

Determination of the self-consistent band structure of CrCl_3 , CrBr_3 , NiCl_2 , and NiBr_2 by the intersecting-spheres model

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It has been proven previously that the intersecting-spheres model is capable of providing accurate band structures for Si and Ge. In order to compute the electronic structure of solids with a complicated unit cell, the intersecting-spheres model has been simply modified to neglect the core states in the secular equations without significant loss of accuracy. The model formulated in this way has been used to determine the self-consistent band structure of CrCl_3 , CrBr_3 , NiCl_2 , and NiBr_2 ; the electronic structures of all these materials turned out to be essentially similar. For NiCl_2 and NiBr_2 , our results were compared with existing photoemission data, and satisfactory agreement between calculated and observed structures was found. For all the crystals investigated, a comparison was also made with ultraviolet optical and thermorefectance data; through this comparison it seems possible to attempt a unique interpretation of the observed spectra in terms of transitions occurring between the occupied halogen p bands and the unoccupied metal s and p bands.

I. INTRODUCTION

In a recent paper it has been shown that the computation of the electronic structure of Si and Ge performed by the intersecting-spheres model¹ gives results in good agreement with orthogonalized-plane-wave calculations performed under the same assumptions for the self-consistent potential.

Further test calculations² of the electronic structure of quite different crystals like $(\text{SN})_x$, Al, and TiO indicate that the model is of rather general applicability. In order to afford the determination of the electronic structure of solids with a complicated unit cell and rather heavy constituent atoms it may be useful to neglect the core states in the calculations. This modification can be easily introduced; in fact the intersecting-spheres model adopts trial functions for the core states and for the valence states which are very nearly mutually orthogonal. This near orthogonality is ensured by the peculiar form of the warped atomiclike orbitals¹: the core states of a given atom are automatically orthogonal to the unwarped valence orbitals of the same atom since all these functions are generated by the solution of an eigenvalue problem with the same boundary conditions; the subsequent warping, while not destroying the orthogonality in a relevant way, does not allow any significant overlap of the warped valence orbitals centered on neighboring atoms with the above-mentioned core states. Due to this near orthogonality not only the overlap matrix elements but also the Hamiltonian matrix elements between core and valence warped orbitals should be negligible. In fact the unwarped functions are solutions of the self-consistent atomic problem and therefore the above-mentioned Hamiltonian matrix elements calculated

with the atomic potential are nearly zero. Since the correction to the potential in the core regions due to the crystalline environment predominantly consists of a constant term we expect that the above-mentioned matrix elements calculated with the actual crystal potential should be negligible.

If this expectation is realized in practice we can simply drop the core states from the secular equations of the intersecting-spheres model without further adjustment. For what concerns the self-consistent potential it is then possible to introduce the frozen-core approximation. We have verified² that the scheme outlined above works well in practice since the neglect of the core states in the secular equations calculated with the atomic potential alters the valence eigenvalues at most 10^{-4} eV, while self-consistent band energies obtained with an all-electron calculation and with the above-mentioned approximations typically differ by 0.02 eV in the valence region. With the modification outlined above the intersecting-spheres model is well suited to the computation of the band structure of solids with a complicated unit cell.

In this article we report on the self-consistent calculation of the electronic structure of CrCl_3 , CrBr_3 , NiCl_2 , and NiBr_2 and we compare the results with existing photoemission and ultraviolet optical data.

II. CrCl_3 AND CrBr_3 BAND STRUCTURE

The low-temperature structure of the layer crystals CrCl_3 and CrBr_3 is reported in Ref. 3. The three-dimensional unit cell of these crystals is rhombohedral and contains two chromium and six halogen atoms.

The length a of the primitive translation vector

for CrCl_3 and CrBr_3 was assumed to be 12.668 and 13.346 a.u., respectively, as inferred from Ref. 4. The primitive vectors and the coordinates of chromium and halogen atoms in the unit cell are reported in Table I(a). It can be recognized that the six halogen atoms nearest neighbors to each chromium atom are arranged on a regular octahedron.

The isogonal point group of the space group of these crystals is C_{3i} and the classification of the levels at the various points and lines of symmetry of the Brillouin zone, shown in Fig. 1, has been performed accordingly (see also Ref. 5). The coordinates of the symmetry points contained in the irreducible segment of the Brillouin zone are reported in Table II(a).

As required by the intersecting-spheres model we surrounded all the atoms of the crystal with spheres, we inserted cylindrical regions between neighboring spheres and we derived a cellular partition into atomic cells and spline regions in the manner indicated in Ref. 1. Assuming a local exchange correlation potential of the Kohn-Sham type we then solved the self-consistent Schrödinger equation within each atomic sphere, thus generating atomiclike orbitals which were warped according

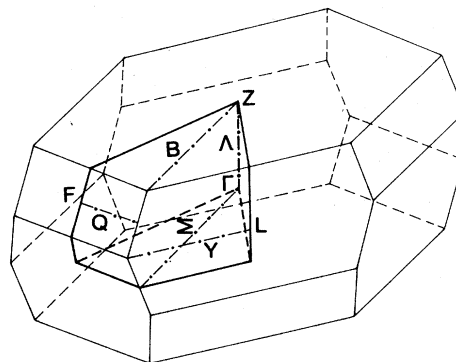


FIG. 1. Sketch of the Brillouin zone occurring for CrCl_3 , CrBr_3 and also for NiCl_2 , NiBr_2 . Γ , Z , L , F are the symmetry points reported in Table II, while Σ , Λ , Y , Q , and B are symmetry lines (dash dotted). The irreducible segment is also drawn.

to the prescriptions described in Ref. 1. We have then built Bloch sums using the $4s$, $5s$, $6s$, $4p$, $5p$, $3d$, $4d$ warped atomic orbitals for Cr, the $3s$, $4s$, $5s$, $3p$, $4p$, $3d$ orbitals for Cl and the $4s$, $5s$, $6s$, $4p$, $5p$, $4d$ orbitals for Br, respectively.

With this choice for the valence functions and with the spatial partition adopted by us the requirement of orthogonality of the valence Bloch sums to the core states is very nearly fulfilled; the above-mentioned choice also allows satisfactory flexibility of the overall trial function. In fact, with such a choice the energy levels associated with the valence and the lower conduction bands of

TABLE I. (a) Unit vectors and position vectors of the atoms in the unit cell of CrCl_3 and CrBr_3 (a units). h stands for halogen. (b) Unit vectors and position vectors of the atoms in the unit cell of NiCl_2 and NiBr_2 (a units). h stands for halogen.

(a)			
	x	y	z
T_1	0	-0.5118	0.8591
T_2	0.4432	0.2559	0.8591
T_3	-0.4432	0.2559	0.8591
$\text{Cr}^{(1)}$	0	0	0.4295
$\text{Cr}^{(2)}$	0	0	-0.4295
h_1	0.1477	-0.2559	0.2206
h_2	0.1477	0.2559	0.2206
h_3	-0.2955	0	0.2206
h_4	-0.1477	-0.2559	-0.2206
h_5	0.2955	0	-0.2206
h_6	-0.1477	0.2559	-0.2206
(b)			
	x	y	z
T_1	0	$-\frac{1}{3}$	$2\sqrt{2}/3$
T_2	$\sqrt{3}/6$	$\frac{1}{6}$	$2\sqrt{2}/3$
T_3	$-\sqrt{3}/6$	$\frac{1}{6}$	$2\sqrt{2}/3$
Ni	0	0	0
h_1	0	0	$\sqrt{2}/2$
h_2	0	0	$-\sqrt{2}/2$

TABLE II. (a) Coordinates of the symmetry points in the irreducible segment of the Brillouin zone for CrCl_3 and CrBr_3 . The coordinates are given in $2\pi/a$ units. (b) Coordinates and weights of the symmetry points in the irreducible segment of the Brillouin zone for NiCl_2 and NiBr_2 . The coordinates are given in $2\pi/a$ units.

(a)				
Point	K_x	K_y	K_z	
Γ	0	0	0	
Z	0	0	0.5820	
L	0.5640	0.3256	0.1940	
F	0.5640	-0.3256	0.3880	
(b)				
Point	K_x	K_y	K_z	Weight
Γ	0	0	0	1
Z	0	0	$3\sqrt{2}/8$	1
L	$\sqrt{3}/2$	$\frac{1}{2}$	$\sqrt{2}/8$	3
F	$\sqrt{3}/2$	$-\frac{1}{2}$	$2\sqrt{2}/8$	3

The dispersion of the individual bands is not larger than 1.6 eV along each symmetry line. The investigation of the eigenvectors shows that these bands have predominantly Cl $3p$ or Br $4p$ character. We find then two groups of six and four bands with very small dispersion (at most 0.3 eV). These levels turn out to be essentially associated with Cr $3d$ atomic orbitals. Since the crystal potential around each Cr site has approximate octahedral symmetry it is reasonable that the bands are clustered in two groups of six and four levels, approximately corresponding to the T_{2g} and E_g representations of the O_h point group (we have two Cr atoms per unit cell). We note that the Fermi level lies within the lower group of bands and that in our spin restricted treatment only the equivalent of three full bands of the lower group is occupied. Above the d bands we find a conduction band of Cr $4s$ character with a minimum at Γ (Γ_1^+). Next in energy we find at Γ a Γ_1^- level associated with Cr $4p$ orbitals and another Γ_1^- level predominantly constituted by Cr $4s$ orbitals. From these levels origin two bands of mixed $s-p$ character, while the remaining four bands shown in Figures 2 and 3 have predominantly Cr $4p$ character.

III. NiCl₂ AND NiBr₂ BAND STRUCTURE

The crystal structure of NiCl₂ and of NiBr₂ is reported in Ref. 6. The Bravais lattice can be assumed to be trigonal primitive and the rhombohedral unit cell contains now one Ni and two halogen atoms. The length a of the primitive translation vectors for NiCl₂ and NiBr₂ was assumed to be 11.584 and 12.217 a.u., respectively.⁶ The primitive vectors and the coordinates of the Ni and of the halogen atoms in the unit cell are reported in Table I(b). Also in this case it can be seen that the halogen atoms are arranged on regular octahedra surrounding each Ni atom. The isogonal point group of the space group for these crystals is now D_{3d} . The three-dimensional Brillouin zone is again shown in Fig. 1. The coordinates of the symmetry points and their weight for integration over the Brillouin zone are reported in Table II(b).

The spatial partition needed by the intersecting-spheres model was performed as indicated previously for CrCl₃ and CrBr₃. We assumed again a local exchange correlation potential of the Kohn-Sham type and we generated the basis set as previously shown. Bloch sums were built in this case using for the metal and for the halogens the same choice of atomiclike orbitals as that adopted for CrCl₃ and CrBr₃. The radii of the spheres and the thickness of the cylinders turned out to be $R_{Ni} = R_{Cl} = 5.6$ a.u., $h_{Ni-Cl} = h_{Cl-Cl} = 0.8$ a.u. for NiCl₂ and $R_{Ni} = R_{Br} = 5.9$ a.u., $h_{Ni-Br} = h_{Br-Br} = 0.8$ a.u. for NiBr₂,

respectively. The expansion of the self-consistent potential in spherical harmonics was truncated at $l=4$ with very satisfactory convergence. Contributions to the potential in an atomic cell due to the other cells were treated through spherical harmonics reexpansion up to four shells of neighbors and the remaining terms were treated through Madelung sums. Since for NiCl₂ and NiBr₂ the self-consistent calculations require a rather small numerical effort we have been able to compute the self-consistent charge density by substituting the integral over the Brillouin zone with a weighted sum over the four points indicated in Table II(b).

We have verified by performing the same calculations with a single point (Γ) in the sum that the convergence is indeed satisfactory, since we did not find changes in the relative position of the valence levels greater than 0.2 eV when the two above mentioned calculations were compared. This result supports also the validity of the single point sampling adopted for CrCl₃ and CrBr₃: the dispersion of the valence bands in that case is in fact smaller than in the case of NiCl₂ and NiBr₂.

In Figs. 4 and 5 the self-consistent band structures of NiCl₂ and NiBr₂ are reported (the low-ly-

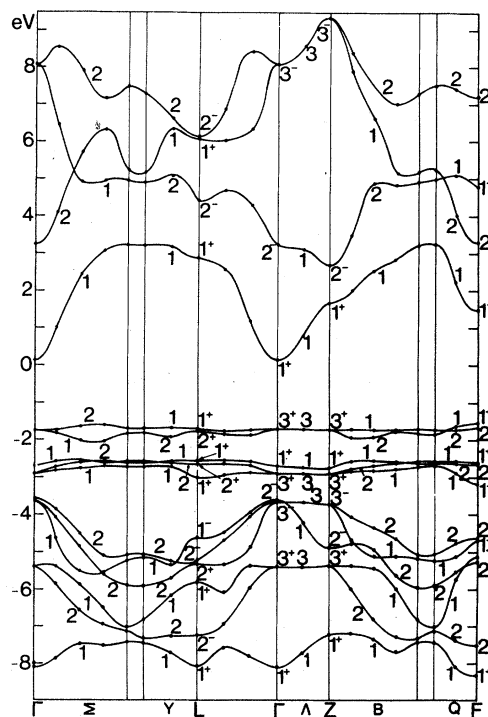


FIG. 4. Self-consistent band structure of NiCl₂ along a contour connecting and including the symmetry lines shown in Fig. 1.

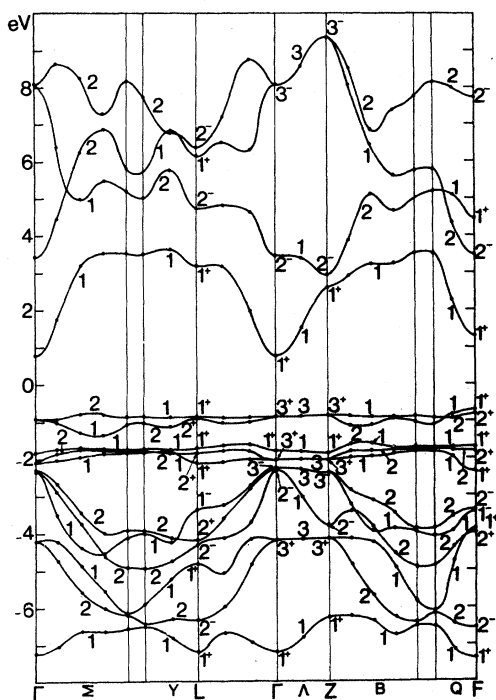


FIG. 5. Self-consistent band structure of NiBr_2 along a contour connecting and including the symmetry lines shown in Fig. 1.

ing bands associated with Cl 3s or Br 4s atomic levels are not shown in Figures 4 and 5; they fall between -18.0 and -16.8 eV for NiCl_2 , and lie between -16.6 and -15.4 eV for NiBr_2 , respectively).

In Figs. 4 and 5 we note six valence bands of essentially halogen 3p or 4p character covering an energy interval of 4.5 and 5.0 eV for NiCl_2 and NiBr_2 , respectively. These bands correspond to the eighteen p valence bands found in the case of CrCl_3 and CrBr_3 . We find then five Ni d bands divided in two groups of three and two as must be expected from the near octahedral symmetry of the Ni site. These bands display very small dispersion; the Fermi level falls now within the upper doublet, which has a population equivalent to one fully occupied band. Above these d levels we find an isolated conduction band predominantly of Ni 4s character with a dispersion of about 3 eV and three more bands of Ni 4p character. Concluding this description of the results we note that, despite the differences in the crystal structure, the electronic structures of NiCl_2 and NiBr_2 are qualitatively very similar to those of CrCl_3 and CrBr_3 .

IV. COMPARISON WITH PHOTOEMISSION DATA

If the valence-band structure of NiCl_2 and NiBr_2 calculated by us is directly utilized to predict the photoemission spectra we should expect in the

high-energy side a doublet structure associated with the Ni d bands, with a separation of about 0.8 eV for both crystals; measuring the binding energies from the top levels of the doublet we should expect structures associated with the halogen p bands ranging from -1.8 to -6.6 eV for NiCl_2 and lying between -1.3 and -6.4 eV for NiBr_2 . We should then find a rather sharp structure associated with the narrow halogen s bands occurring at -15.7 eV for NiCl_2 and at -15.0 eV for NiBr_2 . Experimental photoemission data for NiCl_2 and for NiBr_2 are reported in the literature.^{7,8} Since we have performed only a spin restricted calculation for the ground state within a one-electron formalism we cannot expect to find precise agreement with experimental data in particular for the d levels. Many-body effects influencing the photoemission spectra are neglected in the band description; we therefore caution the reader not to take possible agreement between the results of these calculation and the experimental data as really conclusive. It is however surprising to note that when we compare our results for NiCl_2 with the well resolved spectrum of Ref. 7, shown in Fig. 6, our predictions are fulfilled with rather good accuracy. In particular the separation between the high-

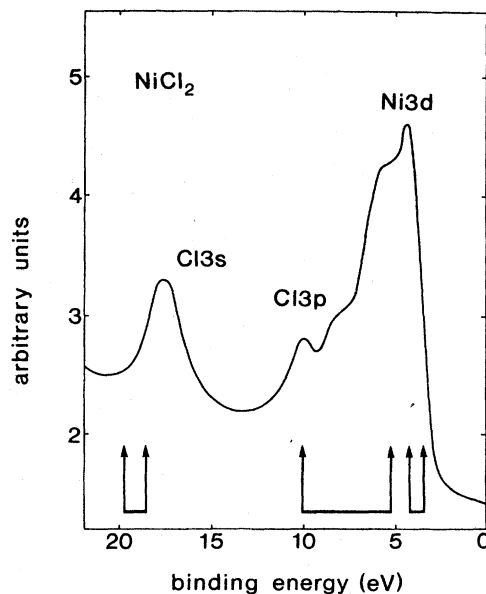


FIG. 6. X-ray photoemission valence-band spectrum of NiCl_2 according to Ref. 7. The peak assignments of Ref. 7 are also reported. The arrows mark, from right to left, the upper and lower limits of the Ni 3d, Cl 3p, and Cl 3s bands, respectively, as resulting from the present calculations. The band structure has been shifted rigidly to put the highest occupied d band in coincidence with the onset of the photoemission spectrum.

energy onset of the photoemission spectrum and the isolated peak marking the position of the s levels is 14 eV according to Ref. 7 and 15–16 eV in Ref. 8. Furthermore, the separation of the middle peak seen in Ref. 7 and presumably marking the bottom of the p valence band from the above mentioned onset is about 7 eV.

In Ref. 8 broad structures are reported for both NiCl_2 and NiBr_2 (apart from the s peaks) and are entirely attributed to d levels spread in an energy interval much larger than predicted by us. It is claimed that such an interpretation is necessary to explain the observed similarity of the photoemission spectra of NiCl_2 and NiBr_2 ; it is argued that, if the spectra were due also to p contributions, they should appear quite different, since the p bands of Cl should be different from those of Br. Our calculations however indicate that NiCl_2 and NiBr_2 have a very similar structure of the valence p bands; therefore the arguments of Ref. 8 seem not to be valid. We believe that the interpretation of the photoemission spectrum of NiCl_2 reported in Ref. 7 is correct and that it should apply also to NiBr_2 . To our knowledge the photoemission spectra of CrCl_3 and CrBr_3 are not present in the literature; according to our calculated band structures they should look very similar to the spectra of the Ni halides (except for the d region).

V. COMPARISON WITH OPTICAL DATA

For what concerns optical measurements much information is present in the literature for all the crystals investigated by us.^{9–12}

The low-energy part of the spectra is dominated by weak $d \rightarrow d$ transitions⁹ and its interpretation is evidently beyond the possibilities of a calculation with a one-electron formalism. For the same reasons we cannot safely predict charge transfer and orbital promotion transitions beyond a generic information concerning their energy range. We can however hope to predict with some more affidability where the interband gaps between the valence p bands and the conduction s and p bands occur in the spectra. In view of the low dielectric constant of these materials we should expect that the exciton interaction may give rise to marked structures associated with interband gaps. We therefore attempt to correlate the sharp structures observed by reflectance and thermorelectance measurements on single crystals^{11,12} with the calculated interband gaps. Since the above mentioned measurements were performed with light incident normally to the layer planes only transitions allowed for this configuration need to be considered at present.

In the case of CrCl_3 and CrBr_3 the first allowed

interband transition is predicted to occur at Γ between the top of the p valence band (Γ_2^-, Γ_3^-) and the bottom of the s conduction band (Γ_1^+) at 4.9 and 4.2 eV, respectively. It seems reasonable to associate this gap with the strong peak observed in thermorelectance at 4.3 eV in CrCl_3 and with the strong doublet (a spin-orbit doublet?) centered at about 3.8 eV in CrBr_3 . The strength of these structures, clearly shown in Fig. 7, where the imaginary part of the experimental dielectric function ϵ_2 is reported, seems to indicate that a true interband transition rather than a charge transfer transition is involved. Orbital promotion to the s level is discarded by symmetry arguments. According to the calculated band structures and apart from possible saddle points we do not expect other strong interband transitions to occur up to 7.5 eV in CrCl_3 and up to 6.5 eV in CrBr_3 ; at these energies we find allowed gaps between the highest Γ_2^+ and Γ_3^+ valence levels and the lowest Γ_1^- conduction level. From these energies on we find many contributions deriving from the p conduction bands which should lead to an appreciable increase of the imaginary part of the dielectric function ϵ_2 and to additional excitonic structure. From the experimental point of view such an increase occurs in CrCl_3 starting from 8 eV and in CrBr_3 starting from 7.3 eV. In CrBr_3 we find also a sharp thermorelectance structure at 7.3 eV. We believe that the correlation found between the experimental structures and the interband gaps is something more than a pure coincidence: corresponding experimental structures are found

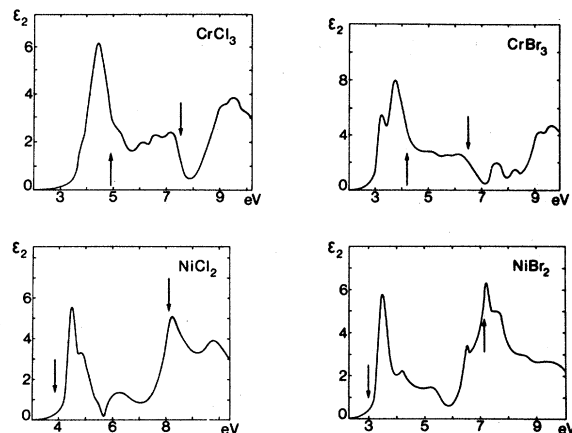


FIG. 7. Imaginary part of the dielectric function ϵ_2 at $T = 80^\circ\text{K}$ for CrCl_3 , CrBr_3 , NiCl_2 , and NiBr_2 according to Ref. 12. The arrows mark the calculated interband gaps corresponding to the onset of allowed transitions between the halogen p bands and the metal s and p bands.

in CrBr_3 at lower energy than in CrCl_3 by about 0.7 eV and the calculated gaps closely follow this trend.

In the case of NiCl_2 and of NiBr_2 we predict that the first interband gap occurs between the top valence level Γ_3^- and the Γ_1^+ conduction level. This gap has a width of 3.8 eV in NiCl_2 and of 3.0 eV in NiBr_2 . The first strong reflectivity peak occurs at 4.4 eV in NiCl_2 and at 3.5 eV in NiBr_2 and can reasonably be associated to the first interband gap (see also the ϵ_2 spectra of Fig. 7 for comparison). According to the calculated band structure and apart from possible saddle points the next allowed interband transition occurs at Z between the Z_3^+ valence level and the $Z_2^- p$ conduction level. This gap has a width of 8.1 eV in NiCl_2 and of 7.1 eV in NiBr_2 . From these energies on we expect that transitions to the p levels should maintain the imaginary part of the dielectric function ϵ_2 to a high level. Experimentally we note that ϵ_2 displays a sharp rise at about 8 eV in NiCl_2 and at about 6.5 eV in NiBr_2 . This rise is accompanied by sharp structures seen in thermoreflectance at 8.5 eV in NiCl_2 and at 7.2 eV in NiBr_2 (the additional sharp structure at 6.6 eV which tends to disappear at low temperature is presently a puzzle). We note that also for the Ni halides corresponding structures shift to lower energy about 1 eV when Cl is re-

placed by Br and that such a behavior is replicated by the theoretical gaps.

VI. CONCLUSIONS

For NiCl_2 (and NiBr_2) the calculations reported in this article support one of the two interpretations of the photoemission spectra reported in the literature. For all the crystals investigated they suggest a unique interpretation for the main structures of the ultraviolet optical spectra. According to this interpretation the peaks occurring in the low-energy side of the ultraviolet spectra should originate from gaps between the halogen p levels and the metal s levels, while the structures in the high-energy side of the spectra should derive from transitions between the halogen p levels and the metal p levels.

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