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Pinning and conductivity of two-dimensional charge-density waves in magnetic fields

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The dynamical properties of the impurity pinning of phasons of the charge-density wave are investigated for two-dimensional electron systems in the presence of magnetic fields applied perpendicular to the system. The effect of magnetic fields is treated classically. It is found that there exists a finite size of domains if the impurity potential overwhelms the long-range part of mutual Coulomb interactions, and the magnetoconductivity tensor is evaluated in such cases. The cyclotron resonance is shown to be shifted and broadened. It is predicted that absorption of electromagnetic waves exists at the pinned-mode frequency though the oscillator strength will be smaller by a factor of $(\omega_c\tau)^{-2}$ than the cyclotron resonance if $\omega_c\tau \gg 1$, where ω_c and τ are the cyclotron frequency and the damping of the resonance, respectively.

I. INTRODUCTION

Electron systems formed at interfaces such as metal-oxide-semiconductors (MOS) and electrons on helium surfaces are now accepted as typical examples of two-dimensional electron systems.[1] In these, the electron number density can be varied in a wide range by simply changing the external electric fields. Consequently, these systems afford us the possibility of quantitative comparison between theory and experiment on the effects of strong correlations between electrons.

If mutual-Coulomb-interaction energies are the main ones in determining electronic properties, a Wigner crystal or a charge-density-wave (CDW) state will be formed at sufficiently low temperatures.[2] Moreover, the formation of CDW's will be encouraged by the application of strong magnetic fields perpendicular to the surface.[3-5] There are recent claims that such ordered states have been observed in MOS.[6] On the other hand, the existence of short-range order of the Wigner crystal has been beautifully demonstrated in electrons on helium surfaces.[7,8]

In view of the fact that observability is more or less within the present experimental conditions, we investigate in this paper the response properties of such ordered phases to external electric fields under the presence of a uniform magnetic field, i.e., we examine the magnetoconductivity. We focus our attention here on the dynamical properties of the state at absolute zero, and the response to an oscillatory electric field. We assume that there exists a two-dimensional sinusoidal CDW in the system, with the charge-density pattern of a square lattice. The periodicity is assumed to be given *a priori*. This CDW is under the influence of static impurity centers distributed randomly over space, which pin and distort the CDW. The magnetoconductivity is determined by the dynamical properties of the phase fluctuations about this distorted CDW pattern.

In Sec. II we define our model, and the pinning is examined in Sec. III. The magnetoconductivity is obtained in Sec. IV. Discussions are given in Sec. V, where we point out some possible relevance of our theoretical results to the experimental results.

II. MODEL

We assume that there exists a CDW with the following spatial variation of charge $\rho(\vec{r})$:

$$\rho(\vec{r}) = n + \rho_0\{\cos[Qx + \phi_x(\vec{r})] + \cos[Qy + \phi_y(\vec{r})]\}, \quad (2.1)$$

where n is the average electron number density, ρ_0 is the amplitude of the CDW taken to be constant in space, and $\phi_\alpha(\vec{r})$ ($\alpha = x$ or y) is the phase that is the only dynamical variable of the problem, i.e., we focus our attention on the phase only. The CDW [Eq. (2.1)], has a periodicity of a square lattice with a spacing of $2\pi/Q$ in each direction. Although the lattice structure of the Wigner crystal is known to be triangular in the limit of small n,[2,9] we assume a square-lattice CDW, since the qualitative properties regarding pinning will not be different between these two lattice structures, and the calculations are simpler and more transparent for the square lattice.

In the following we will define various kinds of energy that determine the dynamics of ϕ_α. We restrict our consideration to the case where the spatial variation is slow compared with the lattice size, i.e., $|\nabla\phi_\alpha| \ll Q$. First the kinetic energy

K, associated with the temporal variation of the phase ϕ_α, is written

$$K=\frac{nm}{2Q^2}\int d\vec{r}\,(\dot{\phi}_x^2+\dot{\phi}_y^2)\ . \tag{2.2}$$

Equation (2.2) is true since $(\dot{\phi}_\alpha/2\pi)(2\pi/Q)$ is the local velocity in the αth direction and since nm is the mass density. Next there exists the elastic energy coming from the spatial variation of the phase, i.e., the local distortion of the CDW. We assume this energy to be written

$$U=\frac{1}{2}C_0\sum_{\alpha\beta}\int d\vec{r}\left(\frac{d\phi_\beta}{dx_\alpha}\right)^2, \tag{2.3}$$

where C_0 is a phenomenological parameter. In Eq. (2.3) we assumed for simplicity that the longitudinal distortion, $d\phi_x/dx$ or $d\phi_y/dy$ [Fig. 1(a)], costs the same energy as the transverse one, $d\phi_x/dy$ or $d\phi_y/dx$ [Fig. 1(b)]. This assumption is equivalent to taking the longitudinal sound velocity to be the same as the transverse one in the absence of long-range Coulomb interactions. The effect of the long-range part of the mutual Coulomb interaction can be written

$$U_c=\tfrac{1}{2}(en/Q)^2\times\int d\vec{r}\,d\vec{r}'\left(\frac{d\phi_x}{dx}+\frac{d\phi_y}{dy}\right)v(\vec{r}-\vec{r}')\left(\frac{d\phi_x}{dx'}+\frac{d\phi_y}{dy'}\right), \tag{2.4}$$

where $v(\vec{r})=1/r$. Equation (2.4) is derived by the following consideration. The spatial variation of ϕ_α changes the area of the unit cell (see Fig. 2) as follows:

$$\left[\frac{2\pi}{Q+d\phi_x/dx}\,\frac{2\pi}{Q+d\phi_y/dy}\right]\simeq\left(\frac{2\pi}{Q}\right)^2\left[1-\frac{1}{Q}\left(\frac{d\phi_x}{dx}+\frac{d\phi_y}{dy}\right)\right]. \tag{2.5}$$

Since the compensating background charge is uniform in space, the local accumulation of electronic charge associated with the contraction of the

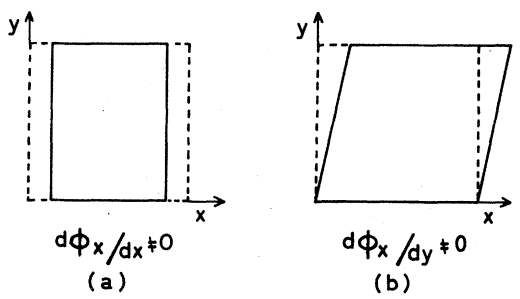

FIG. 1. Schematic representations of the longitudinal mode (a) and the shear mode (b).

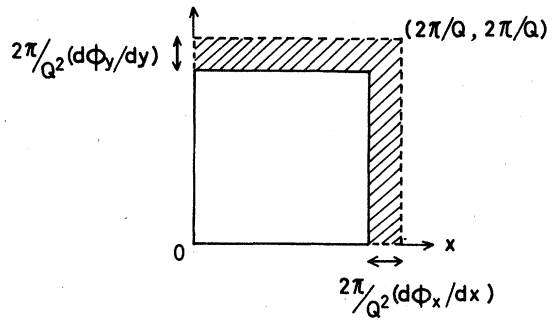

FIG. 2. Change of area of a unit cell by phase variation.

cell is then given by

$$-\left(\frac{en}{Q}\right)\left(\frac{2\pi}{Q}\right)^2\left(\frac{d\phi_x}{dx}+\frac{d\phi_y}{dy}\right). \tag{2.6}$$

Thus the electronic charge density per unit area introduced by the phase distortion is simply given by Eq. (2.6) divided by the area $(2\pi/Q)^2$. This leads to the interaction of Eq. (2.4). Finally, the interaction with impurity potentials will be written

$$V=eV_0\rho_0\sum_i\{\cos[Qx_i+\phi_x(\vec{R}_i)]+\cos[Qy_i+\phi_y(\vec{R}_i)]\}, \tag{2.7}$$

where we assumed that the impurity potential is short ranged and is given by $V_0\delta(\vec{r})$, and that $\vec{R}_i=(x_i,y_i)$ is the coordinate of the ith impurity. There exists also another type of interaction of the CDW with impurities that comes from the local modification of the unit cell:

$$V'=-\frac{en}{Q}V_0\sum_{\vec{R}_i}\left(\frac{d\phi_x}{dx}+\frac{d\phi_y}{dy}\right)_{\vec{r}=\vec{R}_i}\ . \tag{2.8}$$

Since V' in Eq. (2.8) contains only $d\phi_\alpha/dx_\alpha$ compared with V, which represents the coupling of ϕ_α itself to impurities, it is not essential to the pinning problem, and we discard V' in this paper. Equations (2.2)–(2.4) and (2.7) give the energies of our model. By rewriting K in Eq. (2.2) in terms of the variable P_α, which is the canonical conjugate to ϕ_α, our Hamiltonian is written

$$H=H_K+U+U_c+V, \tag{2.9}$$

$$H_K=\frac{1}{2nm}\int d\vec{r}\,(p_x^2+p_y^2), \tag{2.10}$$

$$[P_\alpha(\vec{r}),\,\phi_\beta(\vec{r}')]=-iQ\delta(\vec{r}-\vec{r}')\delta_{\alpha\beta}, \tag{2.11}$$

$$[P_\alpha(\vec{r}),\,P_\beta(\vec{r}')]=-inl^{-2}\delta(\vec{r}-\vec{r}')\epsilon_{\alpha\beta}, \tag{2.12}$$

where U, U_c, and V are given by Eqs. (2.3), (2.4), and (2.7), respectively.

Equation (2.12) represents the effect of magnetic

fields H applied perpendicular to the surface, and $l=\sqrt{c/eH}$ is the cyclotron radius. The components of the antisymmetric tensor $\epsilon_{\alpha\beta}$ are $\epsilon_{xx}=\epsilon_{yy}=0$, $\epsilon_{xy}=1$, and $\epsilon_{yx}=-1$. In Eq. (2.10) $P_\alpha=Q\,\partial K/\partial\dot{\phi}_\alpha=nm\dot{\phi}_\alpha/Q$ is the momentum density, and the commutation relation of Eq. (2.11) follows straightforwardly. Equation (2.12) is derived in the Appendix. It follows from the commutation relation of the canonical momentum variables of an electron in the presence of the vector potential $\vec{A}$.

$$[P_x+(e/c)A_x, P_y+(e/c)A_y]=-il^{-2}.$$

Equations (2.9)–(2.12) determine the dynamical properties of the phase that will be investigated in Sec. III.

III. PINNING OF THE CDW

The dynamical properties are conveniently described by the Green's function, defined by

$$D_{\alpha\beta}(\vec{q},\vec{q}';i\omega_n)=\frac{1}{2}\int_{-\beta}^{\beta}d\tau\, e^{i\omega_n\tau}\langle T_\tau\phi_\alpha(\vec{q},\tau)\phi_\beta(-\vec{q}')\rangle\,, \quad (3.1)$$

where $\beta=T^{-1}$ ($k_B=1$), $\omega_n=2\pi nT$, and T_τ is the chronological operator. The Fourier transform of $\phi_\alpha(\vec{r})$ is defined by

$$\phi_\alpha(\vec{r})=\frac{1}{(2\pi)^2}\int d\vec{q}\, e^{i\vec{q}\cdot\vec{r}}\phi_\alpha(\vec{q})\,, \quad (3.2)$$

and $\phi_\alpha(\vec{q},\tau)=e^{H\tau}\phi_\alpha(\vec{q})e^{-H\tau}$. Correspondingly, the Hamiltonian H [Eq. (2.9)] can be written in terms of $\phi_\alpha(\vec{q})$

$$H=\frac{1}{(2\pi)^2}\int d\vec{q}\left[\frac{1}{2nm}\sum_\alpha P_\alpha(\vec{q})P_\alpha(-\vec{q})+\frac{1}{2}C_0\sum_{\alpha,\beta}q_\alpha^2\phi_\beta(\vec{q})\phi_\beta(-\vec{q})+\frac{1}{2}\left(\frac{en}{Q}\right)^2\left(\sum_\alpha q_\alpha\phi_\alpha(\vec{q})\right)\frac{2\pi}{q}\left(\sum_\beta q_\beta\phi_\beta(-\vec{q})\right)\right]+V, \quad (3.3)$$

where $P_\alpha(\vec{q})$ is the Fourier transform of $P_\alpha(\vec{r})$, and Eqs. (2.11) and (2.12) can be rewritten

$$[P_\alpha(\vec{q}),\phi_\beta(\vec{q}')]=-i(2\pi)^2Q\delta(\vec{q}+\vec{q}')\delta_{\alpha\beta}\,, \quad (3.4a)$$

$$[P_\alpha(\vec{q}),P_\beta(\vec{q}')]=-i(2\pi)^2nl^{-2}\delta(\vec{q}+\vec{q}')\epsilon_{\alpha\beta}\,. \quad (3.4b)$$

The equation of motion of $D_{\alpha\beta}$ can easily be derived by use of Eqs. (3.4a) and (3.4b):

$$\omega_n^2D_{\alpha\beta}+\frac{C_0Q^2}{nm}q^2D_{\alpha\beta}+\frac{2\pi ne^2}{m}\frac{q_\alpha q_{\alpha'}}{q}D_{\alpha'\beta}-\omega_n\omega_c\epsilon_{\alpha\beta}D_{\alpha\beta}-\frac{iQ}{2nm}\int_{-\beta}^{\beta}d\tau e^{i\omega_n\tau}\langle T_\tau[V,P_\alpha(\vec{q})](\tau),\phi_\beta(-\vec{q})\rangle$$
$$=\frac{Q^2}{nm}(2\pi)^2\delta(\vec{q}-\vec{q}')\delta_{\alpha\beta}\,, \quad (3.5)$$

where $D_{\alpha\beta}=D_{\alpha\beta}(\vec{q},\vec{q}';i\omega_n)$, and $e^{H\tau}[V,P_\alpha(\vec{q})]e^{-H\tau}$ is abbreviated as $[V,P_\alpha(\vec{q})](\tau)$. In the absence of the impurity potential V, Eq. (3.5) yields

$$D_{\alpha\beta}(\vec{q},\vec{q}';i\omega_n)=(2\pi)^2\delta(\vec{q}-\vec{q}')\frac{Q^2}{nm}\frac{1}{(\omega_n^2+\omega_+^2)(\omega_n^2+\omega_-^2)}\begin{pmatrix}\omega_n^2+\omega_t^2+\omega_p^2\frac{q_y^2}{q^2} & -\omega_p^2\frac{q_xq_y}{q^2}+\omega_n\omega_c\\ -\omega_p^2\frac{q_xq_y}{q^2}-\omega_n\omega_c & \omega_n^2+\omega_t^2+\omega_p^2\frac{q_x^2}{q^2}\end{pmatrix}.$$

We shall define $D_{\alpha\beta}(\vec{q},\vec{q}';i\omega_n)=(2\pi)^2\delta(\vec{q}-\vec{q}')D^0_{\alpha\beta}(\vec{q},i\omega_n)$. (3.6)

In Eq. (3.6) ω_t, ω_p, and $\omega_\pm$ are defined as follows:

$$\omega_t^2=(C_0Q^2/nm)q^2\,, \quad (3.7)$$

$$\omega_p^2=(2\pi ne^2/m)q\,, \quad (3.8)$$

$$\omega_\pm^2=\tfrac{1}{2}\{\omega_c^2+\omega_p^2+2\omega_t^2\pm[(\omega_c^2+\omega_p^2)^2+4\omega_c^2\omega_t^2]^{1/2}\}\,, \quad (3.9)$$

where ω_t is the eigenfrequency of the phase excitation in the absence of both magnetic field and the Coulomb long-range interaction. On the other hand, ω_p is the plasma frequency. In the limit of the small wave vector q, we have

$$\omega_+\simeq\omega_c+\omega_p^2/2\omega_c,\quad \omega_-\simeq\omega_t\omega_p/\omega_c\,. \quad (3.10)$$

We see that $\omega_\pm$ is essentially the same as those of a Wigner crystal, the only difference being that C_0 here is an arbitrary parameter whereas in the Wigner crystal ω_t is explicitly determined.[10]

Next we will determine the effect of the impurity potential. Even in pure systems the existence of long-range order is impossible, in a rigorous sense, at finite temperatures for our model system, since $T\sum_{\omega_n}\sum_{\vec{q}}D^0_{\alpha\alpha}(\vec{q},i\omega_n)$ is logarithmically

divergent.[11] However, this divergence is extremely weak and in a finite system it is possible to think of the system as possessing long-range order. The existence of impurities will destroy the long-range order,[12] and the density correlation function will decay at large distances. We can define[13] the characteristic distance L_0 of the variation of the phase, i.e., $|d\phi_\alpha/dx_\beta| \sim L_0^{-1}$. (The phase variation will be the same in both x and y directions.) This implies that the phase in the ground state, $\phi_\alpha^0(r)$, is distorted over a distance of L_0. If the magnetic field is treated classically, it does not affect static properties like the correlation length L_0. This L_0 will be determined, following Ref. 13, by minimizing the energy gain per unit area $\delta E(L_0)$ with respect to L_0:

$$\delta E(L_0) = \frac{C_0}{2}\frac{4}{L_0^2} + \frac{1}{2}\left(\frac{en}{Q}\right)^2 (2\pi L_0)\frac{4}{L_0^2} - eV_0\rho_0\frac{(n_i L_0^2)^{1/2}}{L_0^2}$$
$$= 2C_0/L_0^2 - [eV_0\rho_0\sqrt{n_i} - 4\pi(en/Q)^2]1/L_0 , \tag{3.11}$$

where n_i is the impurity number density. From Eq. (3.11) we see that, if $eV_0\rho_0\sqrt{n_i} > 4\pi(en/Q)^2$, L_0 is given by

$$L_0 = 4C_0[eV_0\rho_0\sqrt{n_i} - 4\pi(en/Q)^2]^{-1} . \tag{3.12}$$

If $eV_0\rho_0\sqrt{n_i} \leq 4\pi(en/Q)^2$, Eq. (3.11) implies that $L_0 = \infty$, which in turn will mean that the density correlation function does not decay exponentially, but probably with some power. In the following we will focus our attention on the case of Eq. (3.12), i.e., the impurity potential is strong enough so that

$$eV_0\rho_0\sqrt{n_i} \geq 4\pi(en/Q)^2 . \tag{3.13}$$

In order to determine the spectrum of the excitations above such a distorted ground state, we use the method employed in Ref. 13 and write the phase as $\phi_\alpha(\vec{r}) = \phi_\alpha^0(\vec{r}) + \psi_\alpha(\vec{r})$, where ϕ_α^0 describes the ground state and ψ_α the small fluctuations. Corresponding to this separation of the phase variable, we expand the potential $U[\phi] + U_c[\phi] + V[\phi]$ in terms of $\psi_\alpha(r)$. Since U and U_c are quadratic in $\phi_\alpha(r)$, their functional forms are the same as Eqs. (2.3) and (2.4), except that $\phi_\alpha(\vec{r})$ is replaced by $\psi_\alpha(\vec{r})$. On the other hand, the nonlinear potential V should read

$$V = -\frac{eV_0\rho_0}{2}\sum\{\cos[Qx_i + \phi_x^0(\vec{R}_i)]\psi_x^2(\vec{R}_i) + \cos[Qy_i + \phi_y^0(\vec{R}_i)]\psi_y^2(\vec{R}_i)\} \tag{3.14a}$$

$$= -\frac{1}{2}\frac{1}{(2\pi)^4}\int\int d\vec{q}\,d\vec{q}'\,[\psi_x(\vec{q})\psi_x(-\vec{q}')S_x(\vec{q}-\vec{q}') + \psi_y(\vec{q})\psi_y(-\vec{q}')S_y(\vec{q}-\vec{q}')] , \tag{3.14b}$$

where $\psi_\alpha(\vec{q})$ is the Fourier transform of $\psi_\alpha(\vec{r})$, and

$$S_x(\vec{q}) = eV_0\rho_0\sum e^{i\vec{q}\cdot\vec{R}_i}\cos[Qx_i + \phi_x^0(\vec{R}_i)]. \tag{3.15}$$

By use of Eqs. (3.14b) and (3.5) we find that the Green's function $D_{\alpha\beta}$ redefined in terms of $\psi_\alpha(q)$ instead of $\phi_\alpha(q)$ in Eq. (3.1) obeys the following equation of motion:

$$D_{\alpha\beta}(\vec{q},\vec{q}') = (2\pi)^2\delta(\vec{q}-\vec{q}')D_{\alpha\beta}^0(\vec{q}) + D_{\alpha\beta}^0(\vec{q})\frac{1}{(2\pi)^2}$$
$$\times\int d\vec{q}''S_\alpha(\vec{q}''-\vec{q})D_{\alpha\beta}(\vec{q}'',\vec{q}') . \tag{3.16}$$

We treat the second term on the right-hand side of Eq. (3.16) perturbatively. In order to determine the Green's function averaged over impurity configurations, $\langle D\rangle_{\rm av}$, we define the self-energy function of the phason, Π, by

$$\langle D_{\alpha\beta}(\vec{q},\vec{q}')\rangle_{\rm av} = (2\pi)^2\delta(\vec{q}-\vec{q}')[(D^0)^{-1} - \Pi]^{-1}_{\alpha\beta}$$
$$\equiv (2\pi)^2\delta(\vec{q}-\vec{q}')D_{\alpha\beta}(\vec{q}, i\omega_n) . \tag{3.17}$$

The first two contributions to Π are shown in Fig. 3(a) ($\Pi_{(1)}$) and 3(b) ($\Pi_{(2)}$), where the wavy line and the dotted lines are D and S, respectively, and the cross represents impurities. The first one is given by

$$\Pi_{(1)}^{\alpha\beta} = -(eV_0\rho_0\sqrt{n_i}/L_0)\delta_{\alpha\beta} , \tag{3.18}$$

where we assumed that the pinning potential is weak,[13] since

$$\langle S_\alpha(\vec{q})\rangle_{\rm av} = eV_0\rho_0(\sqrt{n_i}/L_0)(2\pi)^2\delta(\vec{q}) . \tag{3.19}$$

Equation (3.18) gives the following excitation spectrum:

$$\Omega_\pm^2 = \tfrac{1}{2}\{\omega_c^2 + \omega_p^2 + 2(\omega_t^2+\gamma^2)$$
$$\pm[(\omega_c^2+\omega_p^2)^2 + 4\omega_c^2(\omega_t^2+\gamma^2]^{1/2}\} , \tag{3.20}$$

where γ is given by

$$\gamma = [(eV_0\rho_0\sqrt{n_i}/L_0)(Q^2/nm)]^{1/2} . \tag{3.21}$$

At $q = 0$, Eq. (3.20) yields

$$\Omega_\pm^2 = \tfrac{1}{2}[\omega_c^2 + 2\gamma^2 \pm \omega_c(\omega_c^2+4\gamma^2)^{1/2}] \equiv \Omega_{\pm 0}^2 . \tag{3.22}$$

This is shown in Fig. 4, as solid lines, as a function of ω_c/γ. This spectrum has a gap at $q = 0$, which represents the effect of pinning. The q dependence in the small-q region, where $\omega_p, \omega_t < \gamma$ is

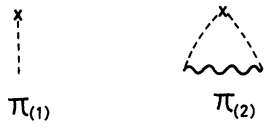

FIG. 3. Diagrammatical representations of the contribution to the phason self-energy in the first order (a) and the second order (b) of the impurity potential.

satisfied, depends on whether ω_c is zero or not. If $\omega_c = 0$, Eq. (3.20) results in

$$\Omega_+^2 = \gamma^2 + \omega_p^2 + \omega_t^2 , \tag{3.23a}$$

$$\Omega_-^2 = \gamma^2 + \omega_t^2 . \tag{3.23b}$$

However, if $\omega_c \neq 0$ and if q is small such that ω_p, $\omega_t < \omega_c$, γ, then

$$\Omega_\pm^2 = \Omega_{\pm 0}^2 + \tfrac{1}{2}(\omega_p^2 + 2\omega_t^2)\{1 \pm [1 + 4(\gamma/\omega_c)^2]^{-1/2}\} \pm \omega_p^4/4\omega_c^2[1 + 4(\gamma/\omega_c)^2]^{-1/2} . \tag{3.24}$$

From Eq. (3.24) we see that in the presence of a magnetic field, i.e., if $\omega_c \neq 0$, $\Omega_{\pm 0}^2$ is quadratic in q as $q \to 0$ as long as the Coulomb interaction U_c exists.

The modes $\Omega_\pm$ given by Eq. (3.20) are actually damped due to the random distribution of impurities. This is seen by looking at $\Pi_{(2)}$ of Fig. 3(b), which is written

$$\Pi_{(2)}^{\alpha\beta}(i\omega_n) = \delta_{\alpha\beta}\frac{n_i(eV_0\rho_0)^2}{(2\pi)^2}\int d\tilde{q}\, D^0_{\alpha\beta}(\tilde{q}, i\omega_n) . \tag{3.25}$$

This is due to the fact that

$$\langle S_\gamma(\tilde{q}) S_{\gamma'}(\tilde{q}')\rangle_{\rm av} = \tfrac{1}{2}\delta_{\gamma\gamma'}(eV_0\rho_0)^2 n_i (2\pi)^2 \delta(\tilde{q} + \tilde{q}') . \tag{3.26}$$

In estimating Eq. (3.25) we can replace D^0 on the right-hand side by $(D^{0^{-1}} - \Pi_{(1)})^{-1}$, e.g.,

$$\Pi_{(2)}^{xx}(i\omega_n) = \frac{n_i}{nm}(eV_0\rho_0 Q)^2 \frac{1}{(2\pi)^2}\int d\tilde{q}\frac{\omega_n^2 + \omega_t^2 + \omega_p^2 q_y^2/q^2 + \gamma^2}{(\omega_n^2 + \Omega_+^2)(\omega_n^2 + \Omega_-^2)}$$
$$= \frac{n_i}{2nm}(eV_0\rho_0 Q)^2 \frac{1}{(2\pi)^2}\int d\tilde{q}\left(\frac{1}{\omega_n^2 + \Omega_+^2} + \frac{1}{\omega_n^2 + \Omega_-^2} - \frac{\omega_c^2}{(\omega_n^2 + \Omega_+^2)(\omega_n^2 + \Omega_-^2)}\right)$$
$$= \Pi_{(2)}^{yy}(i\omega_n) \equiv \Pi_{(2)}(i\omega_n) . \tag{3.27}$$

From Eqs. (3.23) and (3.24) we see that the q integration in Eq. (3.27) is convergent as long as $U_c \neq 0$ and $\omega_c \neq 0$. Even if $U_c = 0$, this integration is only logarithmically divergent, in contrast to the case of quasi-one-dimensional systems.[13-15] This weak divergence is simply removed by renormalizing γ^2 by

$$\gamma^2[1 - \Pi_{(2)}(i\omega_n)(nm/Q^2)]$$

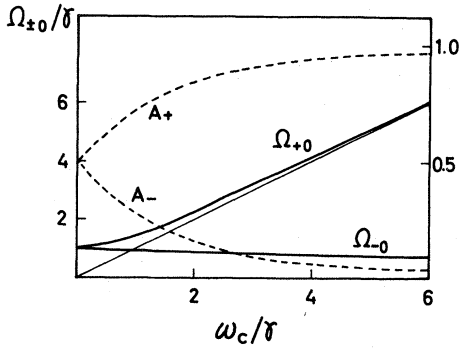

FIG. 4. Magnetic field dependences of the collective modes at $q = 0$, $\Omega_{\pm 0}$, Eq. (3.22) and the spectral weights $A_\pm$, Eq. (4.4b). (Dashed lines refer to the right-hand vertical scale.) The straight line through the origin represents the cyclotron frequency to which Ω_{+0} tends as ω_c/γ increases.

in $\Omega_\pm$, and solving Eq. (3.27) self-consistently for $\Pi_{(2)}(i\omega_n)$. (This is the self-consistent Born approximation.[15]) The result is that the existence of $\Pi_{(2)}(i\omega_n)$ introduces an imaginary part that is essentially independent of frequency. The phason self-energy function is now written

$$\Pi(i\omega_n)_{\alpha\beta}|_{i\omega_n \to \omega + i0} = -\delta_{\alpha\beta}\frac{Q^2}{nm}\gamma^2[1 + ia\ \mathrm{sgn}(\omega)] , \tag{3.28}$$

where a is a constant of order unity.

Thus we determined the phason Green's function [Eq. (3.17) with Eqs. (3.28) and (3.21)] in the presence of pinning. In the following Sec. IV we will investigate the transport properties based on the result obtained here.

IV. MAGNETOCONDUCTIVITY

In this section we will evaluate the frequency-dependent conductivity tensor in the presence of impurity pinning at absolute zero. As has been shown in Ref. 15, the conductivity tensor is given as follows, in terms of the phason Green's function:

$$\sigma_{\alpha\beta}(\omega)=\int_0^\infty dt\, e^{-i\omega t}\langle[J_\alpha(t),P_\beta]\rangle \tag{4.1a}$$

$$=i\omega\left(\frac{en}{Q}\right)^2\int_0^\infty dt\, e^{-i\omega t}\times\langle[\phi_\alpha(q=0,t),\phi_\beta(q=0]\rangle \tag{4.1b}$$

$$=i\omega\left(\frac{en}{Q}\right)^2 D_{\alpha\beta}(q=0,\omega+i0)\,, \tag{4.1c}$$

where J_α and P_β are the total current operator $-(en/Q)\int dr\,\dot{\phi}_\alpha$, and the total polarization operator $-(en/Q)\int dr\,\phi_\beta$, respectively. In Eq. (4.1) $D_{\alpha\beta}(q,\omega+i0)$ is the Green's function defined by Eq. (3.17). We have a word to say about the equality of Eq. (4.1c) regarding the local-field corrections due to the long-range part of the Coulomb interactions. Physically, the conductivity is to be defined with respect to the local electric field, whereas Eq. (4.1c) represents the response to the external electric field, since $D_{\alpha\beta}$ is determined in the presence of the long-range part of the Coulomb interactions U_c. However, in contrast to the case of three-dimensional systems, the uniform ($q=0$) polarization field is absent in the present case of two-dimensional systems for any finite frequency. Thus $\sigma_{\alpha\beta}$ defined by Eq. (4.1c) can be considered as that with respect to the local electric field.

In the absence of impurity scattering, Eq. (4.1c), together with Eq. (3.6), yields[16]

$$\sigma_{xx}(\omega)=\sigma_{yy}(\omega)=(ne^2/m)i\omega/(\omega_c^2-\omega^2)\,, \tag{4.2a}$$

$$\sigma_{xy}(\omega)=(ne^2/m)\omega_c/(\omega_c^2-\omega^2)\,. \tag{4.2b}$$

These are the same as those for noninteracting systems, as they should be. If $\gamma\neq 0$, we obtain, by ignoring damping of the collective modes,

$$\sigma_{xx}(\omega)=\sigma_{yy}(\omega)=-\frac{ne^2}{m}\frac{i\omega(\omega^2-\gamma^2)}{(\Omega_{-0}^2-\omega^2)(\Omega_{+0}^2-\omega^2)} \equiv\sigma(\omega)\,, \tag{4.3a}$$

$$\sigma_{xy}(\omega)=-(ne^2\omega_c/m)\omega^2/(\Omega_{-0}^2-\omega^2)(\Omega_{+0}^2-\omega^2)\,, \tag{4.3b}$$

where $\Omega_{\pm0}$ is given by Eq. (3.22). As $\omega\to 0$, we see that not only σ_{xx} and σ_{yy}, but also σ_{xy} tends to vanish. This is the pinning of the CDW at absolute zero.

The real part of $\sigma(\omega)$ has δ functions at Ω_{-0} and Ω_{+0} whose weights are

$$\mathrm{Re}\sigma(\omega)=\frac{ne^2\pi}{m}\left[A_-\delta(\omega-\Omega_{-0})+A_+\delta(\omega-\Omega_{+0})\right], \tag{4.4a}$$

$$A_\pm=\tfrac{1}{2}\{1\pm[1+4(\gamma/\omega_c)^2]^{-1/2}\}\,. \tag{4.4b}$$

These two weights, $A_\pm$, are shown in Fig. 4 as broken lines. These two peaks of $\mathrm{Re}\sigma(\omega)$ correspond to the absorptions due to the pinned mode (Ω_{-0}) and the cyclotron mode (Ω_{+0}). In the case where $\omega_c\gg\gamma$ is satisfied, $\Omega_{\pm0}$ and $A_{\pm0}$ take the forms

$$\Omega_{-0}\sim\gamma^2/\omega_c,\quad A_-\sim(\gamma/\omega_c)^2\,, \tag{4.5a}$$

$$\Omega_{+0}\sim\omega_c+\gamma^2/\omega_c,\quad A_+\sim 1-(\gamma/\omega_c)^2\,. \tag{4.5b}$$

If we include the effect of damping of the collective modes, Eq. (3.28), the δ-function of Eq. (4.4a) is changed into Lorentzians, i.e., $\delta(\omega-\Omega_{\pm0})\to\Gamma_\pm/\pi[(\omega-\Omega_{\pm0})^2+\Gamma_\pm^2]$, with $\Gamma_\pm$ given by

$$\Gamma_-\sim\gamma^2 a/\omega_c\,, \tag{4.6a}$$

$$\Gamma_+\sim\gamma^2 a/\omega_c\,. \tag{4.6b}$$

We see that the absorption due to the pinned mode is broad and weak, since $\Omega_{-0}\sim\Gamma_-$ and $A_-\ll 1$. On the other hand, the cyclotron resonance is sharp and strong as $\Omega_{+0}\gg\Gamma_+$ and $A_+\sim 1$. An interesting and important fact on the cyclotron resonance in the presence of pinning is that the cyclotron mass gets lighter by the amount δm:

$$\delta m/m\simeq-(\gamma/\omega_c)^2\,. \tag{4.7}$$

Moreover, we see from Eqs. (4.6b) and (4.7) that

$$(\Gamma_+/\omega_c)/(|\delta m|/m)\sim a\sim O(1)\,. \tag{4.8}$$

V. SUMMARY AND DISCUSSIONS

We investigated the phase pinning of the CDW in the presence of magnetic fields at absolute zero. We assumed random impurities for the pinning force. We found that the phase is distorted in the ground state, and we determined the characteristic distance over which the phase is correlated. This distance is independent of the magnetic field, since this is solely determined by the static properties of the system on which the magnetic field has no influence in the classical limit. However, the excitation spectrum is affected by both the pinning force and the magnetic field.

Based on the excitation spectrum, we evaluated the conductivity tensor and found the following results: (i) $\sigma_{xx}=0$, and $\sigma_{xy}=0$ at $\omega=0$. (ii) $\mathrm{Re}\sigma_{xx}(\omega)$ has peaks at the frequencies of the pinning mode and the cyclotron resonance. (iii) The cyclotron mass gets lighter in the presence of pinning. The absolute magnitude of this shift and the width of the resonance are dependent not only on the scattering potential but also on the electron number density, and they satisfy the relation of Eq. (4.8) as long as the pinning frequency in the absence of magnetic field is smaller than the cyclotron frequency.

We will discuss these findings in the following:

(i) The absence of static conductivity is the result of pinning.

(ii) The frequencies $\Omega_{\pm 0}$ [Eq. (3.22)] are dependent on carrier density, scattering potential, and magnetic field. Above all, the carrier number dependence will generally be complicated, since not only ρ_0 but also Q will depend on n. Though we assumed elastic impurity scattering for the existtence of finite Ω_{-0} (or γ), dimples in the electron–helium-surface system[11] and lattice distortions in the MOS will also lead to finite γ as far as these can be treated adiabatically. They will act like a commensurate pinning force. Actually, Shikin[17] obtained similar results as Eq. (3.22) by considering the effect of dimples on the Wigner crystal in the electron-helium surface system. However, in such a case of a classical Wigner crystal, the pinning energy γ will be independent of carrier density,[17] since the average distance between electrons is much larger than the width of dimples.

(iii) The possiblility of a negative mass shift of the cyclotron resonance is itself not new. Such effects are well known in doped semiconductors where the ground Landau state at an impurity site is more localized than the first excited state.[18] A similar effect is also demonstrated recently by Cheng and Platzman[19] who treated dimples in the electron–helium-surface system in comparison with the experiment of Edelman.[20] In these investigations only single-particle effects are considered and then the mass shift is independent of the carrier density n. If the CDW or Wigner crystal is formed, the mass shift will be in general dependent on n through γ. However, the shift due only to static dimples is independent of n even if the Wigner crystal is formed since γ is determined locally as is discussed in (ii).

In the case of MOS, Kennedy *et al.*[5] found the negative mass shift that is independent on both carrier density and magnetic field. They found that if the carrier density is low, $n_s \sim 10^{11}$ cm^{-2}, the width of the cyclotron resonance (τ^{-1}) and the absolute magnitude of the negative mass shift increase as the magnetic field is decreased. These results are qualitatively consistent with ours of Eqs. (4.6b) and (4.7), though the present semiclassical treatment of magnetic fields might not be completely appropriate in this case of relatively strong magnetic fields.

In conclusion, we investigated the conductivity of the pinned CDW in the presence of magnetic fields. The direct consequence of the existence of the CDW is that $\mathrm{Re}\sigma_{xx}(\omega)$ has peaks not only at $\omega \sim \omega_c$ but also at the pinning frequency, though the oscillator strength might not be appreciable in the pinned mode. In the CDW state the cyclotron resonance is also expected to have a particular shift and width. However, in the present stage we could not make detailed comparisons between theoretical predictions and experimental data.

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APPENDIX

Here we will derive Eq. (2.12). We assume that the Hamiltonian of the electronic system that forms the CDW state consists of kinetic energy and interaction energy independent of momentum. The former is written as follows in the presence of uniform external magnetic fields:

$$H_0 = \sum_i \frac{1}{2m}\left(p_i + \frac{e}{c}A_i\right)^2 , \tag{A1}$$

where i is the particle index. In order to define the momentum density per unit area $P_\alpha(r)$, we first divide the whole system into cells with area ΔS whose size is larger than the unit cell of the CDW and is smaller than the characteristic length of the spatial variation of the phase, i.e., within ΔS the phase can be taken as constant in space. Thus $P_\alpha(r)$ is given by

$$P_\alpha(\vec{r}) = \frac{1}{\Delta S}\sum_{i\in\Delta S}\left(p_i + \frac{e}{c}A_i\right), \tag{A2}$$

where the summation is over all electrons in the cell where the coordinate $\vec{r}$ is located. Equation (A2) leads to the following commutation relation:

$$[P_x(r), P_y(r')] = -i[N_i/(\Delta S)^2]eH/c , \tag{A3}$$

if r and r' are in the same cell and $[P_x(r), P_y(r')] = 0$ otherwise. In Eq. (A3) N_i is the number of electrons in the cell. Ignoring the fluctaution of the electron number in the cell, we put $N_i/\Delta S = n$. On the other hand, $(\Delta S)^{-1}$ can be considered equivalent to $\delta(r-r')$ in the limit of small size of the cell, since we are interested in the slow variation of the phase, i.e., $|\nabla\phi|/Q \ll 1$. Consequently, Eq. (2.12) follows.

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