Free-carrier absorption in n-type semiconductors: The inelastic scattering of electrons from ionized impurities†

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We examine the theory of free-carrier absorption in the Drude tail of n-type semiconductors, from the process where absorption of a photon by a conduction electron occurs in concert with scattering from an ionized impurity imbedded in a rigid lattice. The electron may suffer energy loss when scattering from the impurity because a fraction of its kinetic energy may be transferred to the screening cloud which surrounds the impurity. We derive an expression for the frequency-dependent optical relaxation time from this process, and explore its behavior for parameters typical of reasonably heavily doped semiconductors. The physcial origin of structure in the frequency variation of the optical relaxation rate is discussed; we find clear evidence of structure for photon frequencies near the conduction-electron plasma frequency.

I. INTRODUCTORY REMARKS

For frequencies well below the interband absorption edge, optical absorption in doped semiconductors arises primarily from the free carriers through intraband, inverse bremsstrahlung processes. One describes this free-carrier absorption phenomenologically through use of the Drude model, in simple cases where the carriers reside in a nearly parabolic band. If the carriers have effective mass m^*, and number density n, one introduces a phenomenological relaxation time $\tau(\Omega)$ for the carriers, where Ω is the frequency of the radiation that excites them. The absorption rate is controlled by the real part of the frequency-dependent conductivity $\sigma(\Omega)$. When $\Omega\tau(\Omega) \gg 1$, one has

$$\mathrm{Re}[\sigma(\Omega)] = (ne^2/m^*\Omega^2)[1/\tau(\Omega)] . \tag{1.1}$$

The theoretical study of absorption in this "Drude tail" regime reduces to the examination of the various contributions to the optical relaxation rate $1/\tau(\Omega)$.

This area has been the focus of a rather considerable number of theoretical studies.[1] Our attention has been attracted to it because it seems that a number of features of the data remain difficult to explain. An example is the apparent absence of temperature dependence in the absorption coefficient of heavily doped n-type CdTe,[2] under conditions where it has been argued[3] that LO-phonon-mediated processes should provide the dominant contribution to the relaxation rate $1/\tau(\Omega)$. This has led us to extend the earlier theories[1] of the LO-phonon-mediated process, which are really applicable when the electron-plasma frequency ω_p is lower than that of the LO phonon ω_{LO}, into the range of carrier concentrations where ω_p is comparable to or larger than ω_{LO}.

In this paper, we reexamine a second process believed to play an important role in the Drude tail. This is the process where a photon is absorbed by a free carrier, which then scatters off an ionized impurity in the lattice.

In the earlier treatments,[1] the impurity is presumed rigidly imbedded in the lattice, and the incoming electron sees the potential of the charged impurity screened statically by the free carriers. If z is the effective charge of the impurity, and $\epsilon(q)$ the wave-vector-dependent static dielectric constant of the system, the screened potential is given by

$$V_s(q) = \frac{4\pi Ze^2}{q^2\epsilon(q)} \cong \frac{4\pi Ze^2}{\epsilon_\infty(q^2+q_s^2)} , \tag{1.2}$$

where in the second step the Fermi-Thomas model with $\epsilon(q) = \epsilon_\infty(1+q_s^2/q^2)$ has been introduced.

The potential with matrix element $V_s(q)$ in Eq. (1.2) is a static potential, and as a consequence the electron scatters elastically from the impurity. But even if the impurity is rigidly imbedded in the lattice, there can still be inelastic character to the electron-impurity scattering event. This is because the screening cloud which surrounds the impurity can be excited by the incoming electron, to absorb part of its kinetic energy. In essence, in response to the electron impact, the impurity may radiate plasmons, or single particle-hole excitations. A complete description of the impurity contribution to the Drude tail should incorporate these features of the scattering process. The purpose of this paper is to describe such a treatment of the problem, for the simple case of electrons in a parabolic conduction band of a semiconductor. It turns out that the formalism we developed in our first paper can be readily adapted to the present problem. Indeed, we need not carry out a separate derivation of the formula for $1/\tau(\Omega)$; it fol-

lows directly from the expressions in our earlier paper, after use of a simple physical argument. We understand that Mycielski and Mycielski[5] have recently completed a study of the question of the inelastic scattering of electrons from charged impurities, with energy transfer to the screening cloud. We shall comment on the relation of our work to theirs in Sec. II.

In Sec. II, we present a derivation of the expression for $1/\tau(\Omega)$, and explore its behavior in a number of limits. This allows us to assess where the earlier theories apply, and where they breakdown. In Sec. III, we present detailed studies of the frequency variation of $1/\tau(\Omega)$, with emphasis on the new features present here. We make brief remarks on the trends in the frequency dependence of $1/\tau(\Omega)$ in Sec. IV, and a comparison of the present results with our earlier ones for the LO-phonon-mediated process.

II. DERIVATION OF AN EXPRESSION FOR THE OPTICAL RELAXATION RATE, AND EXPLORATION OF ITS GENERAL PROPERTIES

Before we begin, as in our earlier paper[4] (referred to as I in the remainder of the present article), we presume the conduction band is parabolic, with effective mass m^*. In Jensen's extensive study of free-carrier absorption, band-structure effects are included, and we refer the reader there for a discussion of their role.

We start by writing down the second quantized Hamiltonian that describes interaction of an electron with a single impurity of effective charge Z imbedded in a lattice with background dielectric constant ϵ_∞. We shall deduce a form for $1/\tau(\Omega)$ for scattering of an electron from one impurity, then multiply the result by the number of impurities/unit volume to obtain a final result appropriate to a material with finite impurity concentration. If the impurity is placed at the origin, the contribution H_I to the crystal Hamiltonian from the electron-impurity interaction is

$$H_I=\frac{4\pi Ze^2}{V\epsilon_\infty}\sum_{\vec{k},\vec{q}}\frac{1}{q^2}c^\dagger_{\vec{k}+\vec{q}}c_{\vec{k}}, \tag{2.1}$$

where V is the crystal volume, and $c_{\vec{k}}$ ($c^\dagger_{\vec{k}}$) is the annihilation (creation) operator for electrons.

In our previous paper,[4] as remarked in Sec. I, we derived an expression for $1/\tau(\Omega)$, for the case where an LO phonon (coupled to the electron plasma) was absorbed or emitted by an electron, as the photon is absorbed. We take the limit of this expression as the frequency ω_{LO} of the LO phonon approaches zero. This expression, interpreted properly, describes absorption in the presence of frozen-in (i.e., static) lattice disorder. Upon relating the Fröhlich coupling constant to the electron-impurity matrix element in Eq. (2.1) through a physical argument, we are able to extract from the result a formula for $1/\tau(\Omega)$ for the problem of present interest.

As we noted in I, when ω_{LO} is small compared to the electron plasma frequency ω_p, our general expression for $1/\tau(\Omega)$ reduces to a form equivalent to a result of Mahan's,[6] obtained in the discussion of a rather different problem. We have, from Eq. (4.7) of I,

$$\frac{1}{\tau(\Omega)}=\frac{\epsilon_\infty^2}{12\pi^2ne^2m^*\Omega}\int_{-\infty}^{+\infty}d\omega\,[n(\omega)-n(\omega+\Omega)]\times\sum_{\vec{q}}q^4\gamma_0^2(q)\,\mathrm{Im}[D(\omega+i\eta)]\times\mathrm{Im}\left(\frac{1}{\epsilon(q,\omega+\Omega+i\eta)}\right). \tag{2.2}$$

In this expression, n is the number of electrons/cm^3, $n(\omega)=[\exp(\beta\omega)-1]^{-1}$ is the Bose-Einstein function, $\gamma_0(q)$ the Fröhlich coupling constant (we shall not need its precise form), $D(\omega+i\eta)$ the Green's function for the LO phonon and $\epsilon(q,\omega+\Omega+i\eta)$ the dielectric function of the electron gas. An expression for $\mathrm{Im}[D(\omega+i\eta)]$ is readily found from Eq. (3.18) of I:

$$\mathrm{Im}[D(\omega+i\eta)]=\pi[\delta(\omega-\omega_{LO})-\delta(\omega+\omega_{LO})]. \tag{2.3}$$

As we let $\omega_{LO}\to 0$, the factor of ω in $\epsilon(q,\omega+\Omega+i\eta)$ may be set to zero. After evaluating the integral over ω that is left, we have, noting $n(-\omega_{LO})=-1-n(\omega_{LO})$,

$$\frac{1}{\tau(\Omega)}=\frac{\epsilon_\infty^2}{12\pi ne^2m^*\Omega}\sum_{\vec{q}}q^4\gamma_0^2(q)\,\mathrm{Im}\left(\frac{1}{\epsilon(q,\Omega+i\eta)}\right)\times[1+2n(\omega_{LO})-n(\Omega+\omega_{LO})+n(\Omega-\omega_{LO})]. \tag{2.4}$$

As $\omega_{LO}\to 0$, the factor $2n(\omega_{LO})$ is replaced by $2k_BT/\omega_{LO}$ (we use units with $\hbar=1$), and the other quantities in the square brackets are ignored, since they remain of order unity. Thus, as $\omega_{LO}\to 0$, we have

$$\frac{1}{\tau(\Omega)}=\frac{\epsilon_\infty^2k_BT}{6\pi ne^2m^*\omega_{LO}\Omega}\times\sum_{\vec{q}}q^4\gamma_0^2(q)\,\mathrm{Im}\left(\frac{1}{\epsilon(q,\Omega+i\eta)}\right). \tag{2.5}$$

We now require an argument which eliminates $\gamma_0(q)$ in favor of the electron-impurity matrix element. In I, the electron-phonon Hamiltonian was written [combine Eq. (3.3c) with Eq. (3.3a)]

$$H_{e-l}=\sum_{\vec{k}\vec{q}}\gamma_0(\vec{q})c^\dagger_{\vec{k}+\vec{q}}c_{\vec{k}}i\varphi(\vec{q})\,, \tag{2.6}$$

where $\varphi(\vec{q})=b_{\vec{q}}-b^\dagger_{-\vec{q}}$, with $b_{\vec{q}}, b^\dagger_{\vec{q}}$ the phonon annihilation and creation operators.

As $\omega_{LO}\to 0$, the occupation number of each phonon mode diverges, $n_{\vec{q}}\approx k_BT/\omega_{LO}$. In this limit, we need not worry about the quantum-mechanical nature of $\varphi(\vec{q})$. We may treat $\varphi(\vec{q})$ as a classical random variable, which is time independent as $\omega_{LO}\to 0$. In fact, as $\omega_{LO}\to 0$, one recovers the results of conventional electron-phonon scattering theory if in Eq. (2.6), the factor of $i\varphi(\vec{q})$ is replaced by its classical root-mean-square value $(2k_BT/\omega_{LO})^{1/2}$. Thus, in the static limit $\omega_{LO}\to 0$, Eq. (2.6) may be replaced by an equivalent effective Hamiltonian $\tilde{H}_{e-l}$ given by

$$\tilde{H}_{e-l}=\left(\frac{2k_BT}{\omega_{LO}}\right)^{1/2}\sum_{\vec{q}}\gamma_0(\vec{q})c^\dagger_{\vec{k}+\vec{q}}c_{\vec{k}}\,. \tag{2.7}$$

The expression in Eq. (2.7) is identical in form to Eq. (2.1). As a consequence, we obtain the expression for $1/\tau(\Omega)$ for our electron-impurity scattering problem from Eq. (2.5) by making the replacement $\gamma_0(\vec{q})\to(\omega_{LO}/2k_BT)^{1/2}(4\pi Ze^2/V\epsilon_\infty q^2)$. Upon multiplying the expression that results by the number N_I of impurities in the crystal, and denoting their density by n_I, we have the very simple form

$$\frac{1}{\tau(\Omega)}=\frac{4\pi n_IZ^2e^2}{3m^*n\Omega}\frac{1}{V}\sum_{\vec{q}}\mathrm{Im}\left(\frac{1}{\epsilon(q,\Omega+i\eta)}\right). \tag{2.8}$$

It will be useful to recall that in the random-phase approximation (RPA) employed here, one has

$$\epsilon(q,\Omega+i\eta)=\epsilon_\infty-\frac{4\pi e^2}{q^2}\chi_0(q,\Omega)\,, \tag{2.9}$$

where

$$\chi_0(q,\Omega)=\frac{2}{V}\sum_{\vec{k}}\frac{f_{\vec{k}}-f_{\vec{k}+\vec{q}}}{\epsilon_{\vec{k}}-\epsilon_{\vec{k}+\vec{q}}+\Omega+i\eta}\,. \tag{2.10}$$

The factor of 2 in Eq. (2.10) is for spin, $f_{\vec{k}}$ is the Fermi-Dirac function, and $\epsilon_k=k^2/2m^*$.

The results displayed in Eqs. (2.8)–(2.10) form the basis for the discussion in the remainder of the paper. In Eq. (2.8), we have the expression which describes the transfer of energy $\hbar\Omega$ from the incoming photon to the excitations in the electron gas, with the impurity playing the role of a momentum sink. This is just as in the classical theory of the bremsstrahlung process. Physically, in our case, we can think of the process as one where the incoming photon virtually excites an electron, which collides with the impurity center, to transfer part of its kinetic energy to oscillations in the screening cloud around the impurity.

The remainder of this section is devoted to a study of $1/\tau(\Omega)$ in various special limits, and the relation of the form in Eq. (2.8) to earlier static screening approaches to the present problem. For this purpose, it is useful to rearrange Eq. (2.8) to read

$$\frac{1}{\tau(\Omega)}=\frac{32\pi^3n_IZ^2e^4}{3m^*n\Omega}\frac{1}{V^2}\sum_{\vec{q}}\frac{1}{q^2[\epsilon_R^2(q,\Omega)+\epsilon_I^2(q,\Omega)]}\times\sum_{\vec{k}}(f_{\vec{k}}-f_{\vec{k}+\vec{q}})\times\delta(\epsilon_{\vec{k}+\vec{q}}-\epsilon_{\vec{k}}-\Omega)\,, \tag{2.11}$$

or in the equivalent form (with $\beta=1/k_BT$)

$$\frac{1}{\tau(\Omega)}=\frac{32\pi^3n_IZ^2e^4}{3m^*n\Omega}\frac{1-e^{-\beta\Omega}}{V^2}\times\sum_{\vec{q}}\frac{1}{q^2[\epsilon_R^2(q,\Omega)+\epsilon_I^2(q,\Omega)]}\times\sum_{\vec{k}}f_{\vec{k}}(1-f_{\vec{k}+\vec{q}})\,\delta(\epsilon_{\vec{k}+\vec{q}}-\epsilon_{\vec{k}}-\Omega)\,. \tag{2.12}$$

First, consider the behavior in the limit as the photon frequency becomes very large. One can see this by noting that the values of $\vec{k}$ are restricted by the factor of $f_{\vec{k}}$ in Eq. (2.12), so that we must have $|\vec{q}|\to\infty$ as $\Omega\to\infty$. As $\Omega\to\infty$, to conserve energy and wave vector $\vec{q}$ must become very large also. Since $\vec{k}$ is limited by the factor $f_{\vec{k}}$ to values equal to k_F or smaller, where k_F is the Fermi wave vector, (we assume the electrons are degenerate, although the conclusion we draw is quite general), in the energy conserving delta function, $\epsilon_{\vec{k}+\vec{q}}-\epsilon_{\vec{k}}$ may be replaced by simply $\epsilon_{\vec{q}}$. Also, since the final state is surely empty, $1-f_{\vec{k}+\vec{q}}$ may be replaced by unity.

The above argument shows that when Ω becomes large, q necessarily does also. But at large wave vectors (much larger than k_F, for example) and high frequencies (higher than either the Fermi energy E_F or the plasma frequency ω_p), the dielectric function $\epsilon(\vec{q},\Omega)$ reduces simply to ϵ_∞. The electron gas is ineffective in screening the impurity. Thus, Eq. (2.12) can be replaced by, to very good accuracy, recalling the factor of 2 for spin,

$$\frac{1}{\tau(\Omega)}=\frac{16\pi^3n_IZ^2e^4}{3m^*\Omega\epsilon_\infty^2}\frac{1-e^{-\beta\Omega}}{V}\sum_{\vec{q}}\frac{1}{q^2}\,\delta(\epsilon_{\vec{q}}-\Omega). \tag{2.13}$$

This expression is easily evaluated to give

$$\frac{1}{\tau(\Omega)}=\frac{8\pi n_IZ^2e^4}{3\epsilon_\infty^2(2m^*)^{1/2}}\frac{1-e^{-\beta\Omega}}{\Omega^{3/2}}. \tag{2.14}$$

When carried through to the absorption constant, the expression in Eq. (2.14) gives the often quoted $\Omega^{-5/2}$ law for the contribution to the absorption constant from ionized impurity scattering.

From the present view point, the important feature of the argument above is that the factors $\epsilon_R(q,\Omega)$ and $\epsilon_I(q,\Omega)$ which appear in Eq. (2.12) drop out, with the whole expression in square brackets replaced by simply ϵ_∞^2. Thus, the result in Eq. (2.13) is independent of whether the impurity is regarded as dynamically screened, as in the present theory, or whether the screening cloud is treated as rigid and static, as assumed in the expression displayed in Eq. (1.1), which has been utilized in earlier theories, as remarked in Sec. I.

Next consider the limit $\Omega \to 0$, which should give us the dc mobility relaxation time, i.e., the relaxation time appropriate to the dc electrical transport.

Before we turn to the limit $\Omega \to 0$, a word about the significance of the result is in order. The expression we have obtained for the electronic relaxation time $\tau(\Omega)$ utilizes an approach that makes essential use of the assumption $\Omega\tau(\Omega) \gg 1$, as Holstein's classic paper discusses most elegantly.[7] One must question the extrapolation of the expression for $\tau(\Omega)$ to zero frequency as a consequence. However, as Mahan has argued,[6] and we have also,[4] the expression for $\tau(\Omega)$ obtained by the method used in I gives at $\Omega \equiv 0$ an expression for the electronic relaxation time identical to that provided by a simple approximate solution of the Boltzmann equation of dc transport theory. This solution is the one where one presumes that in the external dc electric field, the nonequilibrium electron distribution function is just shifted in velocity space by a drift velocity V_D. While this method of solving the dc transport equation is clearly an approximate one, the variational principal of transport theory[8] insures that this procedure yields an upper bound to the relaxation *rate*; the estimate of the relaxation rate can be quite good, even when the scattering is quite anisotropic.[9]

We examine the dc limit $\Omega = 0$ with the above qualifications in mind. At $\Omega = 0$, $\epsilon_I(q,\Omega) \equiv 0$, and Eq. (2.12) becomes

$$\frac{1}{\tau(0)} = \frac{32\pi^2 n_I Z^2 e^4}{3m^* n k_B T} \frac{1}{V^2} \sum_{\vec{q}} \frac{1}{q^2 \epsilon_R^2(q,0)} \times \sum_{\vec{k}} f_{\vec{k}}(1 - f_{\vec{k}+\vec{q}})\, \delta(\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}}). \tag{2.15}$$

If $\vec{V}_k = \vec{k}/m^*$ is the velocity of an electron with wave vector $\vec{k}$, and $\hat{x}$ is a unit vector in the direction of the applied electric field, then Eq. (2.15) may be rewritten in the form

$$\frac{1}{\tau(0)} = \frac{2\pi n_I m^*}{n k_B T V^2} \sum_{\vec{q}} \sum_{\vec{k}} \left[\frac{4\pi Z e^2}{q^2 \epsilon_R(q,0)}\right]^2 \times [\hat{x} \cdot (\vec{V}_{\vec{k}+\vec{q}} - \vec{V}_{\vec{k}})]^2 \times f_{\vec{k}}(1 - f_{\vec{k}+\vec{q}})\, \delta(\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}}). \tag{2.16}$$

This expression can be recognized as the expression for $1/\tau(0)$ provided by Ziman's variational principal[8] in combination with the "drifting electron gas" ansatz for the nonequilibrium form of the distribution function. The effective potential seen by the electron is just the statically screened potential of Eq. (1.1).

Thus, there is no difference in principal between the earlier theories of ionized impurity scattering and the present dynamical theory in the dc limit $\Omega \to 0$. When this is combined with the result displayed in Eq. (2.14) and the discussion that follows, we see that the present theory has new things to say about the frequency variation of the optical relaxation rate only for frequencies Ω near the characteristic frequencies of the electron gas, most notably near the plasma frequency ω_p.

In the frequency regime allowed to the electron plasma oscillations, the dynamical nature of the screening cloud can affect the behavior of $1/\tau(\Omega)$ in a most pronounced fashion. Let $\omega_p(q)$ be the dispersion relation of the collective excitations (plasmons) in the conduction electron gas. Then $\epsilon_R(q, \omega_p(q))$ vanishes, and if the imaginary part of $\epsilon(q, \Omega + i\eta)$ is small, there will be a strong resonance in the integrand of Eq. (2.11). This will show up as structure in the relaxation rate $1/\tau(\Omega)$ near ω_p.

We can isolate the contribution to $1/\tau(\Omega)$ from these collective modes, if we assume them sharp, well-defined elementary excitations of the electron gas. Let $\omega_p(q)$ be the dispersion relation of these modes, and write Eq. (2.8) in the form

$$\frac{1}{\tau(\Omega)} = \frac{4\pi n_I Z^2 e^2}{3m^* n\Omega} \frac{1}{V} \sum_{\vec{q}} \frac{\epsilon_I(q,\Omega)}{\epsilon_R^2(q,\Omega) + \epsilon_I^2(q,\Omega)} \tag{2.17a}$$

$$= \frac{2n_I Z^2 e^2}{3\pi m^* n\Omega} \int_0^\infty \frac{dq\, q^2 \epsilon_I(q,\Omega)}{\epsilon_R^2(q,\Omega) + \epsilon_I^2(q,\Omega)}. \tag{2.17b}$$

Let $q = q_p(\Omega)$ be the wave vector of the plasmon of frequency Ω [here $\Omega > \omega_p(0)$, the plasmon frequency at $q = 0$], and assume $\epsilon_I(q,\Omega)$ small and smoothly varying, considered as a function of q near $q_p(\Omega)$. Then in Eq. (2.17b), replace $\epsilon_I(q,\Omega)$ by $\epsilon_I(q,\Omega) \equiv \epsilon_I(q_p(\Omega),\Omega)$ with little error, and near $q = q_p(\Omega)$ write $\epsilon_R(q,\Omega) \cong \epsilon_R'[q - q_p(\Omega)]$. The integral over q is then performed readily [after replacing the quan-

tity q^2 in the numerator by $q_p(\Omega)$], and one finds from the collective mode the contribution

$$\frac{1}{\tau(\Omega)} = \frac{2n_I Z^2 e^2}{3m^* n\Omega} \frac{q_p^2(\Omega)}{|\epsilon_R'|}. \tag{2.18}$$

If we take the conduction electrons to be degenerate, then in the region $\Omega \gg v_F q$ where well-defined plasmons exist, we have

$$\epsilon_R(q,\Omega) = \epsilon_\infty - \frac{4\pi ne^2}{m^*\Omega^2} - \frac{12\pi ne^2}{5m^*\Omega^2}\left(\frac{v_F q}{\Omega}\right)^2, \tag{2.19}$$

so near $\Omega = \omega_p = (4\pi ne^2/\epsilon_\infty m^*)^{1/2}$, we have

$$v_F q_p(\Omega) = (\tfrac{5}{3})^{1/2}(\Omega^2 - \omega_p^2)^{1/2} \tag{2.20}$$

and

$$|\epsilon_R'| = \frac{24\pi ne^2}{5m^*\Omega^4} v_F^2 q_p(\Omega). \tag{2.21}$$

Equation (2.18) then becomes, for Ω near but just above ω_p,

$$\frac{1}{\tau(\Omega)} = \tfrac{1}{3}(\tfrac{5}{3})^{3/2}\frac{n_I Z^2 \omega_p e^2}{\epsilon_\infty m^* n v_F^3}(\Omega^2 - \omega_p^2)^{1/2}. \tag{2.22}$$

In Eq. (2.22), we see that as Ω passes through ω_p, one expects a shoulder in the relaxation rate $1/\tau(\Omega)$. This should have the square-root form, familiar from the theory of optical absorption in direct gap semiconductors.

In deriving Eq. (2.22), we have extracted only the contribution to $1/\tau(\Omega)$ only from the plasmons. There is in addition a smooth background ignored here. Also, Eq. (2.22) applies only near ω_p, and we cannot trace out the shape of the entire plasmon induced structure. In Sec. III, we turn to a series of quantitative numerical studies of the frequency variation of $1/\tau(\Omega)$, for parameters typical of the semiconductor CdTe. During the discussion of these numerical calculations, we shall come in contact with the analytic results of the present section.

At this point, we can offer a comparison between the present theory, and that of Mycielski and Mycielski.[5] These authors examine the inelastic scattering of electrons from a static charged impurity, where the scattering is accompanied by plasmon emission. They are led to an expression for $1/\tau(\Omega)$ which near the plasma edge becomes equivalent to our Eq. (2.22). This theory includes only the contribution to $1/\tau(\Omega)$ from plasmon emission, and as a consequence their expression for $1/\tau(\Omega)$ does not reduce properly to Eq. (2.16) in the low-frequency limit, nor does it correctly produce the high-frequency limit of Eq. (2.14). In essence, these authors include only one part of the processes that contribute to the relaxation rate. The calculations we present in Sec. III show a clear plasmon structure in $1/\tau(\Omega)$, but one must use the full formula to obtain results that can be usefully set alongside data.

III. NUMERICAL STUDY OF THE RELAXATION RULE

In Sec. II, we have derived a general expression that describes the electron relaxation rate due to dynamically screened impurities. In this section, we present the results of numerical studies of $1/\tau(\Omega)$ based on the expression displayed in Eq. (2.8). It will also prove instructive to compare the present results with our earlier calculations of the contribution to $1/\tau(\Omega)$ by LO-phonon mediated processes.[4]

As stated above, the starting point for the numerical calculations is Eq. (2.8). It is convenient to write $\chi_0(q,\Omega)$ in the form

$$\chi_0(q,\Omega) = F_1^{(0)}(q,\Omega) + iF_2^{(0)}(q,\Omega), \tag{3.1}$$

Since the free carriers are either fully or nearly degenerate in the regime of concentrations and temperatures of interest here (concentration $n \geq 10^{18}$ cm^{-3} and temperatures $T \leq 300$ K), for $F_1^{(0)}$ we use the well known Lindhard function.[10] As we pointed out earlier,[4] the imaginary part $F_2^{(0)}$ can be evaluated analytically in closed form for arbitrary temperatures, and is given by

$$F_2^{(0)}(q,\Omega) = \frac{(m^*)^{3/2}}{2\sqrt{2}\,\pi\hbar^3\beta^{1/2}}\frac{1}{\kappa} \times\left[\ln\left(\frac{\cosh(\frac{1}{2}W+\alpha)}{\cosh(\frac{1}{2}W-\alpha)}\right) - W\right]. \tag{3.2}$$

Here $\kappa = q(\hbar^2/2m^*k_BT)^{1/2}$, $W = \frac{1}{2}\beta\hbar\Omega$, $\beta = \frac{1}{2}\hbar\Omega$, and $\alpha = \frac{1}{2}(\frac{1}{4}\kappa^2 + W^2/\kappa^2 - \lambda^2)$, with $\lambda^2 = \beta\mu$, where μ is the chemical potential of the fermions. In terms of $F_1^{(0)}(q,\Omega)$ and $F_2^{(0)}(q,\Omega)$, the real and imaginary parts of $\epsilon(q,\Omega)$ are given by

$$\epsilon(q,\Omega+i\eta) = \epsilon_R(q,\Omega) + i\epsilon_I(q,\Omega), \tag{3.3}$$

where

$$\epsilon_R(q,\Omega) = \epsilon_\infty - (4\pi e^2/q^2)F_1^{(0)}(q,\Omega), \tag{3.4a}$$

$$\epsilon_I(q,\Omega) = -(4\pi e^2/q^2)F_2^{(0)}(q,\Omega). \tag{3.4b}$$

Upon carrying out the angular integrations, we are led to the form

$$\frac{1}{\tau(\Omega)} = \frac{2n_I Z^2 e^2}{3m^*\pi n\Omega}\int_0^\infty dq\,\frac{q^2\epsilon_I(q,\Omega)}{\epsilon_R^2(q,\Omega)+\epsilon_I^2(q,\Omega)}. \tag{3.5}$$

In the calculations reported here, we use $m^* = 0.11m_0$, and $\epsilon_\infty = 7.1$. These parameters are characteristic of CdTe, and formed the basis for our earlier study of LO-phonon-mediated processes.

In Fig. 1, we present a plot of Im $[1/\epsilon(q,\omega+i\eta)]$, as a function of q and ω for $n = 10^{19}$ cm^{-3} and

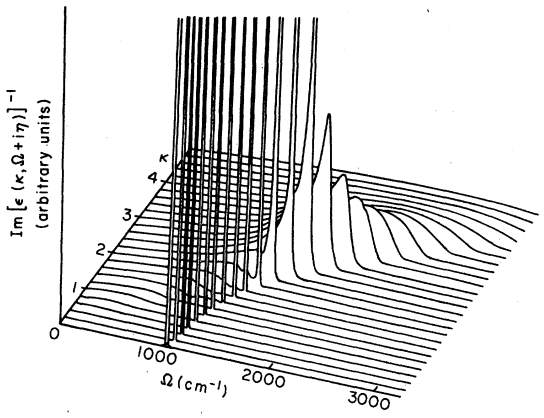

FIG. 1. Plot of $\mathrm{Im}[1/\epsilon(q,\Omega+i\eta)]$ for parameters characteristic of CdTe, with $n=10^9$ electrons cm^{-3} and $T=78$ K. The dimensionless wave vector is $\kappa=q(\hbar^2/2m^*K_BT)^{1/2}$.

$T=78$ K. We see the well-defined plasma excitation at small wave vectors which merges with the particle-hole continuum, to become Landau damped. This plasmon will lead to the appearance of a clearly-defined shoulder in $1/\tau(\Omega)$, as the discussion of Sec. II suggests. We remark again that the theory of Mycielski and Mycielski[5] includes only the contribution from the plasmon excitation, and is unable to include the particle-hole continuum, which makes a substantial contribution to $1/\tau(\Omega)$ at all frequencies.

In Figs. 2 and 3, we present our calculations of $1/\tau(\Omega)$. To evaluate the prefactor in the expression for the relaxation rate, we have assumed $Z=1$, and also that the impurity density n_I and the free-carrier concentration n are equal. Figure 2 shows the frequency variation of $1/\tau(\Omega)$ for several carrier concentrations at room temperature, while Fig. 3 is for liquid-nitrogen temperature. In each case, we see that $1/\tau(\Omega)$ consists of a broad, monotonically decreasing background, upon which a

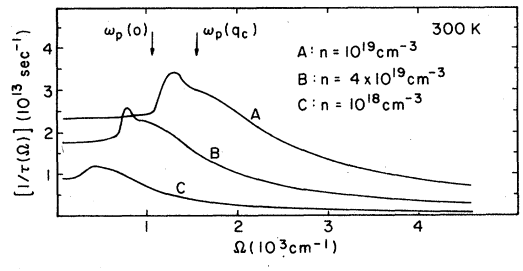

FIG. 2. Frequency variation of $1/\tau(\Omega)$ at room temperature for several carrier concentrations in CdTe. The $q=0$ plasma frequency $\omega_p(0)$ and the cutoff plasma frequency $\omega_p(q_c)$ for which the plasmon merges with the Landau damping region are indicated for $n=10^{19}$ electrons cm^{-3}.

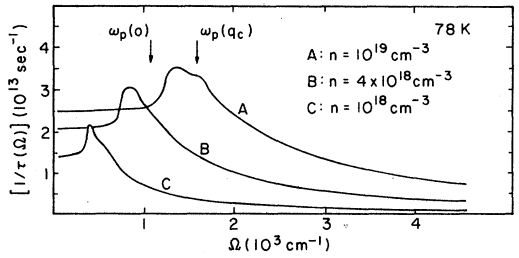

FIG. 3. Frequency variation of $1/\tau(\Omega)$ at liquid-nitrogen temperature, for several carrier concentrations in CdTe. $\omega_p(0)$ and $\omega_p(p_c)$ are indicated for $n=10^{19}$ electrons cm^{-3}.

clear and well-defined structure has been superimposed. This structure originates from inelastic scattering of electrons from charged impurities accompanied by plasmon emission. In each case, the onset of the structure is at the $q=0$ plasma frequency $(4\pi ne^2/e_\infty m^*)^{1/2}$, and it cuts off at the frequency where the plasmon emerges with the particle-hole continuum. It is intriguing that study of this mechanism provides information on the width of the plasmon band, i.e., one can determine the frequency where Landau damping sets in. In all curves, at high frequencies we find the $\Omega^{-3/2}$ frequency variation as expected from the arguments that lead to Eq. (2.14).

As one would expect from Sec. II, and from the nature of the process examined here, $1/\tau(\Omega)$ is insensitive to temperature at high frequencies, quite in contrast to the LO-phonon-mediated process.

In fact we can contrast the contributions to the total relaxation rate $1/\tau_T(\Omega)$ by adding the contributions from the individual processes so that

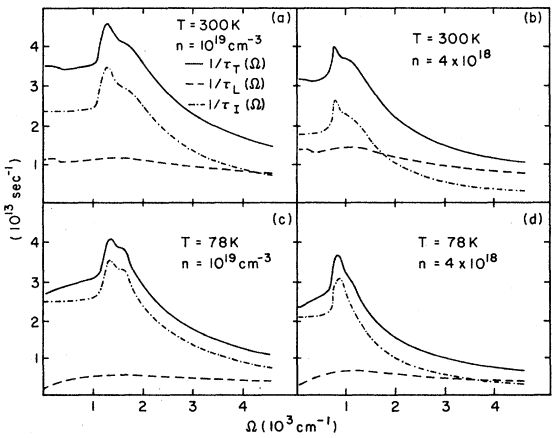

FIG. 4. Frequency variations of the total relaxation rate $1/\tau_T(\Omega)$ and its components $1/\tau_L(\Omega)$ and $1/\tau_I(\Omega)$ for several carrier concentrations and temperatures in CdTe. In (a) and (b), $T=300$ K, while (c) and (d) are for $T=78$ K. In (a) and (c), the electron and impurity densities are $n=10^{19}$ cm^{-3}, while (b) and (d) are for $n=4\times10^{18}$ cm^{-3}.

$$1/\tau_T(\Omega) = 1/\tau_L(\Omega) + 1/\tau_I(\Omega),$$

where $1/\tau_I(\Omega)$ is the relaxation rate due to dynamically screened ionized impurity induced processes, the subject of this paper, and $1/\tau_L(\Omega)$ is the relaxation rate due to coupled LO-phonon–electron-gas-mediated processes, the topic of our previous paper. In Figs. 4(a)–(d) are plotted the frequency variation of the various damping rates for several densities and temperatures. Figures (a) and (b) are for room temperature, (c) and (d) for liquid-nitrogen temperature; (a) and (c) are for electron and impurity densities $n = 10^{19}$ cm^{-3}, (b) and (d) for $n = 4 \times 10^{18}$ cm^{-3}.

It may well be that in the experimental studies of Spitzer *et al.*, the insensitivity of the absorption constant to temperature results simply becauce ionized impurity scattering rather than LO-phonon-mediated processes make the dominant contribution to $1/\tau(\Omega)$.

IV. CONCLUDING REMARKS

In the literature on free-carrier absorption in semiconductors, it has proved difficult to unambiguously determine in any particular case the mechanism that makes the dominant contribution to $1/\tau(\Omega)$. We believe this is because, as in Ref. 2, the focus is placed on high frequencies where one is in the asymptotic regime, as in Eq. (2.14). The various contributions to the absorption constant do not differ greatly in the frequency dependence expected from the simple parabolic band model ($\Omega^{-3/2}$ for LO-phonon-mediated processes and $\Omega^{-5/2}$ for ionized impurity scattering), and the issue is clouded further by the importance of band-structure effects, as Jensen has emphasized.[1,3]

We believe it much more fruitful to study $1/\tau(\Omega)$ at lower frequencies where, as one can see from the present work and from our previous paper, substantial and clear differences in the frequency variation of $1/\tau(\Omega)$ from different contributing mechanisms are expected. For the LO-phonon-mediated process, we had a broad feature for photon frequencies in the vicinity of $E_F/\hbar$, where E_F is the Fermi energy of the free carriers. This feature is broad at room temperature, and sharpens up in a characteristic manner as the temperature is lowered. For the ionized impurity scattering, a dominant feature appears just above the plasma frequency; its observation serves as a clear "marker" of the presence of ionized impurity scattering.

It is more difficult to study the free-carrier relaxation time at these lower frequencies, because one may be in the near vicinity of the restrahl absorption. In our view, however, there is a great deal to be gained from such an analysis.

We note that a recent analysis of reflectivity data for n-type PbSe by Mycielski *et al.*[11] suggests that near the plasma frequency, structure occurs similar to that in Figs. 2 and 3 of the present work. It is difficult to compare this data quantitatively with our calculations, because of the multivalley character of the conduction band in PbSe. It would be intriguing to see similar data for n-type CdTe.

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[1]A recent very complete theoretical study of a number of these processes has been presented by B. Jensen [Ann. Phys. (N.Y.) 80, 284 (1973)]. In Jensen's paper, a large number of references to the earlier literature may be found.

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[10]See, for example, A. F. Fetter, and J. D. Walecka, *Quantum Theory of Many Particle Systems* (McGraw-Hill, New York, 1971).

[11]A. Mycielski, A. Aziza, J. Mycielski, and M. Balkanski, Phys. Status Solidi B 65, 737 (1974).