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Free-carrier magnetoabsorption by photon-ionized impurity-magnetoplasmon processes in polar semiconductors*

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The contribution of collective plasma excitations to the free-carrier absorption is known to be important in polar semiconductors with high static (lattice) dielectric constant and high carrier concentration, in the presence of ionized impurities (defects). In the present paper, the effect of an external magnetic field on the free-carrier absorption by plasmon generation is studied. The theoretical results are compared with the experimental data on plasmon generation in n-PbSe in the infrared.

I. INTRODUCTION

In this paper we consider the free-electron plasma in a cubic semiconductor of scalar, energy-independent effective mass. We allow the existence of several equivalent conduction-band minima. The results will apply also to a free-hole plasma.

Suppose the crystal lattice is perfect and undeformed (as a consequence, all charge-charge interactions involve only the high-frequency lattice dielectric constant) and charges of ionized donors and acceptors are smeared out to give a uniform charge density. If, moreover, the radiation electric field is assumed to be uniform, as will be done in the following, the free-carrier absorption vanishes in both the absence and the presence of an external uniform magnetic field (except at cyclotron frequency).[1-6] This follows from the fact that the uniform electric field influences only the center-of-mass position and velocity but not the relative positions or velocities of the interacting electrons. Therefore, it cannot excite the electron system.

The electron interactions with crystal imperfections (or deformations) make the free-carrier absorption possible. This absorption consists in individual-carrier excitations, as well as in collective-carrier excitations (plasmon or magnetoplasmon generation). In this paper we are concerned only with the latter contribution and we extend the results of the recent paper of Mycielski and Mycielski[7] (denoted hereafter by I) to the case when the magnetic field is present. Higher-order processes involving plasmons, e.g., photon–plasmon–free-carrier–impurity processes,[5,6,8,9] are not considered. We limit our considerations to the case of a degenerate plasma with plasma frequency much higher than LO-phonon frequency, since it was shown in paper I that the plasmon generation may be important only at high electron concentrations.

In Sec. II we derive a formula for the contribution to power absorption given by photon-magnetoplasmon processes following from the interaction of magnetoplasma with an arbitrary time-independent perturbation in a nonideal (and/or deformed) crystal. In Sec. III we apply this general formula to the case of photon–magnetoplasmon–ionized-impurity (defect) processes. The numerical results are discussed in Sec. IV. The free-carrier magnetoabsorption by magnetoplasmon generation is important in polar semiconductors with high static lattice dielectric constant and high carrier concentration (as in the case of absence of magnetic field).

The experimental evidence for the free-carrier absorption due to plasmon generation in[10] n-PbSe and[11] n-$Pb_{1-x}Sn_xSe$ was obtained in recent years using the magnetoreflection method. In Sec. IV we compare our results with experimental findings for n-PbSe. The agreement is much better than that achieved in paper I in which the effect of magnetic field on plasmon generation was neglected.

II. GENERAL FORMALISM FOR MAGNETOPLASMON GENERATION

Let us consider, as in paper I, an electron plasma in a semiconductor with N_e electrons per unit volume. The number of equivalent conduction-band minima among which these electrons are distributed is w, and the minima are assumed to be spherical and parabolic. The high- and low-frequency dielectric constants of the crystal are ϵ_∞ and ϵ_0, respectively.

In the present paper we are interested in the effect of an external uniform magnetic field $\vec{H}$. We assume that this field is rather low, so that

$$\hbar\omega_c \ll E_F , \quad (1)$$

where E_F is the Fermi energy and $\omega_c = eH/cm^*$ is the cyclotron frequency. $-e$ and m^* are the electron charge and effective mass, respectively. We assume the magnetic field to be low also in the sense that

$$\omega_c \leq \tfrac{1}{2}\omega_{p\infty}(0) . \tag{2}$$

$\omega_{p\infty}(q)$ is the frequency of a plasmon with wave-vector $\vec{q}$ in the medium of the dielectric constant ϵ_∞ and in the absence of magnetic field. Thus, $\omega_{p\infty}(0)$ is the plasma frequency

$$\omega_{p\infty}^2(0) = 4\pi e^2 N_e / \epsilon_\infty m^* . \tag{3}$$

We limit our considerations to the case

$$\omega, \omega_{p\infty}(0) \gg \omega_{\rm LO} , \tag{4}$$

where ω and $\omega_{\rm LO}$ are the radiation and LO-phonon frequencies, respectively. Under this assumption and in the absence of magnetic field plasmons are decoupled from LO phonons and the plasmon frequency in the considered semiconductor is approximately $\omega_{p\infty}(q)$. Since the frequency $\omega_{{\rm mp}\infty}(\vec{q})$ of the upper-branch magnetoplasmon (in the medium of the dielectric constant ϵ_∞) is higher than $\omega_{p\infty}(0)$,[12] the magnetoplasmons are also decoupled from LO phonons because of the assumption (4) and the magnetoplasmon frequency is $\omega_{{\rm mp}\infty}(\vec{q})$. If no optical phonons are present, the ionic (polar) part of the crystal polarization (i.e., the one connected with the difference $\epsilon_0 - \epsilon_\infty$) and the corresponding polarization charge density $\rho(\vec{r})$ are constant in time.

In the present paper we limit our considerations to the case of a degenerate electron plasma. We assume that $w \lesssim 4$ and that $\hbar\omega_{p\infty}(0)$ is not much higher than E_F. For such a plasma the following assumptions were made in paper I to calculate the generation of plasmons in the absence of magnetic field:

$$r_e \lesssim a_\infty^* , \tag{5}$$

$$r_e \lesssim (\epsilon_0/\epsilon_\infty)(\pi/12w)^{2/3} a_\infty^* , \tag{6}$$

$$\omega/\omega_{p\infty}(0) - 1 < \tfrac{1}{4} , \tag{7}$$

where

$$r_e = (3/4\pi N_e)^{1/3} \tag{8}$$

and $a_\infty^* = \hbar^2\epsilon_\infty/e^2 m^*$ is the effective Bohr radius for the electron. The assumptions (5) and (6) justified the use of the weak-coupling plasma theory and the Thomas-Fermi formula for static screening by an electron plasma, respectively. With assumption (7), the Landau damping of plasmons was neglected. Assumptions (5) and (7) justified also the use of the "jellium model" to describe plasmons and their interactions with high-frequency perturbations.

If the conditions (1) and (2) are fulfilled, the magnetic field does not change very seriously the plasma properties. Therefore, we do not change assumptions (5)–(7) in the present paper, and we use them for the same purposes as in paper I.

Considering the magnetoplasma dynamics it is convenient to introduce, as it was also done in paper I, two artificial, mutually compensating uniform charge densities $+eN_e$ and $-eN_e$. Our model consists now, first of all, of an "ideal magnetoplasma" in the medium of dielectric constant ϵ_∞ [assumption (4)], i.e., of electrons and of the uniform charge density $+eN_e$.

There are two perturbations of this ideal magnetoplasma. The first is connected with the electron potential energy in the presence of crystal imperfections, e.g., neutral or ionized impurities, defects, dislocations etc. (we are interested here in time-independent imperfections). The mean electric charge density of these crystal imperfections is $+eN_e$ (from the electric-neutrality requirement). We denote the electron potential energy in the presence of crystal imperfections and of the uniform charge density $-eN_e$ (in the medium of dielectric constant ϵ_∞) by $U(\vec{r})$.

The second perturbation of the ideal magnetoplasma is the field given by the polarization charge density $\rho(\vec{r})$. Of course, the average of this charge density vanishes. Because of assumption (4), $\rho(\vec{r})$ is constant in time. The electron potential energy due to the charge density $\rho(\vec{r})$ (in the medium of dielectric constant ϵ_∞) will be denoted by $U_P(\vec{r})$.

As we are interested only in the high-frequency conductivity due to generation of magnetoplasmons of wavelengths shorter than that of the radiation, we can assume that the radiation electric field is uniform, i.e., is of the form

$${\rm Re}(\vec{E}e^{-i\omega t}) , \tag{9}$$

where $\vec{E}$ is a complex vector. It is then convenient to introduce a noninertial reference system

$$\vec{r}' = \vec{r} - (e/m^*\omega^2)\,{\rm Re}(\vec{G}e^{-i\omega t}) , \tag{10}$$

where

$$\vec{G} = \vec{E} + \frac{\omega_c\,\omega}{\omega_c^2 - \omega^2}\left[i(\vec{E}\times\hat{H}) + \frac{\omega_c}{\omega}(\vec{E}\times\hat{H})\times\hat{H}\right] , \tag{11}$$

and $\hat{H} = \vec{H}/H$. In the presence of the electric field (9), the equations of motion of the electrons of the ideal magnetoplasma in the noninertial reference system (10) are exactly the same as the equations of motion in the rest reference system in the absence of radiation. In other words, in the noninertial reference system (10) the electron-radiation interaction is eliminated from the ideal-magnetoplasma Hamiltonian.

On the other hand, in the noninertial reference system the electron potential energy $U + U_P$ is no longer time independent. Assuming a weak radiation field and expanding up to the linear term in $\vec{E}$, we obtain

$$U(\vec{r}',t)+U_P(\vec{r}',t) = U(\vec{r}')+U_P(\vec{r}')+(e/m^*\omega^2)\times\mathrm{Re}(\vec{G}e^{-i\omega t})\cdot[\vec{\nabla}U(\vec{r}')+\vec{\nabla}U_P(\vec{r}')]. \quad (12)$$

The collective-excitation part of the ideal-magnetoplasma Hamiltonian (in the noninertial reference system) can be written in the form

$$H_0=\sum_{\vec{q}}{}' \hbar\,\omega_{\mathrm{mp}\infty}(\vec{q})(b^\dagger_{\vec{q}}b_{\vec{q}}+\tfrac{1}{2}), \quad (13)$$

where $b^\dagger_{\vec{q}}$ and $b_{\vec{q}}$ are the creation and annihilation operators, respectively, for the magnetoplasmon mode of the wave vector $\vec{q}$. The summation is over all magnetoplasmon modes of the upper branch ($\vec{q}=0$ is excluded). Contributions of other branches to H_0 were omitted as in this paper we are interested only in processes involving upper-branch (plasmonlike) modes. The $\vec{q}$ vectors of the magnetoplasmon modes are distributed in $\vec{k}$ space with the density $(2\pi)^{-3}V$, where V is the volume of the crystal (or rather of the periodicity box). Periodic boundary conditions are assumed also on U and U_P.

The part of the magnetoplasma Hamiltonian describing the interaction of the upper-branch magnetoplasmon modes with the crystal imperfections and polarization charges (in the noninertial reference system) is obtained as follows. Let us denote by $d(\vec{r}')$ the change in electron density at the point $\vec{r}'$, due to magnetoplasmon modes. Using the jellium model one obtains for small values of q,

$$d(\vec{r}')=\sum_{\vec{q}}{}'\left(\frac{\hbar N_e}{2Vm^*}\right)^{1/2}\omega_{p\infty}(0)\omega_{\mathrm{mp}\infty}^{-3/2}(0,\hat{q})\times[\omega^2_{\mathrm{mp}\infty}(0,\hat{q})-\omega_c^2]^{-1/2}\times[\omega^2_{\mathrm{mp}\infty}(0,\hat{q})-\omega_c^2(\hat{q}\cdot\hat{H})^2]\times[2\omega^2_{\mathrm{mp}\infty}(0,\hat{q})-\omega^2_{p\infty}(0)-\omega_c^2]^{-1/2}\times q[-i\exp(i\vec{q}\cdot\vec{r}')b_{\vec{q}}+\mathrm{H.c.}], \quad (14)$$

where $\hat{q}=\vec{q}/q$, and $\omega_{\mathrm{mp}\infty}(0,\hat{q})$ is the magnetoplasmon frequency for a given direction of $\vec{q}$ and $q\to 0$:

$$\omega^2_{\mathrm{mp}\infty}(0,\hat{q})=\tfrac{1}{2}[\omega^2_{p\infty}(0)+\omega_c^2]+\tfrac{1}{2}\{[\omega^2_{p\infty}(0)+\omega_c^2]^2-4\,\omega^2_{p\infty}(0)\omega_c^2(\hat{q}\cdot\hat{H})^2\}^{1/2}. \quad (15)$$

As we are interested only in calculating radiation-induced transitions (in the lowest order), we take into account only the last term of expression (12). On the basis of the jellium model we multiply it by the expression (14) and integrate over V. The resulting perturbation H' can be written in the form

$$H'=-\left(\frac{\hbar e^2N_e}{2Vm^{*3}\omega^4}\right)^{1/2}\omega_{p\infty}(0)\,\mathrm{Re}(\vec{G}e^{-i\omega t})\cdot\sum_{\vec{q}}{}' q\vec{q}\,\omega^{-3/2}_{\mathrm{mp}\infty}(0,\hat{q})[\omega^2_{\mathrm{mp}\infty}(0,\hat{q})-\omega_c^2]^{-1/2}\times[\omega^2_{\mathrm{mp}\infty}(0,\hat{q})-\omega_c^2(\hat{q}\cdot\hat{H})^2][2\omega^2_{\mathrm{mp}\infty}(0,\hat{q})-\omega^2_{p\infty}(0)-\omega_c^2]^{-1/2}\times\left[\left(\int_V e^{i\vec{q}\cdot\vec{r}'}[U(\vec{r}')+U_P(\vec{r}')]d^3r'\right)b_{\vec{q}}+\mathrm{H.c.}\right]. \quad (16)$$

We obtain the net power absorbed by the mode $\vec{q}$ which is initially in the state n by multiplying $\hbar\,\omega_{\mathrm{mp}\infty}(\vec{q})$ by the difference of the rates for the $n\to n+1$ and $n\to n-1$ transitions (only these transitions are allowed by the perturbation H'). The result is independent of n. Summing it over all modes $\vec{q}$ we obtain the total power absorption p in the volume V of the crystal, due to magnetoplasmon processes:

$$p=\frac{\pi e^2N_e}{4Vm^{*3}\omega^3}\,\omega^2_{p\infty}(0)\sum_{\vec{q}}{}'\delta(\omega-\omega_{\mathrm{mp}\infty}(\vec{q}))q^2|\vec{G}\cdot\vec{q}|^2\omega^{-3}_{\mathrm{mp}\infty}(0,\hat{q})[\omega^2_{\mathrm{mp}\infty}(0,\hat{q})-\omega_c^2]^{-1}[\omega^2_{\mathrm{mp}\infty}(0,\hat{q})-\omega_c^2(\hat{q}\cdot\hat{H})^2]^2\times[2\omega^2_{\mathrm{mp}\infty}(0,\hat{q})-\omega^2_{p\infty}(0)-\omega_c^2]^{-1}\left|\int_V e^{i\vec{q}\cdot\vec{r}}[U(\vec{r})+U_P(\vec{r})]d^3r\right|^2. \quad (17)$$

We have replaced $\vec{r}'$ by $\vec{r}$ in the integral over V.

It should be noted that (for $\vec{q}\neq 0$),

$$\int_V e^{i\vec{q}\cdot\vec{r}}[U(\vec{r})+U_P(\vec{r})]d^3r=-q^{-2}\int_V e^{i\vec{q}\cdot\vec{r}}[\Delta U(\vec{r})+\Delta U_P(\vec{r})]d^3r. \quad (18)$$

By definition,

$$\Delta U_P(\vec{r})=(4\pi e/\epsilon_\infty)\rho(\vec{r}). \quad (19)$$

Suppose q is small enough to fulfill the conditions

$$q < \omega_{p\infty}(0)/v_F \,, \tag{20}$$

$$q < \omega_c/v_F \,, \tag{21}$$

where v_F is the electron velocity at the Fermi level of the degenerate plasma. For such q and neglecting quantum effects [assumption (1)], the dispersion relation for the upper-branch magnetoplasmon is[12]

$$\begin{aligned}\omega^2_{\mathrm{mp}\infty}(\vec{q}) = {} & \omega^2_{\mathrm{mp}\infty}(0,\hat{q}) + \tfrac{3}{10}\,\omega^4_{p\infty}(0)v_F^2 q^2 \\ & \times\{(\hat{q}\cdot\hat{H})^4\omega^{-4}_{\mathrm{mp}\infty}(0,\hat{q}) + [1-(\hat{q}\cdot\hat{H})^2]^2[\omega^2_{\mathrm{mp}\infty}(0,\hat{q}) - 4\omega_c^2]^{-1}[\omega^2_{\mathrm{mp}\infty}(0,\hat{q}) - \omega_c^2]^{-1} + \tfrac{1}{3}(\hat{q}\cdot\hat{H})^2[1-(\hat{q}\cdot\hat{H})^2] \\ & \times[6\omega^4_{\mathrm{mp}\infty}(0,\hat{q}) - 3\omega_c^2\omega^2_{\mathrm{mp}\infty}(0,\hat{q}) + \omega_c^4]\omega^{-2}_{\mathrm{mp}\infty}(0,\hat{q})[\omega^2_{\mathrm{mp}\infty}(0,\hat{q}) - \omega_c^2]^{-3}\} \\ & \times(1 + [\omega^2_{p\infty}(0) + \omega_c^2 - 2\omega_c^2(\hat{q}\cdot\hat{H})^2]\,\{[\omega^2_{p\infty}(0)+\omega_c^2]^2 - 4\omega_c^2(\hat{q}\cdot\hat{H})^2\omega^2_{p\infty}(0)\}^{-1/2})\,.\end{aligned} \tag{22}$$

This formula is valid for an arbitrary w.

The generated magnetoplasmons have the frequency $\omega_{\mathrm{mp}\infty}(\vec{q}) = \omega$. Using Eqs. (15) and (22) and the assumptions (2) and (7), one can show that the condition (20) is fulfilled for these magnetoplasmons.

The condition (21) is not fulfilled—for a given q—if the magnetic field is too weak. On the other hand, however, for vanishing magnetic fields (i.e., for $\omega_c \to 0$) Eq. (22) turns into the proper dispersion relation for plasmons:

$$\omega^2_{p\infty}(q) = \omega^2_{p\infty}(0) + \tfrac{3}{5}\,v_F^2 q^2 \,. \tag{23}$$

Therefore, we will use Eq. (22) as an approximate dispersion relation for magnetoplasmons also in the range of weak magnetic fields.

III. IONIZED IMPURITIES (DEFECTS) IN POLAR SEMICONDUCTORS

In this paper, as in paper I, we are interested only in the case of $U(\vec{r})$ given by ionized impurities or ionized point defects and by the uniform charge density $-eN_e$. Suppose there are S types of such ions in a semiconductor. $Z_l e$ and $N_l (l=1,\ldots,S)$ are the charge and concentration of the l-type ions, respectively (Z_l may be a positive or negative integer). The neutrality requirement yields the condition

$$\sum_{l=1}^{S} Z_l N_l = N_e \,. \tag{24}$$

Therefore, we have

$$\Delta U(\vec{r}) = \frac{4\pi e^2}{\epsilon_\infty}\left[-N_e + \sum_{l=1}^{S} Z_l \sum_{\vec{R}_l} \delta(\vec{r} - \vec{R}_l)\right] , \tag{25}$$

where the sum over $\vec{R}_l$ denotes summation over the positions of the l-type ions (the VN_l ions in volume V).

The polarization charge density $\rho(\vec{r})$ is

$$\rho(\vec{r}) = [(\epsilon_0 - \epsilon_\infty)/4\pi]\Delta\varphi_{S0}(\vec{r}) \,. \tag{26}$$

$\varphi_{S0}(\vec{r})$ denotes the macroscopic electric potential in the absence of radiation field and magnetoplasma oscillations, i.e., the potential produced by ionized impurities (defects) in the medium of dielectric constant ϵ_0 and in the presence of free carriers in the magnetic field. Thus $\varphi_{S0}(\vec{r})$ is the potential $(\epsilon_\infty/\epsilon_0)U(\vec{r})/-e$ screened by the free carriers in the magnetic field in the medium of dielectric constant ϵ_0. Because of the assumption (1) we will neglect the effect of the magnetic field on free-carrier screening. Thus, for $\vec{q} \neq 0$,

$$\int_V e^{i\vec{q}\cdot\vec{r}}\varphi_{S0}(\vec{r})\,d^3r = -(\epsilon_\infty/e\epsilon_0)\left[1 + \left(\frac{q_{\mathrm{TF}0}}{q}\right)^2\right]^{-1} \times \int_V e^{i\vec{q}\cdot\vec{r}}U(\vec{r})\,d^3r \,, \tag{27}$$

where $q_{\mathrm{TF}0}$ is the inverse of the Thomas-Fermi screening radius with dielectric constant ϵ_0,

$$q^2_{\mathrm{TF}0} = (\epsilon_\infty/\epsilon_0)(12w/\pi)^{2/3}(a^*_\infty r_e)^{-1} \,. \tag{28}$$

From Eqs. (26) and (27) it follows

$$\int_V e^{i\vec{q}\cdot\vec{r}}\rho(\vec{r})\,d^3r = -\frac{\epsilon_\infty(\epsilon_0 - \epsilon_\infty)}{4\pi e\epsilon_0}\left[1 + \left(\frac{q_{\mathrm{TF}0}}{q}\right)^2\right]^{-1}\int_V e^{i\vec{q}\cdot\vec{r}}\Delta U(\vec{r})\,d^3r \,. \tag{29}$$

Using Eqs. (18), (19), (25), and (29) we obtain from Eq. (17),

$$p=\frac{4\pi^3e^6N_e}{V\epsilon_\infty^2 m^{*3}\omega^3}\,\omega_{p\infty}^2(0)\sum_{\vec q}{}'\;\delta(\omega-\omega_{\mathrm{mp}\infty}(\vec q))\,|\vec G\cdot\hat q|^2\omega_{\mathrm{mp}\infty}^{-3}(0,\hat q)[\omega_{\mathrm{mp}\infty}^2(0,\hat q)-\omega_c^2]^{-1}$$
$$\times[\omega_{\mathrm{mp}\infty}^2(0,\hat q)-\omega_c^2(\hat q\cdot\hat H)^2]^2[2\omega_{\mathrm{mp}\infty}^2(0,\hat q)-\omega_{p\infty}^2(0)-\omega_c^2]^{-1}$$
$$\times\left\{1-(\epsilon_0-\epsilon_\infty)\epsilon_0^{-1}\left[1+\left(\frac{q_{\mathrm{TF0}}}{q}\right)^2\right]^{-1}\right\}^2\left|\sum_{l=1}^{S}Z_l\sum_{\vec R_l\in V}e^{i\vec q\cdot\vec R_l}\right|^2. \tag{30}$$

We assume a perfectly uncorrelated and random distribution of ionized impurities (defects) in the crystal and replace the last term on the right-hand side of Eq. (30) by its average value, for $\vec q\neq 0$,

$$\left\langle\left|\sum_{l=1}^{S}Z_l\sum_{\vec R_l\in V}e^{i\vec q\cdot\vec R_l}\right|^2\right\rangle_{\mathrm{av}}=V\sum_{l=1}^{S}Z_l^2N_l\,. \tag{31}$$

Replacing now the summation over $\vec q$ by integration and using Eqs. (8), (11), (22), (28), and the relation

$$N_e=wm^{*3}v_F^3/3\pi^2\hbar^3\,, \tag{32}$$

we find after some algebra,

$$p=(VDe^2N_e/2wm^*)\omega_{p\infty}(0)[\hbar\omega_{p\infty}(0)/E_F]^3\omega^{-2}$$
$$\times\{|\vec E\cdot\hat H|^2F_\parallel(\omega/\omega_{p\infty}(0),\,\omega_c/\omega_{p\infty}(0),\,\epsilon_\infty/\epsilon_0)+\omega^2(\omega^2-\omega_c^2)^{-2}$$
$$\times[(\omega^2+\omega_c^2)(|\vec E|^2-|\vec E\cdot\hat H|^2)+2i\omega\omega_c(\vec E\times\vec E^*)\cdot\hat H]F_\perp(\omega/\omega_{p\infty}(0),\,\omega_c/\omega_{p\infty}(0),\,\epsilon_\infty/\epsilon_0)\} \tag{33}$$

for $\omega>\omega_{p\infty}(0)$, and $p=0$ for $\omega\leq\omega_{p\infty}(0)$. We have denoted

$$D=\sum_{l=1}^{S}Z_l^2(N_l/N_e) \tag{34}$$

and, for $\alpha>1$, $0<\beta\leq\frac12$, $0<\gamma\leq 1$,

$$F_\parallel(\alpha,\beta,\gamma)=\frac{5^{3/2}\pi}{3^{1/2}32\beta^3}\int_1^{\min[\alpha,(1+\beta^2)^{1/2}]}dx\,(x^2-\beta^2)(1+\beta^2-x^2)^{1/2}\left[1-(1-\gamma)\left(1+\frac{9\gamma}{5}A(x,\beta)x^2(\alpha^2-x^2)^{-1}\right)^{-1}\right]^2$$
$$\times(\alpha^2-x^2)^{1/2}A^{-3/2}(x,\beta)\,, \tag{35}$$

$$F_\perp(\alpha,\beta,\gamma)=\frac{5^{3/2}\pi}{3^{1/2}64\beta^3}\int_1^{\min[\alpha,(1+\beta^2)^{1/2}]}dx\,(x^2-1)x^{-2}(x^2-\beta^2)^2(1+\beta^2-x^2)^{-1/2}$$
$$\times\{1-(1-\gamma)[1+(9\gamma/5)A(x,\beta)x^2(\alpha^2-x^2)^{-1}]^{-1}\}^2(\alpha^2-x^2)^{1/2}A^{-3/2}(x,\beta)\,, \tag{36}$$

$$A(x,\beta)=\beta^{-4}(x^2-\beta^2)(2x^2-1-\beta^2)^{-1}[(1+\beta^2-x^2)^2+(x^2-1)^2(x^2-\beta^2)(x^2-4\beta^2)^{-1}+(\tfrac13)(x^2-1)(1+\beta^2-x^2)$$
$$\times(6x^4-3\beta^2x^2+\beta^4)(x^2-\beta^2)^{-2}]\,. \tag{37}$$

It should be noted that $i(\vec E\times\vec E^*)\cdot\hat H$ is real and vanishes for linearly polarized radiation.

From Eq. (24) it follows that $D\geq 1$.

For $\beta\to 0$ we have

$$\lim_{\beta\to0}F_\parallel(\alpha,\beta,\gamma)=\lim_{\beta\to0}F_\perp(\alpha,\beta,\gamma)$$
$$=(5^{3/2}\pi/3^{3/2}32)(\alpha^2-1)^{1/2}$$
$$\times\{1-(1-\gamma)[1+(9\gamma/5)(\alpha^2-1)^{-1}]^{-1}\}^2. \tag{38}$$

The power absorption p is related to the high-frequency conductivity tensor given by magneto-plasmon processes by the formula

$$p=\tfrac12V[|\vec E\cdot\hat H|^2\,\mathrm{Re}\sigma_{zz}+(|\vec E|^2-|\vec E\cdot\hat H|^2)\,\mathrm{Re}\sigma_{xx}$$
$$-i(\vec E\times\vec E^*)\cdot\hat H\,\mathrm{Im}\sigma_{xy}]\,, \tag{39}$$

where σ_{xx}, σ_{xy}, and σ_{zz} are components of the conductivity tensor for $\vec H$ parallel to the z axis.

Comparing Eqs. (33) and (39) we obtain, first of all,

$$\mathrm{Im}\sigma_{xy}=-2\omega\omega_c(\omega^2+\omega_c^2)^{-1}\,\mathrm{Re}\sigma_{xx}\,. \tag{40}$$

This relation is fulfilled also in the Drude theory of free-carrier absorption, if

$$\omega \gg \tau^{-1}, \quad (41)$$

where τ is the momentum relaxation time of that theory. If the inequality (41) holds the Drude theory gives also

$$\mathrm{Re}\sigma_{xx} = (e^2N_e/m^*)(\omega^2+\omega_c^2)(\omega^2-\omega_c^2)^{-2}\tau^{-1}, \quad (42)$$

$$\mathrm{Re}\sigma_{zz} = (e^2N_e/m^*)\omega^{-2}\tau^{-1}. \quad (43)$$

For the conductivity given by magnetoplasmon processes, the ratio $\mathrm{Re}\sigma_{xx}/\mathrm{Re}\sigma_{zz}$ is different from that following from Eqs. (42) and (43). However, we can use those equations to define two "relaxation times" $\tau_\perp$ and $\tau_\parallel$:

$$\mathrm{Re}\sigma_{xx} = (e^2N_e/m^*)(\omega^2+\omega_c^2)(\omega^2-\omega_c^2)^{-2}\tau_\perp^{-1}, \quad (44)$$

$$\mathrm{Re}\sigma_{zz} = (e^2N_e/m^*)\omega^{-2}\tau_\parallel^{-1}. \quad (45)$$

Using Eqs. (39), (44), and (45), we can present our result (33) as Eq. (40) and

$$\tau_{\perp,\parallel}^{-1} = (D/w)\omega_{p\infty}(0)[\hbar\omega_{p\infty}(0)/E_F]^3 \times F_{\perp,\parallel}(\omega/\omega_{p\infty}(0), \omega_c/\omega_{p\infty}(0), \epsilon_\infty/\epsilon_0) \quad (46)$$

for $\omega > \omega_{p\infty}(0)$, and $\tau_{\perp,\parallel}^{-1}=0$ for $\omega \le \omega_{p\infty}(0)$.

It should be stressed that in our problem relations (44) and (45) are nothing but definitions of some frequency-dependent, "collective" momentum relaxation times $\tau_\perp$ and $\tau_\parallel$ corresponding to plasmon processes. Of course, they have nothing to do with relaxation times for individual-carrier scattering. Moreover, frequency dependence of relaxation time violates the dispersion relations of Drude's theory. Therefore, it would be inaccurate to use the Drude formula for, e.g., $\mathrm{Im}\sigma_{xx}$ with the relaxation time defined by Eq. (44).

The relaxation rates $\tau_\perp^{-1}$ and $\tau_\parallel^{-1}$ due to plasmon processes are additive to those given by other momentum-relaxation processes (in particular, individual-carrier scatterings) since the corresponding parts of the high-frequency conductivity tensor are additive.

The condition (41) is not only necessary to justify the simple forms (42) and (43) of the components of the conductivity tensor in Drude's theory, but it is also needed to justify our weak-perturbation treatment, as one can see from the following argument. The mean kinetic energy of the magnetoplasma moving in the electric field (9) is

$$(Ve^2N_e/4m^*\omega^2)|\vec{G}|^2. \quad (47)$$

The ratio of p multiplied by the period $2\pi/\omega$ to the expression (47) should be small. From Eqs. (39), (40), (44), and (45), this ratio equals $4\pi/\omega$ times a weighted average of $\tau_\perp^{-1}$ and $\tau_\parallel^{-1}$. Therefore, it is sufficient to impose the condition

$$\omega \gg \tau_\perp^{-1}, \tau_\parallel^{-1}. \quad (48)$$

For given m^*, ϵ_∞, ϵ_0, D, and N_e, $\tau_{\perp,\parallel}^{-1}$ are temperature independent. Changing the electron concentration N_e we change the ω and ω_c scales for $\tau_\perp^{-1}$ and $\tau_\parallel^{-1}$, as they depend on the ratios $\omega/\omega_{p\infty}(0)$ and $\omega_c/\omega_{p\infty}(0)$. However, the magnitudes of $\tau_\perp^{-1}$ and $\tau_\parallel^{-1}$ (for fixed D) are independent of N_e.

If there are both positive and negative ionized impurities (defects) in the crystal, of charges Ze and $-Ze$ and concentrations N_+ and N_-, respectively, we have

$$D = Z(1+K)/(1-K), \quad (49)$$

where

$$K = N_-/N_+. \quad (50)$$

From Eqs. (38) and (46) we obtain

$$\lim_{H\to 0}\tau_\perp^{-1} = \lim_{H\to 0}\tau_\parallel^{-1} = \frac{5^{3/2}\pi D}{3^{3/2}32w}\,\omega_{p\infty}(0)\left(\frac{\hbar\omega_{p\infty}(0)}{E_F}\right)^3\left[\left(\frac{\omega}{\omega_{p\infty}(0)}\right)^2-1\right]^{1/2} \times\left(1-\left(1-\frac{\epsilon_\infty}{\epsilon_0}\right)\left\{1+\frac{9\epsilon_\infty}{5\epsilon_0}\left[\left(\frac{\omega}{\omega_{p\infty}(0)}\right)^2-1\right]^{-1}\right\}^{-1}\right)^2. \quad (51)$$

This is identical with Eq. (44) of paper I except for an additional factor $\omega/\omega_{p\infty}(0)$ (≈ 1). The difference will disappear if in Eq. (21) of paper I [the formula for $d(\vec{r}')$] $\omega_{p\infty}(0)$ will be used instead of $\omega_{p\infty}(q)$.

IV. NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENT

The functions $F_\perp(\alpha,\beta,\gamma)$ and $F_\parallel(\alpha,\beta,\gamma)$ were calculated numerically from Eqs. (35)–(38) for different values of $1<\alpha\le 1.25$, $0\le\beta\le 0.5$, and $0<\gamma\le 1$, and plotted on Fig. 1. One can observe that with decreasing γ, i.e., increasing ϵ_0, $\tau_\perp^{-1}$ and $\tau_\parallel^{-1}$ decrease and a maximum is formed for small values of α. It should be noted, however, that the decrease of τ^{-1} due to individual carrier scattering is much faster as ϵ_0 increases (see paper I). Therefore, the free-carrier magnetoabsorption by magnetoplasmon generation may be comparable to that due to individual-carrier transitions in polar

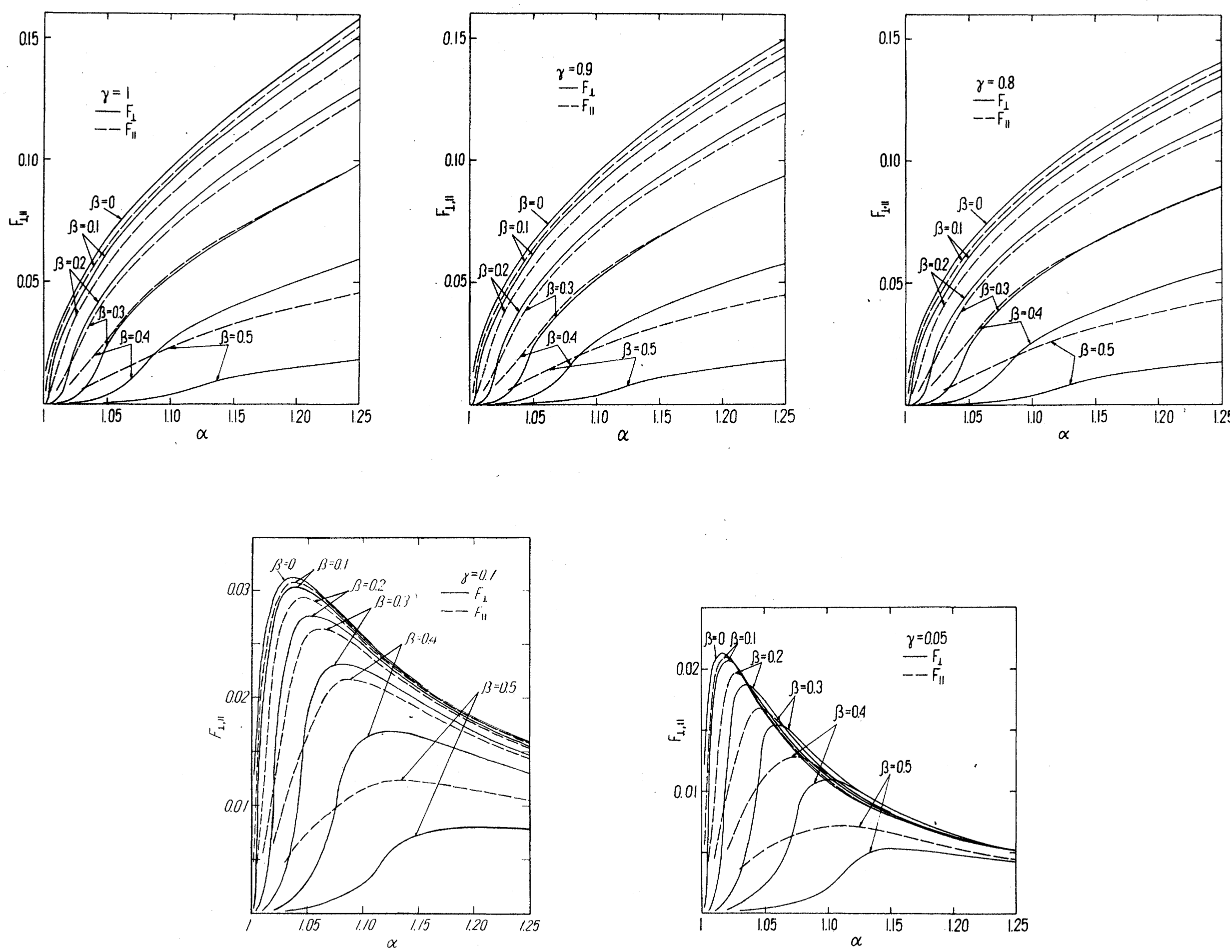

FIG. 1. Functions $F_\perp(\alpha,\beta,\gamma)$ and $F_\parallel(\alpha,\beta,\gamma)$ [see Eqs. (35)–(38)] for different values of α, β, and γ.

semiconductors of high static lattice dielectric constant (and with high concentration of carriers).

The effect of the magnetic field is to reduce the absorption by plasmon generation, and is stronger for $\vec{E}\perp\vec{H}$ than for $\vec{E}\parallel\vec{H}$. A rather sharp increase of $\tau_\perp^{-1}$ can be observed below $\alpha=(1+\beta^2)^{1/2}$. It can be easily interpreted because for $\alpha<(1+\beta^2)^{1/2}$ the radiation cannot generate plasmons with all possible directions of $\vec{q}$ [see Eqs. (15) and (22)].

Suppose that D is of the order of 1. It follows from Eq. (46) and from the values of $F_\perp$ and $F_\parallel$ (see Fig. 1) that the condition (48) is fulfilled, at least if $\hbar\omega_{p\infty}(0)$ is not much higher than E_F.

In paper I the calculated values of τ^{-1} corresponding to plasmon generation in the absence of magnetic field were fitted to the experimental data for n-type PbSe, a semiconductor with a very high ϵ_0. Only an order-of-magnitude agreement was obtained. PbSe does not fit well the model used in the paper I and in this paper since its four conduction-band minima (located at L points) are nonparabolic and anisotropic. However, the most important reason for the poor agreement between the theory of paper I and experiment consists probably in having neglected the magnetic field effect. Experimentally, $\tau_\perp^{-1}(\omega)$ defined as in the present paper was obtained from magnetoreflectivity measurements in Voigt configuration at low temperatures.[10,13] $\tau_\perp^{-1}(\omega)$ seems to be a superposition of a rather flat curve which can be interpreted as due to individual electron–optical-phonon processes,[14,15] and a bump with an edge corresponding roughly to $\omega_{p\infty}(0)$, which may be due to magnetoplasmon generation processes.

Using the parameters $w=4$, $\epsilon_\infty=26$, $\epsilon_0=323$, and $D=10$ [as, according to Eq. (49), for $Z=1$, $K=0.82$, and for $Z=2$, $K=0.67$], we have calculated $\tau_\perp^{-1}(\omega)$ from Eq. (46) for the n-PbSe sample B studied in Refs. 10 and 13. For this strongly de-

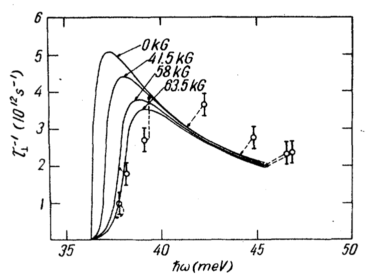

FIG. 2. Comparison of the theoretical relaxation rate $\tau_\perp^{-1}$ calculated for the n-PbSe sample B of Refs. 10 and 13 (solid lines) with the experimental data for four values of the magnetic field intensity. The correspondence between the experimental points and the theoretical curves is indicated.

generated sample $N_e = 1.33\times10^{18}$ cm^{-3}, $E_F = 33$ meV, $\hbar\omega_{p\infty}(0) = 36.3$ meV, and $\hbar\omega_c = 11.6$ meV at $H = 63.5$ kG. The results are plotted on Fig. 2 for four values of the magnetic field intensity used in experiments.

The experimental values of $\tau_\perp^{-1}$ obtained from fitting the magnetoplasma minima observed at 30 K and at different magnetic-field intensities[10] are also plotted in Fig. 2, after subtracting 1.5 $\times10^{12}$ s^{-1} for individual electron scattering. The agreement of these experimental data with the present theory is fair, much better than with the theory of paper I ($H = 0$ curve in Fig. 2).

Comparing the theory with experiment we have used two fitting parameters. We have chosen $D = 10$, and subtracted 1.5×10^{12}s^{-1} from the experimental values of $\tau_\perp^{-1}$. The latter is justified by the results of Szymański.[15] He has shown that the LO-phonon polar scattering of individual electrons gives for sample B $\tau^{-1} \cong 1.1\times10^{12}$ s^{-1}, nearly independent of frequency, in our frequency range. Other individual electron-scattering mechanisms, in particular one- or two-optical-phonon nonpolar scattering, were shown to depend rather weakly on frequency in our frequency range. They will contribute together $\tau^{-1} \cong 0.4\times10^{12}$ s^{-1} if the very high deformation-potential constants used by Szymański are reduced by a factor of 3, which seems reasonable.

There exists evidence for twofold charged defects, at least in PbTe, and for rather high compensations.[16-18] It seems, therefore, that our value for D is of the proper order of magnitude.

For the material considered, assumption (6) is weaker than assumption (5). The latter inequality is well fulfilled for sample B. As $\hbar\omega_{LO} = 19$ meV, assumption (4) is rather poorly fulfilled. Assumptions (1) and (2) are satisfied even for the highest magnetic-field intensity used. It should be also noted that we have calculated the theoretical values of $\tau_\perp^{-1}$ only for the frequency range limited by inequality (7).

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