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Fluctuating forces in turbulent He II

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We have measured a fluctuating force on small probes immersed in turbulent He II thermal counterflow. The force is consistent with neither periodic normal fluid vortex shedding from the probe, nor with a secondary normal fluid flow in the counterflow channel. Instead, the force is shown to be a random function of time, which is most simply interpreted in terms of an exponential power spectrum. The data thus provide the dependence of the force correlation time on temperature and velocity. This dependence is shown to agree with that of the correlation time for vortex line density fluctuations given by the Vinen equation. Theoretical and experimental implications of this result are discussed.

In the past twenty years considerable progress has been made in understanding He II thermal counterflow. Vinen[1] showed that at large flow velocities there is an excess dissipation present in the counterflow which can be accounted for by imagining the fluid to be permeated by a tangled mass of quantized vortex lines. He was able to devise a simple phenomenological equation that accurately accounted for the observed dissipation. Brewer and Edwards[2] later showed that, at low velocities, the flow is described by the laminar two-fluid equations of motion. Several subsequent experiments[3–6] have confirmed the essential features of the Vinen equation, including its prediction of a critical velocity for the onset of the turbulence.[3]

Recent work has emphasized that our understanding of turbulent thermal counterflow is far from complete. Ladner, Childers, and Tough[7] have found that a new flow state, which they interpret as secondary flow in the normal fluid, appears at very high velocities. In another important experiment, Hoch, Busse, and Moss[8] have discovered fluctuations in the vortex line density in turbulent counterflow. Finally, Schwarz[9] has developed a theory of turbulent counterflow from consideration of the detailed dynamics of the vortex lines. He has shown that the Vinen equation follows as an accurate consequence of the theory.

We report here an experiment in which a small resonantly mounted plate was used to probe thermal counterflow in a rectangular channel. The plate was mounted parallel to the counterflow velocity in a manner allowing motion perpendicular to the flow. Any time-dependent forces on the probe resulting from, for example, a secondary flow of the viscous normal fluid, vortex shedding in the normal fluid, or vortex line density fluctuations would then result in a displacement of the probe. By studying the probe response as a function of counterflow velocity, temperature, and geometry, information about the state of the fluid in turbulent counterflow could be inferred.

The thermal counterflow cell is shown schematically at the top of Fig. 1. (Note that the top has been removed for clarity.) Details of the channel and probe construction are given in Table I. The cell was immersed in a temperature regulated He II bath. Flow could be initiated in the rectangular channel by resistively heating the ^{4}He in a small reservoir. The probe was resonantly mounted on a niobium loop to allow motion transverse to the flow. This mechanical oscillator had a resonance frequency ω_0 and a Q of about 100 in the liquid. Motion of the probe in the small ($\sim$10 G) magnetic field $\vec{B}$ resulted in changes in the flux through the niobium loop. These flux changes were coupled into a superconducting quantum interference device (SQUID), the output of which was a voltage proportional to the probe displacement $x(t)$. An analog squarer and digital signal averager were used to compute $\langle x^2\rangle$. Measurements of $\langle x^2\rangle$ were performed using an averaging time of 500 sec or more. An interval of this length resulted in acceptable scatter within a daily data run and ensured day-to-day reproducibility to within this scatter. With the small magnetic field employed here we were able to resolve probe motions of a few angstroms. Great precautions were taken to isolate the cryostat from external vibrations.

A typical example of the probe displacement versus time $x(t)$ is shown in Fig. 1. At low heat currents the rms displacement amplitude is independent of heat current to within the scatter of the points. At higher heat currents the displacement amplitude increases with heat current. Ignoring for the moment the obvious modulation of $x(t)$, we consider the possibility that this response may be driven by normal fluid vor-

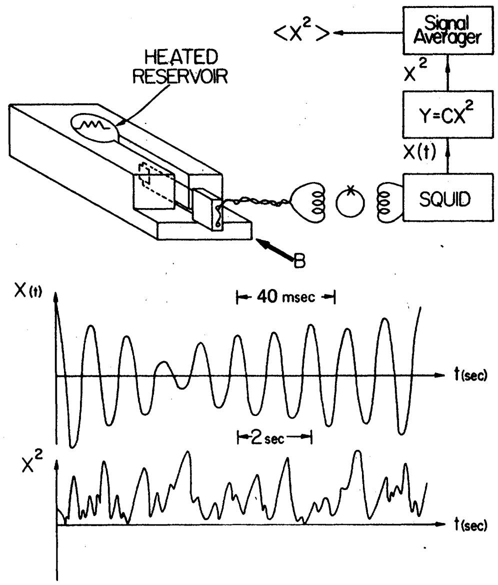

FIG. 1. Schematic representation of the counterflow cell, instrumentation, and response of the probe to a large counterflow velocity V. The cell, shown without its top, was machined from epoxy. Only the envelope of the x^2 vs time curve is shown.

tex shedding. Considerable data[10] exist in the literature of fluid mechanics on the shedding of vortices in the wake of bluff bodies. A characteristic feature of the phenomenon is that the frequency of shedding is related to the flow velocity V and the thickness of the body W by the simple expression $\omega_s = CV/W$. The parameter C, which depends somewhat on geometry and the fraction of the channel blocked by the probe, is about 1 for this apparatus. It is also well established that the shedding produces a periodic lift force on the body at frequency ω_s. If this force is responsible for the oscillations of the probe in the counterflow channel (Fig. 1) then we expect a resonance to occur when $\omega_0 = \omega_s$, or when $V_n = V_{sh} \equiv \omega_0 W/C$, where V_n is the normal fluid velocity. The signature of normal fluid vortex shedding is thus unmistakeable: the amplitude of oscillation should exhibit a maximum when the normal fluid velocity is near V_{sh}.[11]

In Fig. 2 we show data for $\langle x^2\rangle$ as a function of the normal fluid velocity V_n taken in two different geometries at the temperature $T = 1.6$ K.[12] The velocity V_{sh} corresponding to resonant shedding is indicated by an arrow. The data do bear a superficial resemblance to the low frequency portion of a resonance curve. In no case, however, was it possible to reduce $\langle x^2\rangle$ by continued increase of V_n. Moreover, if there were a maximum value of $\langle x^2\rangle$, in cell No. 1 it would certainly occur for $V_n > V_{sh}$ while for cell No. 4 it would be for $V_n < V_{sh}$. It seems clear that these data cannot be understood in terms of the resonant response of the probe to a periodic shedding force. Finally, we note that even when $\langle x^2\rangle$ is very large, $x(t)$ continues to be highly modulated.

The lack of a resonance in certain data (such as for cell No. 1 in Fig. 2) might suggest that normal fluid vortex shedding simply does not occur in turbulent counterflow. This would certainly not be unreasonable since the mass of quantized vortex line in the superfluid could destroy the spatial coherence needed for shedding. Unfortunately there is some ambiguity as to whether the Reynolds number is sufficiently large for a fluctuating lift force to occur even in the absence of turbulence. The value of $R\,(=\rho V_n l/\eta)$ based on the plate lenght l, total fluid density ρ, viscosity η, and the velocity at resonance (arrow in Fig. 2) is about 8500 and surely large enough. The Reynolds number $R_n(=\rho_n V_n l/\eta)$, using the normal fluid density ρ_n rather than the total density ρ, is only about 1400, however, and is marginal. Since it is not certain which is the appropriate Reynolds number, it is not possible unambiguously to demonstrate from these data that normal fluid vortex shedding is suppressed by the turbulence. Experimental con-

TABLE I. Details of channel and probe construction.

	Cell No. 1 $f_0 = 28$ Hz	Cell No. 2 $f_0 = 120$ Hz	Cell No. 4 $f_0 = 35$ Hz	Cell No. 6 $f_0 = 41$ Hz
Channel size (cm)				
width d	0.20	0.20	0.20	0.30
height	1.0	1.0	1.0	1.0
length	2.5	2.5	2.5	2.5
Probe size				
thickness W	0.038	0.015	0.061	0.061
length l	0.12	0.20	0.24	0.24
height	0.70	0.65	0.60	0.60

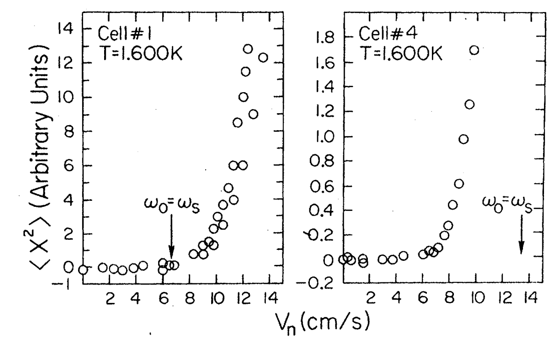

FIG. 2. Mean-squared probe displacement $\langle x^2\rangle$ vs the normal fluid velocity V_n as measured in two different experimental geometries. The arrows indicate the expected velocity for resonant vortex shedding.

siderations unfortunately do not allow a search for shedding in laminar counterflow.[13]

Having eliminated vortex shedding as a source of the probe oscillations we now consider a secondary flow such as suggested by Ladner, Childers, and Tough.[7] In conventional fluids such flows are both time dependent and spatially inhomogeneous, and could be expected to produce a force on a small probe. Unfortunately, $\langle x^2 \rangle$ does not increase above background until V_n is much greater than the onset velocity suggested by Ref. 7. Further, the probe response would be expected to scale with a Reynolds number R $(=\rho V_n d/\eta)$ based upon the channel width d. This is not observed. The rapid increase of $\langle x^2 \rangle$ at $V_n \approx 6$ cm/sec for cells No. 1 and No. 4 (Fig. 2) corresponds to $R \approx 14\,000$. The same feature in the data obtained with the wide channel (cell No. 6) occurs at $V_n \approx 7.5$ cm/sec, or $R \approx 24\,000$. We conclude that a secondary flow is not responsible for the motion of the probe. We note, however, that the original observations interpreted as a secondary flow were made in channels of circular cross section. More recent results[14] show that the secondary flow is suppressed in rectangular geometries. It is therefore not surprising that we see no evidence for this phenomenon.

In order to understand the probe motion produced by the turbulent counterflow it is necessary to account first for the slow modulation of the oscillation amplitude that is obvious in Fig. 1. Suppose that the probe is driven by a random force $F(t)$ which has a zero mean value, $\langle F(t) \rangle = 0$. If $F(t)$ acts on a simple harmonic oscillator, the displacement of the oscillator with time will be[15] $x(t) = E(t)\{\cos[\omega_0 t + \phi(t)]\}$. Here, ω_0 is the resonant frequency of the oscillator and $\phi(t)$ is a random function of time. $E(t)$ has a Rayleigh distribution: it is white for frequencies less than the oscillator bandwidth and zero for higher frequencies. The displacement $x(t)$ of the oscillator has a Gaussian probability distribution. The function $x(t)$ will thus appear to be a slowly modulated cosine at the frequency ω_0. This is precisely the response of our probe to the heat current (Fig. 1), and we conclude it is in fact being driven by a random force.

The power spectrum $S_F(\omega)$ of this force is related to its mean-squared amplitude by[16] $\langle F^2 \rangle = \int S_F(\omega)\, d\omega$. A power spectrum can also be defined for the displacement x. $S_x(\omega)$ is related to $S_F(\omega)$ through an oscillator response function $\Phi(\omega, \omega_0)$ by $S_x(\omega) = \Phi(\omega, \omega_0) S_F(\omega)$. For a narrow bandwidth oscillator, Φ will be very sharp and may be approximated by a δ function, so that

$$\langle x^2 \rangle = \int S_x(\omega)\, d\omega \approx \int \delta(\omega, \omega_0) S_F(\omega)\, d\omega \quad .$$

We finally find

$$\langle x^2 \rangle = S_F(\omega_0) \quad . \tag{1}$$

A measurement of $\langle x^2 \rangle$ therefore gives the power spectrum of the random driving force at the resonant frequency of the oscillator.

Figure 3 shows data for $S_F(\omega_0)$ versus the counterflow velocity V obtained in cell No. 6 at $T = 1.4$ and 1.6 K. Similar results were obtained for all cells and at all temperatures [see, for example, Fig. 2, where now $\langle x^2 \rangle = S_F(\omega_0)$]. The monotonic increase of amplitude with velocity is quite obvious. The lower limit for the superfluid turbulence given by the Vinen equation[1,3] is shown in the figure as V_c, and is much lower than any apparent onset for $\langle x^2 \rangle$. In fact it is not possible to define an onset velocity that corresponds to a value of either R or R_n which is independent of probe size and temperature.

It is possible to understand the strong dependence of the power spectrum on V and T by assuming it has an exponential form

$$S_F(\omega) = S_0 \exp[-\omega\tau(V,T)] \quad , \tag{2}$$

where the dependence of S_F on V is contained in the correlation time τ. All of the dynamical information about the random force is thus contained in a correlation time that is determined by our data. Of course, this choice for the form of the power spectrum is not unique. It would be possible, for example, to use $S_F(\omega) = S_0\tau/(1+\omega^2\tau^2)$, but the data would then require τ to vary exponentially with V. Since this dependence seems rather unphysical, we prefer the power spectrum given by Eq. (2). The Weiner-Khintchine theorem[17] then gives the

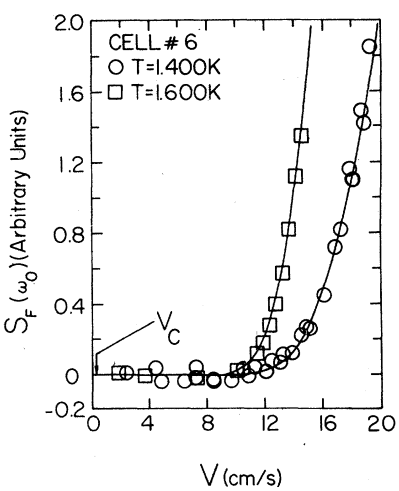

FIG. 3. Measured fluctuation power spectrum $S_F(\omega_0)$ vs the counterflow velocity V. The solid lines are fits of Eqs. (2) and (5) to the data.

corresponding time correlation function as Lorentzian. While exponential power spectra are not common, they have been observed. One recent example is the concentration fluctuations in a phase separating critical binary mixture.[18]

It is very tempting to associate the fluctuating force with random fluctuations in the vortex line density. There are two problems, however. First, the observed[8] power spectrum of the line density fluctuations is not exponential, but varies approximately as $1/\omega^3$. This form does not have a Fourier transform (time correlation function) and thus the fluctuations cannot be characterized by a single correlation time.[17] Second, the detailed coupling of line density fluctuations to the probe are unknown. (We note, however, that turbulent fluctuations in classical fluids have been studied using similar techniques.[19]) These uncertainties make it impossible unambiguously to relate the two fluctuating quantities. However, as we shall demonstrate below, the correlation time for line density fluctuations derived from the Vinen equation[1] is in excellent agreement with the correlation time determined by our data.

The Vinen equation for the time rate of change of the line density is

$$\frac{dL}{dt}=\frac{1}{2}\chi_1 B\frac{\rho_n}{\rho}L^{3/2}V\left(1-\frac{\alpha}{L^{1/2}d}\right)-\chi_2\frac{\kappa}{2\pi}L^2 \quad . \quad (3)$$

B is a dimensionless parameter of order unity determined from experiments on rotating helium, ρ_n/ρ is the normal fluid fraction, d is a characteristic dimension of the flow channel, and κ is the quantum of circulation. The dimensionless constants α, χ_1, and χ_2, all of order unity, have been determined by fitting temperature gradient[2,3] or second sound attentuation[1,6] data to the results given by the model for the steady state. The relationship between the steady-state line density L_0 and V, obtained by setting Eq. (3) equal to zero, is

$$V=bL_0^{1/2}/a\,(1-\alpha/dL_0^{1/2})\approx bL_0^{1/2}/a \quad , \quad (4)$$

where $a\equiv\frac{1}{2}\chi_1 B\rho_n/\rho$, $b\equiv\chi_2\kappa/2\pi$, and the approximation is valid at large line densities.

We now investigate the relaxation time of the steady-state L_0 following a perturbation $\Delta L << L_0$. Writing $L=L_0+\Delta L$ in Eq. (3), linearizing about L_0, and requiring $dL_0/dt=0$, we obtain the correlation time for line density fluctuations

$$1/\tau=-(\tfrac{3}{2}aL_0^{1/2}V-2bL_0-a\alpha V/d)\approx a^2V^2/2b \quad , \quad (5)$$

where the approximation holds at high line densities. In large channels this approximation is valid even at low velocities.

The solid lines in Fig. 3 are fits of Eq. (2) to our data, with τ given by Eq. (5). Clearly the functional dependence of the observed correlation time on the velocity V is exactly that given for vortex line density fluctuations by the Vinen equation. The quantities S_0 and $2b/a^2$ are determined by the fit and are collected for all sets of data in Table II. No particular significance should be ascribed to the quantity S_0 since it involves an unknown overall calibration of our apparatus as well as the (possibly temperature- and geometry-dependent) amplitude of the quantity relating the fundamental fluctuations in the flow to our observed random force. The quantity $2b/a^2$, which is determined absolutely from this fit, is given by the definitions of a and b as

$$2b/a^2=4\kappa\chi_2/\pi B^2\chi_1^2(\rho_n/\rho)^2 \quad . \quad (6)$$

Precise quantitative comparison of this result with our data is impossible since reliable values of χ_1 and χ_2 do not exist, though they are known to be of order unity. The *ratio* χ_1/χ_2 is determined in many counterflow experiments, but published values show variations with geometry of about a factor of 3. Given these uncertainties our values for $2b/a^2$ agree with Eq. (6). To emphasize further the agreement of our results with the functional form of Eq. (5) over the wide range of parameters studied, we have reduced our data in a manner suggested by Eq. (2). Using the values of S_0 and $2b/a^2$ given in Table II along with our data for the mean-squared displacement $S_F(\omega_0)$ produces the results shown in Fig. 4. The points clearly define a straight line of unit slope through the origin in agreement with the Vinen model calculation. We emphasize that Fig. 4 includes data from every temperature and every cell studied, as well as data taken on different days at the same temperature with the same cell.

TABLE II. Values of S_0 and $2b/a^2$ that fit Eqs. (2) and (5) to the fluctuation data.

Cell	T(K)	S_0	$2b/a^2$
	1.300	55	3.70
1	1.400	255	6.09
	1.500	177	4.28
	1.600	175	3.65
2	1.400	39735	16.7
	1.350	9.2	1.86
4	1.400	28.7	2.66
	1.500	44.5	2.38
	1.600	37.6	2.03
	1.400	27.1	4.03
6	1.500	26.8	2.89
	1.600	65.8	3.17

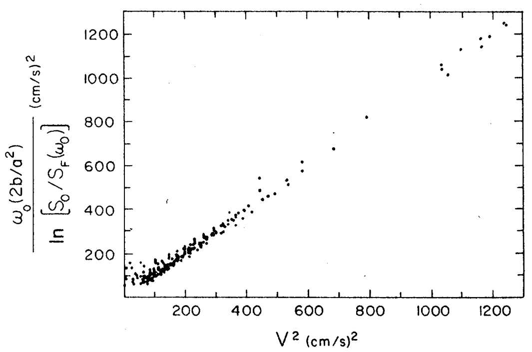

FIG. 4. Universal plot illustrating the success of Eqs. (2) and (5) in fitting the data. Shown are data for $S_F(\omega_0)$ obtained in every cell and at all temperatures.

It is difficult to believe that the excellent agreement between the correlation time for the fluctuating force and that calculated from the Vinen equation is fortuitous. However, to suggest that the line density fluctuations are the *source* of the fluctuating force requires that the force spectrum $S_F(\omega)$ results from a synthesis of the line spectrum $S_L(\omega)$ and some coupling function. Since the line spectrum given by the Vinen equation, and corresponding to Eq. (5), is Lorentzian it is unlikely *any* coupling function can result in an exponential force spectrum. The only experiment[8] which purports to measure $S_L(\omega)$ directly gives $S_L(\omega) \approx 1/\omega^3$. This result is not only in disagreement with the Vinen result, but is no more easily related to our exponential spectrum than is the Lorentzian. Much of this confusion is eliminated when it is recognized that the Vinen equation can only describe spatially homogeneous vortex line density fluctuations. It seems quite unlikely that fluctuations in L_0 would be homogeneous. Our conclusion then is that the fluctuating force observed in the turbulent counterflow is probably the result of inhomogeneous vortex line density fluctuations, and that the experimentally observed correlation time does not differ significantly from that for homogeneous fluctuations as given by the Vinen equation.

These experiments have provided considerable new information about turbulent thermal counterflow in HeII. While we cannot rule out the occurrence of normal fluid vortex shedding, our data show no evidence of the expected resonant response even though one might have thought all the required conditions were roughly satisfied. We found instead that the probe motion $x(t)$ was consistent with a random driving force having a power spectrum $S_F(\omega)$. The rapid increase of $\langle x^2\rangle$ with counterflow velocity V can be fit using an exponential power spectrum $S_F(\omega)=S_0e^{-\omega\tau}$. The correlation time τ determined by the fit is in good agreement with the correlation time computed from the Vinen equation. There is no reason to expect this agreement, yet it is unlikely to be fortuitous. A better theoretical understanding of the vortex line density fluctuations will clearly be needed before these results can be fully understood.

ACKNOWLEDGMENT

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