsilon)$ is exactly Maxwellian,

$$f(\epsilon)\sim e^{-\epsilon/kT_{\rm eff}} . \tag{34}$$

On one hand in the absence of collision broadening, we would have expected, from the principle of detailed balancing, that $\rho(\epsilon)\sim e^{-\epsilon/kT}$ rather than the result of Eq. (34), $f(\epsilon)=\rho(\epsilon)g(\epsilon)\sim e^{-\epsilon/kT}$. However, this is not very important because our calculations have always neglected the dependence of g on ϵ as compared to the exponential behavior of $\rho(\epsilon)$.

On the other hand, Eq. (34) shows that for $\Gamma\neq0$ a heating for the electron gas appears ($T_{\rm eff}>T$). This result can be recovered directly from the master equation and detailed calculations can be found in Appendix B. We mention here that the mechanism leading to this heating is exactly the same as that discussed in Ref. 15 for the electric field action. The electric field blurs the energy conservation, as does the collision broadening,[13] and we know that as a result the electron temperature must rise.

Another interesting feature of the behavior of $f(\epsilon)$ can be noted on Figs. 1(a) and 1(b): at constant η the discontinuity of the slope increases with ϵ_P/kT. This can be explained by remembering that η^{-1} is the average energy lost by an electron during its lifetime (measured in units of $kT_{\rm eff}$). As ϵ_P increases, the ratio of this average energy to ϵ_P decreases so that the dispersion of the electron energies becomes less and less important. An equivalent way to express this is to say that the initial δ distribution becomes more and more weakly broadened by acoustical phonons interaction and hence the slope discontinuity becomes larger and larger.

It is possible to calculate this discontinuity; we get immediately

$$R=\left(\frac{d\varphi}{dx}\right)_{x=1^+}-\left(\frac{d\varphi}{dx}\right)_{x=1^-}=\frac{\alpha I a^{-1}\gamma t\epsilon_P}{f(\epsilon_P)g(\epsilon_P)} . \tag{35}$$

Since this expression depends only on η at given B, T, and ϵ_P, it is possible to deduce a value for the ratio τ/τ_{ac} from it.

V. MEAN ENERGY

In a previous work[9] we have derived an expression for the mean energy per electron (or electronic temperature) by assuming a Maxwellian distribution function and by using an energy balance equation. We will now compare the results of this simplified model with the results obtained by using the preceding expression for $f(\epsilon)$, as given in Eq. (30).

In the case of low-energy excitation, where we neglect optical phonons effects, we obtain

$$\langle\epsilon\rangle=\frac{\int_0^\infty \epsilon f(\epsilon)d\epsilon}{\int_0^\infty f(\epsilon)d\epsilon}$$

$$=\frac{1}{\gamma}\,\frac{B+\gamma\epsilon_P+1-\gamma/t+[\gamma^2/t(\gamma+t)]e^{-t\epsilon_P}}{B+(\gamma+t)^{-1}(\gamma+t-\gamma e^{-t\epsilon_P})} , \tag{36}$$

with $B=Ag(\epsilon_P)/\alpha I a^{-1}\gamma$, where A is given by Eq.

(31).

The dependence of $\langle\epsilon\rangle/kT$ on ϵ_P/kT is plotted on Figs. 2(a)–(c) where the broadening Γ is, respectively, taken equal to zero and to $\frac{1}{2}kT$.

As expected, Fig. 2(a) shows that the mean energy increases with the laser excitation energy ϵ_P and that for energies ϵ_P smaller than kT the mean energy per electron is smaller than the equilibrium thermal energy kT. It is also obvious, in the same figure that this heating (or cooling) becomes more and more important as the ratio τ/τ_{ac} is lowered. If this ratio is lowered the electrons are able to exchange less and less energy with the phonon thermal bath during their lifetime, and thus depart more and more from thermal equilibrium. As for the influence of the magnetic field, Fig. 2(b) shows that it cools the electron gas. The interpretation of this feature is the same as given in Ref. 9: the power lost via acoustical phonons is enhanced by the presence of B. Although these features are similar to those predicted in Ref. 9, a more precise comparison between the present and previous results is not possible, since the distribution function and the relaxation-time energy dependence are not the same anymore.

The influence of a level broadening is shown on Fig. 2(c); in this case, as pointed out in Sec. IV, the thermal equilibrium temperature is no longer T but T_{eff} as defined by Eq. (29). T_{eff} is equal to 3 T in the case plotted in Fig. 2(c). This explains the overall increase of the mean energies in Fig. 2(c), as compared to Fig. 2(a). Also when the excitation energy ϵ_P is equal to kT_{eff} the mean energy per electron is nearly equal to kT_{eff} independent of the ratio τ/τ_{ac}.

Except for this scale variation of $\langle\epsilon\rangle$ due to the change of T into T_{eff}, we find the same features of the curves at $\Gamma=0$ and at $\Gamma=\frac{1}{2}kT$; that is, $\langle\epsilon\rangle$ increases with ϵ_P, it reduces with τ/τ_{ac} for $\epsilon_P > kT_{\text{eff}}$, and increases with τ/τ_{ac} for $\epsilon_P < kT_{\text{eff}}$.

We may also note that at a given ratio τ/τ_{ac}, $\langle\epsilon\rangle$ increases faster with ϵ_P at $\Gamma=0$ than at $\Gamma=\frac{1}{2}kT$; this is not surprising if we remember that the distribution function flattens as Γ is increased [see Figs. 1(b) and 1(c)] and so will do the curves $\langle\epsilon\rangle$ versus ϵ_P.

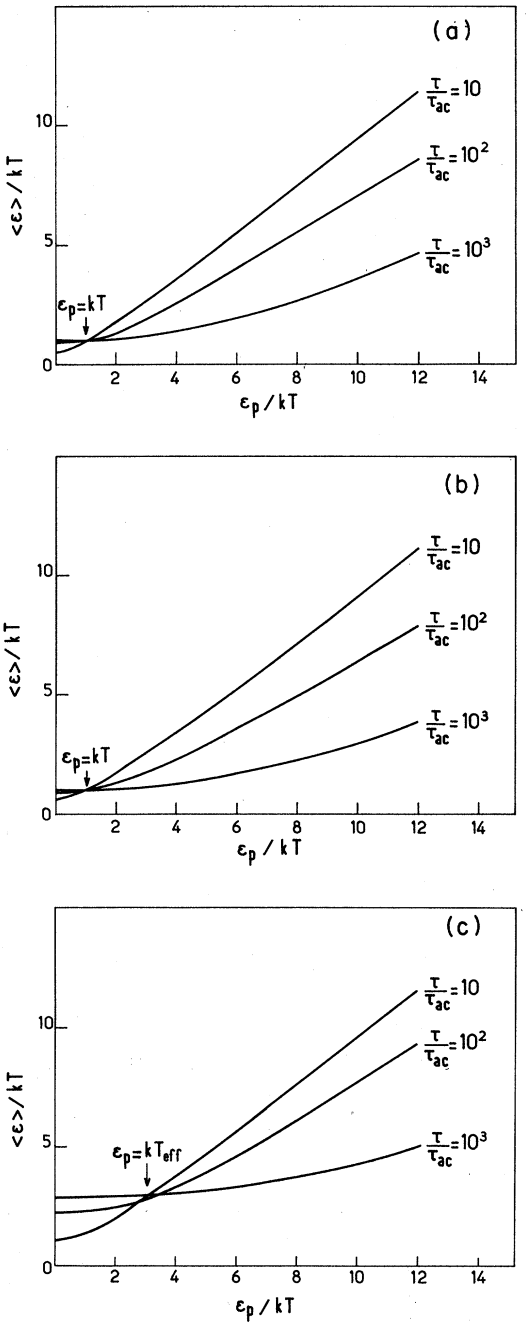

FIG. 2. Dependence of the reduced mean electron energy $\langle\epsilon\rangle/kT$ on the ratio ϵ_P/kT for different values of τ/τ_{ac}; $T=10$ K, $s=3.8\times10^3$ m sec^{-1}. (a) $\Gamma=0$, $B=5$ T; (b) $\Gamma=0$, $B=8$ T; (c) $\Gamma=\frac{1}{2}kT$, $B=5$ T.

VI. INFLUENCE OF OPTICAL PHONONS

In this section we extend our results qualitatively in order to include optical phonons and their relaxation processes. We will discuss the simple case of a zero collision broadening ($\Gamma=0$); detailed calculations performed for the case $\Gamma\neq0$ are presented in Appendix C.

We will use the three following assumptions that are derived from the EQL condition and from the fact that $\hbar\omega_0$ is large compared to the mean energy of the electrons. (i) The temperature is sufficiently low to allow us to neglect the absorption of optical phonons. Hence in the expression (4) of $W_{vv'}$ the term proportional to N_q may be neglected,

whereas $1+N_q$ is reduced to 1. (ii) The density of states decreases when ϵ increases. (iii) Of the two electronic levels differing by $\hbar\omega_0$, only the lowest is occupied with an appreciable probability.

Due to assumptions (ii) and (iii) the term $\sum_{\nu_0'} \rho_{\nu_0'} W_{\nu_0'\nu_0}$ of Eq. (12) gives a negligible contribution (see Appendix C) and will be omitted. Under these conditions,

$$\mathcal{C}^{\mathrm{op}}(\epsilon) \simeq -W(\epsilon)\rho(\epsilon)\Theta(\epsilon-\hbar\omega_0), \tag{37}$$

with

$$W(\epsilon) = \frac{\delta^2 L}{\hbar}\, g(\epsilon-\hbar\omega_0) \times \int d^3q c^2(q) e^{-q_\perp^2\delta^2/2} \times \delta\left[q_z+\left(\frac{2m}{\hbar^2}\right)^{1/2}[\sqrt{\epsilon}-\sqrt{(\epsilon-\hbar\omega_0)}]\right]. \tag{38}$$

We may recast $\mathcal{C}^{\mathrm{op}}(\epsilon)$ into the equivalent form,

$$\mathcal{C}^{\mathrm{op}}(\epsilon) = -\frac{\rho(\epsilon)}{\tau_{\mathrm{op}}(\epsilon)}\,\Theta(\epsilon-\hbar\omega_0), \tag{39}$$

where

$$\tau_{\mathrm{op}}(\epsilon) = W^{-1}(\epsilon). \tag{40}$$

Thus the influence of optical phonons amounts to add the term (39) to Eq. (21); this term is nonvanishing only for energies larger than the phonon energy. It has exactly the same structure as the recombination term, $-\rho(\epsilon)/\tau(\epsilon)$. However, it differs from it by its energy dependence. Thus the introduction of optical phonons reduces the lifetime of the electrons by a factor $\tau(\epsilon)/[\tau(\epsilon)+\tau_{\mathrm{op}}(\epsilon)]$. Qualitatively we may say that this is equivalent to increasing the parameter $\eta \sim \tau_{\mathrm{ac}}/\tau$ for energies ϵ such that $\epsilon \gtrsim \hbar\omega_0$. In that region, and always with the assumption of low-energy excitation, the distribution function behaves as $f(\epsilon) \sim e^{-r\epsilon}$ (for $\epsilon > \epsilon_P$) so that the appearance of optical phonon relaxation processes leads to a slope discontinuity at $\epsilon = \hbar\omega_0$ and to a more rapidly decreasing distribution function for higher energies. This result can be compared with that of Ref. 18, where $f(\epsilon)$ is crudely taken equal to zero for $\epsilon > \hbar\omega_0$.

A finite collision broadening Γ does not change those qualitative results (cf. Appendix C), except that the emission of an optical phonon can now occur at an energy lower than $\hbar\omega_0$ (of order $\hbar\omega_0 - \Gamma$). Also, the slope discontinuity of the distribution function at $\epsilon = \hbar\omega_0$ will be smeared out and the change of slope spread over a region of width Γ around $\hbar\omega_0$.

VII. CONCLUSION

In this paper we determined the distribution function of electrons photoexcited by a laser in the conduction band of a semiconductor. We assumed EQL conditions and modeled in a crude way the recombination processes.

This electron distribution function is a solution of the Pauli master equation in which the collision term with acoustic phonons is transformed using a Fokker-Planck analysis. The influence of optical phonons is shown to be damped by the EQL condition and by the assumption of low-energy excitation. This is discussed qualitatively at the end of the paper.

We give a detailed discussion of the energy dependence of the energy distribution function that was obtained. As the ratio between recombination time and acoustic collision time (i.e., τ/τ_{ac}) is increased, the distribution function passes continuously from a sharply peaked shape around the excitation energy ϵ_P, to an almost Maxwellian function which exhibits only a slope discontinuity at $\epsilon = \epsilon_P$, where it remembers the initial distribution. The value of this discontinuity is related to the ratio τ/τ_{ac} and hence can be determined experimentally.

The interaction with optical phonons leads to a second slope discontinuity at $\epsilon = \hbar\omega_0$ (phonon energy) corresponding to the appearance of energy relaxations processes caused by the emission of optical phonons.

Using the obtained electron distribution function we calculated the mean energy per electron and studied its variations with ϵ_P, τ/τ_{ac}, and the magnetic field in the absence as well as in the presence of collision broadening. As expected, $\langle\epsilon\rangle$ increases with ϵ_P and decreases with τ/τ_{ac}. Moreover, the magnetic field acts to cool the electron gas. These last conclusions are qualitatively the same as those obtained in an earlier paper[9] by using a Maxwellian shape for the electron distribution function.

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APPENDIX A

Let us first derive an expression for $P(\epsilon, \epsilon')$ in the case of acoustic phonons. From Eqs. (4), (5), and (11) the transition probability from the state ν_0 to the state ν_0' can be written

$$W_{\nu_0\nu_0'}=\frac{2\pi}{\hbar}\sum_{\vec{q}} e^{-q_\perp^2\delta^2/2}\,\delta_{k_x',k_x+q_x}\delta_{k_y',k_y+q_y} \times c^2(q)[N_q A(\epsilon-\epsilon'+\hbar sq) +(1+N_q)A(\epsilon-\epsilon'-\hbar sq)], \tag{A1}$$

where $A(\epsilon-\epsilon')$ is given by Eq. (8).

By using the momentum and energy conservation laws it is possible to show[9,19] that, if the condition

$$\hbar s\delta^{-1}\ll kT \tag{A2}$$

is fulfilled, the component q_z of the wave vector $\vec{q}$ is negligible compared to $q_\perp[=(q_x^2+q_y^2)^{1/2}]$. Then we may write

$$\frac{2\pi\delta^2}{\Omega^{2/3}}\sum_{k_y'} W_{\nu_0\nu_0'} =\frac{4\pi^2\delta^2}{\hbar\Omega^{2/3}}\sum_{q_\perp} e^{-q_\perp^2\delta^2/2}c^2(q_\perp) [N_{q_\perp}A(\epsilon-\epsilon'+\hbar sq_\perp) +(1+N_{q_\perp})A(\epsilon-\epsilon'-\hbar sq_\perp)]. \tag{A3}$$

It is then obvious, from Eq. (A3), that $\sum_{k_y'}W_{\nu_0\nu_0'}$ is dependent on the difference $\epsilon-\epsilon'$ only. It can easily be brought into the form

$$\mathcal{P}(\epsilon-\epsilon')=\frac{2\pi\delta^2}{\Omega^{2/3}}\sum_{k_y'}W_{\nu_0\nu_0'} =\int_{-\infty}^{+\infty}\mathcal{P}^0(\epsilon-\epsilon'-x)A(x)\,dx. \tag{A4}$$

$\mathcal{P}(\epsilon-\epsilon')$ appears as the convolution of the broadened spectral density $A(x)$ with the "transition probability" $\mathcal{P}^0(\epsilon-\epsilon')$ at $\Gamma=0$. This last quantity can be written

$$\mathcal{P}^0(\epsilon-\epsilon')=\frac{2\pi\delta^2}{\hbar\Omega^{2/3}}\sum_{q_\perp} e^{-q_\perp^2\delta^2/2}c^2(q) \times[N_{q_\perp}\delta(\epsilon-\epsilon'+\hbar sq_\perp) +(1+N_{q_\perp})\delta(\epsilon-\epsilon'-\hbar sq_\perp)], \tag{A5}$$

which is transformed into

$$\mathcal{P}^0(\xi)=\frac{2\pi}{\hbar}\,\frac{e^{-\delta^2\xi^2/2\hbar^2s^2}}{1-e^{-\xi/kT}}\,\frac{\xi}{(\hbar s\delta^{-1})^2} \times\left[c^2\left(\frac{\xi}{\hbar s}\right)\Theta(\xi)+c^2\left(-\frac{\xi}{\hbar s}\right)\Theta(-\xi)\right]. \tag{A6}$$

Equation (A6) shows that the range of $\mathcal{P}^0(\xi)$ is of order $\hbar s\delta^{-1}$. On the other hand, the range of $A(\xi)$ is Γ, so that the range of $\mathcal{P}(\xi)$ will be the smaller of these two quantities. In practice Γ is of order kT,[20] hence due to the condition (A2) the range of $\mathcal{P}(\xi)$ is of order $\hbar s\delta^{-1}$. By comparing Eqs. (16) and (A4) we may write

$$P(\epsilon,\epsilon')=g(\epsilon)g(\epsilon')\mathcal{P}(\epsilon-\epsilon'). \tag{A7}$$

With the help of Eqs. (12) and (14) and neglecting the variation of the density of states $g(\epsilon)$ over the range of $\mathcal{P}(\epsilon-\epsilon')$ we may rewrite the collision term in the form

$$\mathcal{C}^{ac}(\epsilon)=\int_0^\infty d\epsilon'\,[f(\epsilon')\mathcal{P}(\epsilon'-\epsilon)-f(\epsilon)\mathcal{P}(\epsilon-\epsilon')], \tag{A8}$$

where

$$f(\epsilon)=g(\epsilon)\rho(\epsilon). \tag{A9}$$

For energies ϵ much larger than $\hbar s\delta^{-1}$ and due to the short range of $\mathcal{P}(\epsilon-\epsilon')$ we may replace the lower limit of the integral by $-\infty$. Moreover, if we suppose that $f(\epsilon)$ is a slowly varying function compared to $\mathcal{P}(\epsilon-\epsilon')$, we may expand $f(\epsilon)$ in a Taylor series and obtain

$$\mathcal{C}^{ac}(\epsilon)=M_1\frac{df}{d\epsilon}+M_2\frac{d^2f}{d\epsilon^2}. \tag{A10}$$

Where M_n is the moment of order n of $\mathcal{P}(\xi)$,

$$M_n=\frac{1}{n!}\int_{-\infty}^{+\infty}\xi^n\mathcal{P}(\xi)\,d\xi. \tag{A11}$$

Let us recall that Eq. (A10) is valid only if

(i) $\epsilon\gg\hbar s\delta^{-1}$,

(ii) $\hbar s\delta^{-1}\ll kT$,

(iii) the range of $f(\epsilon)$ is much larger than that of $\mathcal{P}(\epsilon-\epsilon')$. (A12)

Before calculating M_1 and M_2 based on the deformation potential and on the piezoelectric coupling, let us stress that the moment of order zero M_0 is related to the acoustical collision time of an electron in a state of energy ϵ_{ν_0}. More precisely,

$$\frac{1}{\tau_{ac}(\epsilon_{\nu_0})}=\sum_{\nu_0'}W_{\nu_0\nu_0'}=\int d\epsilon'\,g(\epsilon')\mathcal{P}(\epsilon_{\nu_0}-\epsilon') \simeq g(\epsilon_{\nu_0})M_0, \tag{A13}$$

so that the quantity $\tau_{ac}(\epsilon)g(\epsilon)=M_0^{-1}$ is energy independent.

We may also note that by using the form (A4) of $\mathcal{P}(\epsilon-\epsilon')$ the moments M_n of $\mathcal{P}(\epsilon-\epsilon')$ can be easily expressed in terms of the moments M_n^0 of $\mathcal{P}^0(\epsilon-\epsilon')$. For example,

$$M_0=M_0^0,\quad M_1=M_1^0,\quad\text{and}\quad M_2=M_2^0+\tfrac{1}{2}\Gamma^2M_0^0. \tag{A14}$$

In what follows we shall first calculate M_n^0 and then obtain M_n.

For the deformation potential, with the same notation as in Ref. 9, we have

$$c^2(q) = D^{(3)} q . \tag{A15}$$

By using Eq. (A11) together with the fact that $\xi \simeq \hbar s\delta^{-1} \ll kT$ we obtain

$$\begin{aligned} M_0 &= 4\pi kT D^{(3)}/\hbar^2 s , \\ M_1 &= M_0(\hbar s\delta^{-1})^2/kT , \\ M_2 &= M_0(\hbar s\delta^{-1})^2[1+\Gamma^2/2(\hbar s\delta^{-1})^2] . \end{aligned} \tag{A16}$$

For piezoelectric coupling

$$c^2(q) = \frac{D^{(1)}}{q+q_D} , \tag{A17}$$

where q_D^{-1} is the screening length. Then using again Eqs. (A11), (A14), and the approximation $q_D\delta \ll 1$, we get

$$M_0 = \frac{2\pi D^{(1)} kT}{\hbar^2\delta^{-2}s}\,\alpha , \tag{A18}$$

with $\alpha = \ln(2/c\delta^2 q_D^2)$, where lnc is equal to Euler's constant. Then we have

$$\begin{aligned} M_1 &= M_0 \frac{(\hbar s\delta^{-1})^2}{\alpha kT} , \\ M_2 &= M_0 \frac{(\hbar s\delta^{-1})^2}{\alpha}\left(1+\frac{\alpha}{2}\frac{\Gamma^2}{(\hbar s\delta^{-1})^2}\right) . \end{aligned} \tag{A19}$$

In both cases the relation between M_1, M_2, and M_0 has the form (A19), where $\alpha = 1$ for deformation potential and $\alpha = \ln(2/c\delta^2 q_D^2)$ for the piezoelectric coupling.

APPENDIX B

In the limit $\eta = 0$ the electrons stay an infinite time in the conduction band and hence they reach thermodynamical equilibrium. The distribution function must then obey the principle of detailed balancing, that is,

$$\rho_{\nu_0} W_{\nu_0\nu_0'} = \rho_{\nu_0'} W_{\nu_0'\nu_0} . \tag{B1}$$

As we want a spatial homogeneous distribution function, $\rho_{\nu_0'}$ must be independent of k_y'. By summing Eq. (B1) over k_y' and using the definition (A4) we obtain

$$\rho(\epsilon)/\rho(\epsilon') = \mathcal{P}(\epsilon'-\epsilon)/\mathcal{P}(\epsilon-\epsilon') \tag{B2}$$

In order to find $\rho(\epsilon)$ we may in the above equation approximate $\mathcal{P}_{(\epsilon-\epsilon')}$ by the function $F(\epsilon-\epsilon')$ such that $\mathcal{P}(\xi)$ and $F(\xi)$ have their three first moments (M_0, M_1, M_2) equal. In the limit $M_1/M_0 \ll (M_2/M_0)^{1/2}$, $F(\xi)$ can be written

$$\mathcal{P}(\xi) \simeq F(\xi) = \frac{M_0^{3/2}}{2\pi^{1/2}M_2^{1/2}} \exp\left[-\frac{M_0}{4M_2}\left(\xi-\frac{M_1}{M_0}\right)^2\right] . \tag{B3}$$

Thus Eq. (B2) reduces to

$$\rho(\epsilon)/\rho(\epsilon') = \exp[-(M_1/M_2)(\epsilon-\epsilon')] . \tag{B4}$$

Hence the solution

$$\rho(\epsilon) \sim \exp\left[-\frac{\epsilon}{kT\{1+(\alpha/2)[\Gamma^2/(\hbar s\delta^{-1})^2]\}}\right] . \tag{B5}$$

Let us note that from Eq. (30) we obtain

$$f(\epsilon) \sim \exp\left[-\frac{\epsilon}{kT\{1+(\alpha/2)[\Gamma^2/(\hbar s\delta^{-1})^2]\}}\right] \tag{B6}$$

In the limit case, where we neglect the variations of $g(\epsilon)$ as compared to the exponential decreasing of $\rho(\epsilon)$, Eqs. (B5) and (B6) are equivalent.

APPENDIX C

The following is an evaluation of $\mathcal{C}^{op}(\epsilon)$. If we neglect the absorption of optical phonons, the transition probability at finite Γ reduces to

$$W_{\nu_0\nu_0'} = \frac{2\pi}{\hbar}\sum_{\vec{q}} c^2(q) e^{-q_\perp^2\delta^2/2}\delta_{k_y',k_y+q_y}\delta_{k_z',k_z+q_z} \times A(\epsilon_{\nu_0}-\epsilon_{\nu_0'}-\hbar\omega_0) , \tag{C1}$$

where $A(\epsilon)$ is given by Eq. (8) and $c^2(q) = D^{(2)}/q^2$. Then the calculation of the new transition probability as defined by Eq. (14) leads to

$$P^{op}(\epsilon,\epsilon') = g(\epsilon)g(\epsilon')\varphi(\epsilon,\epsilon')A(\epsilon-\epsilon'-\hbar\omega_0) , \tag{C2}$$

with

$$\varphi(\epsilon,\epsilon') = \frac{\pi\delta^2 D^{(2)}}{\hbar} e^{\alpha(\epsilon,\epsilon')} E_1(\alpha(\epsilon,\epsilon')) , \tag{C3}$$

where

$$\alpha(\epsilon,\epsilon') = \frac{m\delta^2}{\hbar^2}(\sqrt{\epsilon}-\sqrt{\epsilon'})^2 , \tag{C4}$$

and where $E_1(y)$ is the exponential integral function.[17] This result on $P^{op}(\epsilon,\epsilon')$ allows us to write

$$\mathcal{C}^{op}(\epsilon) = \int dx \frac{e^{-x^2/2\Gamma^2}}{(2\pi\Gamma^2)^{1/2}} [\Theta(\epsilon+\hbar\omega_0+x)\varphi(\epsilon+\hbar\omega_0+x,\epsilon)g(\epsilon+\hbar\omega_0+x)\rho(\epsilon+\hbar\omega_0+x) - \Theta(\epsilon-\hbar\omega_0-x)\varphi(\epsilon,\epsilon-\hbar\omega_0-x)g(\epsilon-\hbar\omega_0-x)\rho(\epsilon)] . \tag{C5}$$

Let us first note that the values of x that contribute significantly to the above integral are of order Γ ($\ll \hbar\omega_0$). For $\epsilon > \hbar\omega_0$ both terms of the integral contribute to $\mathcal{C}^{op}$, but the first one is negligible compared to the second because it involves $\rho(\epsilon+\hbar\omega_0-x)$, which is vanishingly small compared to $\rho(\epsilon)$. On the contrary, for $\epsilon < \hbar\omega_0$ only the first term exists; it accounts for the transitions of electrons from states with energy $\epsilon+\hbar\omega_0$ to states with energy ϵ, the former states being populated

with the probability $\rho(\epsilon+\hbar\omega_0)$. Again this probability is much smaller than $\rho(\epsilon)$ and the contribution of the first term of $\mathcal{C}^{op}(\epsilon)$ to the master equation will be much smaller than the variations of the distribution function, due to the interaction with acoustic phonons. Under these conditions,

$$\mathcal{C}^{op}(\epsilon)\simeq -\frac{\rho(\epsilon)}{(2\pi\Gamma^2)^{1/2}}\int dx\, e^{-x^2/2\Gamma^2}\Theta(\epsilon-\hbar\omega_0-x)\times\varphi(\epsilon,\epsilon-\hbar\omega_0-x)\, g(\epsilon-\hbar\omega_0-x). \quad \text{(C6)}$$

By noting that φ and g are slowly varying functions of x, we obtain

$$\mathcal{C}^{op}(\epsilon)=-\rho(\epsilon)\varphi(\epsilon,\epsilon-\hbar\omega_0)g(\epsilon-\hbar\omega_0)\psi(\epsilon-\hbar\omega_0), \quad \text{(C7)}$$

with

$$\psi(\epsilon-\hbar\omega_0)=\tfrac{1}{2}\left[1+\mathrm{erf}\left(\frac{|\epsilon-\hbar\omega_0|}{\sqrt{2}\,\Gamma}\right)\times[\Theta(\epsilon-\hbar\omega_0)-\Theta(\hbar\omega_0-\epsilon)]\right]. \quad \text{(C8)}$$

For $\Gamma=0$, $\psi(\epsilon-\hbar\omega_0)$ reduces to a step function, which means that only electrons of ϵ larger than $\hbar\omega_0$ may relax their energy via an optical phonon.

At finite Γ, $\psi(\epsilon-\hbar\omega_0)$ is a smooth function of $\epsilon-\hbar\omega_0$, varying from 0 to 1 around $\epsilon=\hbar\omega_0$ over a range of order Γ. We find, as expected, that an electron having an energy around $\hbar\omega_0-\Gamma$ can emit an optical phonon.

In all cases,

$$\mathcal{C}^{op}(\epsilon)=-\rho(\epsilon)/\tau_{op}(\epsilon), \quad \text{(C9)}$$

where $\tau_{op}^{-1}(\epsilon)$ is nonvanishing only for $\epsilon\geq\hbar\omega_0-\Gamma$.

The result (C9) is formally the same with and without collision broadening so that the discussion of Sec. VI, performed in the case $\Gamma=0$, can be repeated for the case $\Gamma\neq 0$.

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