PHYSICAL REVIEW B VOLUME 18, NUMBER 10 15 NOVEMBER, 1978

Nonlinear interaction of acoustic waves with microwave electric fields in piezoelectric semiconductors

John E. Economou and Harold N. Spector
Department of Physics, Illinois Institute of Technology, Chicago, Illinois 60616
(Received 20 April 1978)

We present a phenomenological approach to the nonlinear interaction between acoustic waves and microwave electric fields in piezoelectric semiconductors. This paper is concerned specifically with the ω-2ω three-wave parametric process in which the interaction of a pump (radiation) field at frequency 2ω with an acoustic wave at frequency ω generates a backward-traveling acoustic wave at frequency ω. The threshold microwave field that is required for acoustic gain is found to be higher in the presence of charge carriers than in their absence. With the application of a dc electric field, however, the sound waves can be amplified at relatively low microwave-field strengths. In fact, under certain conditions a threshold field is not required. The nature of the amplification process, however, is found to be dependent upon the polarization of the microwave field and, in particular, it is sensitive to the nonlinear contributions to the current density. Furthermore, we find that in a low-mobility semiconductor, amplification can also occur when the linear electronic gain exceeds the lattice loss.

I. INTRODUCTION

Several experiments have been performed in recent years that have been concerned with the nonlinear interaction between microwave electric fields and sound waves in crystals.[1-3] In a typical experiment, a crystal placed in a microwave cavity is subjected at $t=0$ to a pulsed microwave electric field of frequency ω that is essentially uniform over the dimensions of the crystal. As a result, a sound wave of frequency ω and wave vector q is generated within the crystal volume. At $t=\tau$, a second pulse of frequency 2ω is applied. The nonlinear interaction between the forward-traveling acoustic wave generated by the first pulse and the microwave electric field constituting the second pulse (sometimes referred to as the pump) results in the generation of a backward-traveling acoustic wave of frequency ω and wave vector $-q$ and in the possible amplification of the forward-traveling wave. The backward-traveling wave, which is commonly referred to as a phonon echo, is detected at $t=2\tau$. In quantum-mechanical terms, the interaction is that of the conversion of a photon into two oppositely traveling phonons in the presence of a phonon population (stimulated emission of phonons).[4] A variation of the above experiment involves applying the two pulses at the same frequency. Experiments consisting of a single pulse or of a sequence of three pulses have also been studied and yield interesting results. These experiments have shown that the echoes can be generated either parametrically, where the coupling of the fields arises via the parameters of the solid, or by exciting a static, space-charge hologram.

The study of such interactions is of importance for a number of reasons.[1-3] By observing the decrease in the amplitude of the echo in a two-pulse experiment as the time τ between pulses is increased, one can obtain information about the inelastic scattering of phonons from crystal impurities and thermal phonons. Furthermore, the three-pulse experiments can give information about inelastic scattering processes in a crystal and it has been suggested that such experiments should be especially useful at low temperatures, where ultrasonic attenuation experiments are ineffective. Finally, the phenomenon can be of importance for the generation and amplification of acoustic waves and for the processing of signals in engineering applications (e.g., delay lines and the convolution or correlation of signals).

The theoretical work up to the present time has been concerned primarily with a description of the interaction in insulators,[5] although Shiren has discussed the acoustically induced charge transfer process.[3] The purpose of this paper is to provide a treatment of the parametric process in piezoelectric semiconductors. We shall consider in particular the ω-2ω three-wave interaction described above. With the aid of a specific solution to a set of coupled-mode equations, the condition for net amplification of the forward-traveling acoustic wave and overall growth of the echo wave with time is determined. This condition specifies that acoustic gain is possible only above a certain value of the microwave field, which we refer to as the threshold field. We find that in the absence of a dc electric field, the effect of the linear and nonlinear conductivities is to increase slightly the value of the threshold microwave field over its value in the absence of carriers. With the application of a dc electric field, however, the sound waves can be amplified at relatively low microwave field strengths and, in some cases, a threshold field is not required at all. For this latter situation, the microwave field is needed only to couple the forward- and backward-traveling acoustic waves. Thus, under certain conditions, the presence of both the dc field and the microwave field will tend to enhance the growth of the waves. The interesting feature of our solutions is that the oppositely traveling acoustic waves satisfy the same amplification condition; that is, the asymmetry introduced by the dc field is not present in the gain coefficient. It is found, however, that the gain coefficient is dependent upon the polarization of the microwave field.

II. THEORY

A necessary condition for the observation of the phonon echo phenomenon is the existence of nonlinearities in the medium. We shall assume that the nonlinear interactions that occur among the sound waves and the electric fields in the solid arise from the anharmonicities in the interatomic forces. The anharmonic terms serve to couple the microwave field to the acoustical waves. Our approach will be classical and phenomenological. The crystal will be considered as a continuous medium, which places a restriction on the ultrasonic frequencies that can be treated. Specifically, the phonon wavelength is required to be much greater than a lattice spacing. We shall work in terms of the free energy F which we expand as a function of the electric field $\vec{E}$ and the strain tensor S_{ij} up to terms of the third order in the product of these quantities

$$F = F_0 + a_i E_i + b_{ij} S_{ij} + \frac{1}{2!} e_{ij} E_i E_j - \beta_{ijk} E_i S_{jk} + \frac{1}{2!} C_{ijkl} S_{ij} S_{kl} + \frac{1}{3!} d_{ijk} E_i E_j E_k + \tfrac{1}{2} h_{ijkl} E_i E_j S_{kl} + \tfrac{1}{2}\gamma_{ijklm} E_i S_{jk} S_{lm} + \frac{1}{3!}\lambda_{ijklmn} S_{ij} S_{kl} S_{mn} . \tag{2.1}$$

The stress tensor T_{ij} and the electric induction $\vec{D}$ can be obtained from the free energy by means of the following expressions[6]:

$$T_{ij} = \frac{\partial F}{\partial S_{ij}} + \frac{1}{8\pi}(E_i D_j + E_j D_i) \tag{2.2a}$$

and

$$\vec{D} = -4\pi \frac{\partial F}{\partial \vec{E}} . \tag{2.2b}$$

We shall assume that the coefficients have the following symmetry properties:

$$\begin{aligned} &e_{ij} = e_{(ij)}, \qquad d_{ijk} = d_{(ijk)}, \\ &\beta_{ijk} = \beta_{i(jk)}, \qquad h_{ijkl} = h_{(ij)(kl)}, \\ &C_{ijkl} = C_{(ij)(kl)} \quad \gamma_{ijklm} = \gamma_{i(jk)(lm)} \\ &\quad = C_{klij}, \qquad\quad = \gamma_{ilmjk}, \\ &\lambda_{ijklmn} = \lambda_{(ij)(kl)(mn)} \\ &\quad = \lambda_{klijmn} \cdots , \end{aligned} \tag{2.3}$$

where ( ) denotes taking the symmetric part of the tensor. Expressions (2.2a) and (2.2b) then yield

$$D_i = \epsilon'_{ij} E_j + 4\pi \beta'_{ijk} S_{jk} \tag{2.4}$$

and

$$T_{rs} = C'_{rskl} S_{kl} - \beta'_{irs} E_i + \frac{1}{8\pi}(\epsilon_{sj} E_r + \epsilon_{rj} E_s) E_j + \tfrac{1}{2}(\beta_{sjk} E_r + \beta_{rjk} E_s) S_{jk} , \tag{2.5}$$

where

$$\epsilon'_{ij} \equiv \epsilon_{ij} - 2\pi d_{ijk} E_k - 2\pi h_{ijkl} S_{kl} , \tag{2.6a}$$

$$\beta'_{ijk} \equiv \beta_{ijk} - \tfrac{1}{2} h_{iljk} E_l - \tfrac{1}{2}\gamma_{ijklm} S_{lm} , \tag{2.6b}$$

$$C'_{rskl} \equiv C_{rskl} + \tfrac{1}{2}\gamma_{irskl} E_i + \tfrac{1}{2}\lambda_{rsklmn} S_{mn} , \tag{2.6c}$$

and where we have taken $\epsilon_{ij} = -4\pi e_{ij}$ and $a_i = 0 = b_{ij}$, since we are not interested in materials that are polarized or stressed in the absence of the fields and strains. The quantities ϵ'_{ij}, β'_{ijk}, and C'_{rskl} represent, respectively, the effective dielectric, piezoelectric, and elastic constants of the crystal. It is seen, therefore, that in the presence of a sound wave and an electric field the equilibrium properties of the solid are modified; in other words, the crystal parameters are modulated by the disturbances. From the above expressions we can say something about the physical significance of the nonlinear coefficients. The terms in D_i involving d_{ijk}, h_{ijkl}, and γ_{ijklm} represent contributions to the nonlinear polarization, while the tensors λ_{ijklmn} and γ_{ijklm} represent the higher-order contributions to the elastic constants of the crystal. At optical frequencies, the coefficients d_{ijk} are related to the quantities describing the electro-optical or nonlinear optical effects, while the coefficients h_{ijkl} correspond to the quantities describing the photoelastic (elasto-optical) or electrostrictive effects. The piezo-optical coefficients can also be related to the h_{ijkl}.

The nonlinear interaction between a microwave electric field and an acoustic wave in a piezoelectric semiconductor can be described theoretically by Maxwell's equations, the equation of motion of an elastic continuum, an expression for the induced electronic current density $\vec{J}$, and expressions (2.4) and (2.5) for the electric induction $\vec{D}$ and the stress tensor T_{ij} (we are assuming that the material is nonmagnetic so that there is no net magnetization; that is, $\vec{H}=\vec{B}$). In particular, we must satisfy the equations

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{H}}{\partial t}\,,\quad \vec{\nabla}\times\vec{H}=\frac{1}{c}\frac{\partial\vec{D}}{\partial t}+\frac{4\pi}{c}\vec{J}\,, \qquad (2.7)$$

$$\vec{\nabla}\cdot\vec{H}=0\,,\quad \vec{\nabla}\cdot\vec{D}=-4\pi e(n-n_0)\,,$$

and

$$\rho\frac{\partial^2\xi_i}{\partial t^2}=\frac{\partial T_{ij}}{\partial X_j}\,, \qquad (2.8)$$

where n is the electron density, n_0 is the equilibrium carrier density, ρ is the density of the material, and $\vec{\xi}$ is the lattice displacement vector. In addition, there is the equation of continuity

$$\vec{\nabla}\cdot\vec{J}-e\frac{\partial n}{\partial t}=0\,, \qquad (2.9)$$

which derives from Maxwell's equations.

For the induced electronic current density, we take a phenomenological approach which assumes a local relationship between the electronic current and the fields and gradients

$$\vec{J}=ne\mu\vec{E}+eD\vec{\nabla}n\,, \qquad (2.10)$$

where μ is the mobility and D is the diffusion constant, which is related to μ by the Einstein relation $D=(\mu K_B T/e)$, where T is the temperature. This assumption is valid for low-mobility semiconductors where the electron mean free path is much less than a phonon wavelength ($ql\ll 1$). This condition implies that an electron undergoes many collisions over the distance of an acoustic wavelength, so that its motion under the influence of the external field is averaged out.

For the purposes of this paper, we shall concentrate on the description of a two-pulse experiment. We note that because of the nature of this type of experiment, the forward-traveling waves generated by the second pulse will not in any way contribute to the interaction that results in the formation of the echo. It is the nonlinear interaction between the microwave electric field and the fields induced by the first pulse which gives rise to the echo. The electric field and lattice displacement induced by the first pulse are represented by

$$\vec{E}_1=\vec{\mathscr{E}}_1(\vec{r},t)\exp[i(\vec{q}\cdot\vec{r}-\omega t)]+\text{c.c.}\,, \qquad (2.11a)$$

$$\vec{\xi}_1=\vec{u}_1(\vec{r},t)\exp[i(\vec{q}\cdot\vec{r}-\omega t)]+\text{c.c.}\,, \qquad (2.11b)$$

while the disturbances associated with the echo wave are written

$$\vec{E}_e=\vec{\mathscr{E}}_e(\vec{r},t)\exp[-i(\vec{q}\cdot\vec{r}+\omega t)]+\text{c.c.}\,, \qquad (2.11c)$$

$$\vec{\xi}_e=\vec{u}_e(\vec{r},t)\exp[-i(\vec{q}\cdot\vec{r}+\omega t)]+\text{c.c.}\,. \qquad (2.11d)$$

The strains S_{ij1} and S_{ije} accompanying the sound waves can be obtained from $\vec{\xi}_1$ and $\vec{\xi}_e$ by means of the relation $S_{ij}=\frac{1}{2}(\partial\xi_i/\partial X_j+\partial\xi_j/\partial X_i)$. The microwave electric field is taken to be given by the expression

$$\vec{E}_M=\vec{\mathscr{E}}_{M0}e^{-2i\omega t}+\text{c.c.}\,, \qquad (2.11e)$$

where $\vec{\mathscr{E}}_{M0}$ is a constant. We have neglected the spatial variation of this field, because it is essentially uniform over the dimensions of the interaction volume. Furthermore, by assuming $\vec{\mathscr{E}}_{M0}$ to be a constant, we are not taking into account the depletion of the pump field. This, however, is a higher-order effect, which we need not consider at the present time.

It is known from previous work that the transverse electric fields induced by a sound wave in a piezoelectric material are negligible compared to the longitudinal electric fields.[7] Therefore, as long as the amplitude of the microwave field is assumed to be a constant, we need only consider Gauss's law and the equation of continuity. However, since the microwave field is a radiation field, the full set of Maxwell's equations must be used when the amplitude of this field is allowed to vary.

The total electric field in the crystal is given by

$$\vec{E}=\vec{E}_1+\vec{E}_e+\vec{E}_M+\vec{E}_{\text{dc}}\,, \qquad (2.12)$$

where we have included the contribution from a dc electric field. We can expand $\vec{J}$ and n in a similar manner. If we take the $\hat{z}$ direction as the direction of propagation, then according to the discussion in the previous paragraph $\vec{E}_1=\hat{z}E_1$ and $\vec{E}_e=\hat{z}E_e$. We also require the dc field to lie along the $-\hat{z}$ direction. If the microwave field is assumed to be polarized along the $\hat{z}$ direction, then substitution into Eq. (2.10) and use of Eq. (2.9) gives

$$j_{1z}=\sigma_1\mathscr{E}_{1z}+2\Lambda_1\mathscr{E}_{ez}\mathscr{E}_{Mz}\,, \qquad (2.13a)$$

$$j_{ez}=\sigma_e\mathscr{E}_{ez}+2\Lambda_e\mathscr{E}^*_{1z}\mathscr{E}_{Mz}\,, \qquad (2.13b)$$

$$j_{Mz}=\sigma_0\mathscr{E}_{Mz}+2\Lambda_M\mathscr{E}_{1z}\mathscr{E}_{ez}\,, \qquad (2.13c)$$

where

$$\sigma_1=\sigma_{zz}(\omega_1)=\frac{\sigma_0}{(1-v_d/v_s+i\omega/\omega_D)}\,, \qquad (2.14a)$$

$$\sigma_e=\sigma_{zz}(\omega_e)=\frac{\sigma_0}{(1+v_d/v_s+i\omega/\omega_D)}\,, \qquad (2.14b)$$

and

$$\Lambda_1 = \Lambda_{zzz}(-\omega_e, \omega_M) = \frac{\sigma_0\mu/2v_s}{(1 - v_d/v_s + i\omega/\omega_D)(1 + v_d/v_s - i\omega/\omega_D)} = -\Lambda_e^* \tag{2.15a}$$

$$\Lambda_M = \Lambda_{zzz}(\omega_1, \omega_e) = \frac{-(\sigma_0\mu/v_s)(v_d/v_s)}{(1 - v_d/v_s + i\omega/\omega_D)(1 + v_d/v_s + i\omega/\omega_D)} . \tag{2.15b}$$

In these expressions, $\sigma_0 = n_0 e\mu$ is the dc conductivity, $v_d = \mu E_{\rm dc}$ is the drift velocity of the electrons, v_s is the sound velocity, and $\omega_D = v_s^2/D$ is the diffusion frequency. On the other hand, the polarization $\vec{E}_M = \hat{y}E_M$ gives

$$j_{1z} = \sigma_1 \mathcal{E}_{1z}, \quad j_{1y} = 2\Lambda_1' \mathcal{E}_{ez}^* \mathcal{E}_{My}, \tag{2.16a}$$

$$j_{ez} = \sigma_e \mathcal{E}_{ez}, \quad j_{ey} = 2\Lambda_e' \mathcal{E}_{1z}^* \mathcal{E}_{My}, \tag{2.16b}$$

$$j_{Mz} = 2\Lambda_M \mathcal{E}_{1z} \mathcal{E}_{ez}, \quad j_{My} = \sigma_0 \mathcal{E}_{My}, \tag{2.16c}$$

where σ_1, σ_e, and Λ_M are the same as before and

$$\Lambda_1' = \Lambda_{yzy}(-\omega_e, \omega_M) = \frac{\sigma_0\mu/2v_s}{1 + v_d/v_s - i\omega/\omega_D} , \tag{2.17a}$$

$$\Lambda_e' = \Lambda_{yzy}(-\omega_1, \omega_M) = \frac{\sigma_0\mu/2v_s}{1 - v_d/v_s - i\omega/\omega_D} . \tag{2.17b}$$

From Gauss's law and the equation of continuity, we can derive the equation

$$\frac{\partial D_z}{\partial t} + 4\pi J_z = 0 . \tag{2.18}$$

We shall take a perturbative approach and write the induced electric fields as the first-order fields plus second-order corrections due to the interactions between waves

$$\mathcal{E}_1 = \mathcal{E}_1^{(1)} + \mathcal{E}_1^{(2)}, \quad \mathcal{E}_e = \mathcal{E}_e^{(1)} + \mathcal{E}_e^{(2)} . \tag{2.19}$$

We also assume that the amplitudes of the waves are slowly varying functions of space and time over the distance of a wavelength and the interval of a period; that is,

$$\frac{\partial \mathcal{E}}{\partial z} \ll iq\mathcal{E}, \quad \frac{\partial \mathcal{E}}{\partial t} \ll i\omega\mathcal{E} . \tag{2.20}$$

Making use of these relations in expressions (2.4), (2.10), and (2.18) and taking $\vec{E}_M = \hat{z}E_M$, we find that the first-order fields are given by

$$\mathcal{E}_1^{(1)} = -\frac{4\pi}{\epsilon}\frac{1}{\Gamma_1}(\beta + hE_{\rm dc})\left(iqu_1 + \frac{\partial u_1}{\partial z}\right), \tag{2.21a}$$

$$\mathcal{E}_e^{(1)} = -\frac{4\pi}{\epsilon}\frac{1}{\Gamma_e}(\beta + hE_{\rm dc})\left(-iqu_e + \frac{\partial u_e}{\partial z}\right), \tag{2.21b}$$

where

$$\Gamma_1 = 1 + \frac{4\pi d}{\epsilon} E_{\rm dc} + i\frac{4\pi\sigma_1}{\epsilon\omega} , \tag{2.22a}$$

$$\Gamma_e = 1 + \frac{4\pi d}{\epsilon} E_{\rm dc} + i\frac{4\pi\sigma_e}{\epsilon\omega} , \tag{2.22b}$$

while the second-order corrections are

$$\mathcal{E}_1^{(2)} = -i\frac{4\pi}{\epsilon\omega}\frac{\delta_1}{\Gamma_1\Gamma_e^*}\mathcal{E}_{M0}\left(iqu_e^* + \frac{\partial u_e^*}{\partial z}\right), \tag{2.23a}$$

$$\mathcal{E}_e^{(2)} = -i\frac{4\pi}{\epsilon\omega}\frac{\delta_e}{\Gamma_1^*\Gamma_e}\mathcal{E}_{M0}\left(-iqu_1^* + \frac{\partial u_1^*}{\partial z}\right), \tag{2.23b}$$

where

$$\delta_1 = -\frac{8\pi}{\epsilon}(\beta + hE_{\rm dc})\Lambda_1 + i\omega h\Gamma_e^* - i\omega\frac{4\pi d}{\epsilon}(\beta + hE_{\rm dc}) , \tag{2.24a}$$

$$\delta_e = -\frac{8\pi}{\epsilon}(\beta + hE_{\rm dc})\Lambda_e + i\omega h\Gamma_1^* - i\omega\frac{4\pi d}{\epsilon}(\beta + hE_{\rm dc}) . \tag{2.24b}$$

In the above expressions, we have set $\epsilon = \epsilon_{ZZ}$, $d = d_{ZZZ}$, $h = h_{ZZZZ}$, and $\beta = \beta_{ZZZ}$. From these expressions, it is seen that in the presence of the dc field the term $h_{ijkz}E_{\rm dc}$ acts as an effective piezoelectric tensor and consequently the induced fields remain nonvanishing even in a nonpiezoelectric material ($\beta = 0$). It is also interesting to note that although the first-order fields vanish in a nonpiezoelectric material when $E_{\rm dc} = 0$, the second-order fields need not vanish. This strictly nonlinear effect arises from the fact that the tensor h_{ijkl} serves to couple the microwave field to the acoustic waves.

The lattice equation of motion can be written

$$\rho\frac{\partial^2 \xi_r}{\partial t^2} = \frac{\partial}{\partial z}\left(-\beta_{jrz}E_j + C_{rzjk}S_{jk} + \tfrac{1}{2}h_{jkrz}E_jE_k + \frac{1}{8\pi}\epsilon_{zj}E_rE_j + \frac{1}{8\pi}\epsilon_{rj}E_zE_j + \gamma_{irzkl}E_iS_{kl} + \tfrac{1}{2}\beta_{zjk}E_rS_{jk} + \tfrac{1}{2}\beta_{rjk}E_zS_{jk} + \tfrac{1}{2}\lambda_{rzklmn}S_{kl}S_{mn}\right). \tag{2.25}$$

Making use of this expression and expressions (2.21a), (2.21b), (2.23a) and (2.23b), we find that the amplitudes u_1 and u_e must satisfy the following set of coupled equations:

$$\Delta_1\frac{\partial u_1}{\partial z} + \left(\frac{\rho}{C}\frac{\omega}{q}\right)\frac{\partial u_1}{\partial t} = (\alpha_1 - \alpha_l)u_1 - \eta_1\mathcal{E}_{M0}u_e^* , \tag{2.26a}$$

$$\Delta_e\frac{\partial u_e}{\partial z} - \left(\frac{\rho}{C}\frac{\omega}{q}\right)\frac{\partial u_e}{\partial t} = -(\alpha_e - \alpha_l)u_e + \eta_e\mathcal{E}_{M0}u_1^* , \tag{2.26b}$$

where

$$\alpha_1 = \frac{2\pi q}{\epsilon C}(\beta + hE_{dc})\left[\beta + \left(h + \frac{\epsilon}{2\pi}\right)E_{dc}\right]\mathrm{Im}\left(\frac{1}{\Gamma_1}\right), \tag{2.27}$$

$$\eta_1 = -\frac{2\pi\beta}{\epsilon C}\frac{1}{\Gamma_1\Gamma_e^*}\left\{\delta_1\left(\frac{q}{\omega}\right)\left[1 + \frac{1}{\beta}\left(h + \frac{\epsilon}{2\pi}\right)E_{dc}\right] + \frac{iq}{\beta}\Gamma_1\left(h + \frac{\epsilon}{2\pi}\right)(\beta + hE_{dc}) - \frac{iq\epsilon}{4\pi\beta}\Gamma_1\Gamma_e^*(\gamma+\beta)\right\}, \tag{2.28}$$

and

$$\Delta_1 = 1 - \frac{\gamma+\beta}{C}E_{dc} + \frac{4\pi}{\epsilon C}\frac{1}{\Gamma_1}(\beta + hE_{dc}) \times \left[\beta + \left(h + \frac{\epsilon}{2\pi}\right)E_{dc}\right]. \tag{2.29}$$

The interchange $1 \leftrightarrow e$ will give the expressions for α_e, η_e, and Δ_e. We also find that the sound waves satisfy the following dispersion relations:

$$\frac{\omega_1^2}{q_1^2} = \left(\frac{C}{\rho}\right)\left\{1 - \frac{\gamma+\beta}{C}E_{dc} + \frac{4\pi}{\epsilon C}(\beta + hE_{dc}) \times \left[\beta + \left(h + \frac{\epsilon}{2\pi}\right)E_{dc}\right]\mathrm{Re}\left(\frac{1}{\Gamma_1}\right)\right\} \tag{2.30a}$$

$$\frac{\omega_e^2}{q_e^2} = \left(\frac{C}{\rho}\right)\left\{1 - \frac{\gamma+\beta}{C}E_{dc} + \frac{4\pi}{\epsilon C}(\beta + hE_{dc}) \times \left[\beta + \left(h + \frac{\epsilon}{2\pi}\right)\cdot E_{dc}\right]\mathrm{Re}\left(\frac{1}{\Gamma_e}\right)\right\}. \tag{2.30b}$$

In deriving the above expressions, we have assumed that the dielectric tensor is diagonal and we have taken $C = C_{zzzz}$ and $\gamma = \gamma_{zzzzz}$. These relations show that the corrections to the wave vectors of the forward-traveling wave and the echo wave differ in the presence of a dc electric field. In Sec. III, however, we shall find that the electromechanical coupling constant $4\pi\beta^2/\epsilon C$ is small compared to unity and that $\mathrm{Re}(1/\Gamma_1) \lesssim 1$. Furthermore, for reasonable dc field strengths in nonferroelectric materials, it turns out that $hE_{dc} \ll \beta$ and $(\gamma+\beta)E_{dc} \ll C$. (Preliminary calculations indicate that $(\gamma+\beta)E_{dc}$ can become comparable in magnitude to C in certain ferroelectric materials, thus giving rise to the situation where $\omega \sim 0$ for finite values of the wave vector q. We intend to examine this possibility in greater detail in a forthcoming paper.). Thus to a good approximation

$$\omega^2/q^2 \approx C/\rho = v_s^2, \tag{2.31}$$

where v_s denotes the velocity of sound in the absence of piezoelectric coupling. Equations (2.26a) and (2.26b) can now be written in the form

$$\Delta_1\frac{\partial u_1}{\partial z} + \frac{1}{v_s}\frac{\partial u_1}{\partial t} = (\alpha_1 - \alpha_l)u_1 - \eta_1\mathcal{E}_{M0}u_e^*, \tag{2.32a}$$

$$\Delta_e\frac{\partial u_e}{\partial z} - \frac{1}{v_s}\frac{\partial u_e}{\partial t} = -(\alpha_e - \alpha_l)u_e + \eta_e\mathcal{E}_{M0}u_1^*. \tag{2.32b}$$

The terms involving α_l in these coupled-mode equations have been introduced in order to take lattice losses into account.[8] These lattice losses arise from phonon-phonon interactions, that is, from the interactions between the induced sound waves and the thermal phonons. Such interactions can be important at the frequencies with which we shall concern ourselves. These losses are expected to be dependent on the frequency ω of the ultrasonic wave, but independent of the carrier concentration. An explicit frequency dependence for α_l will be assumed at a later stage in this paper.

The above expressions have been derived for the polarization $\vec{E}_M = \hat{z}E_M$. If we require that $\vec{E}_M$ be polarized along the $\hat{y}$ direction, on the other hand, and if we neglect the tensor character of the nonlinear coefficients, then we find that the relevant expressions can be obtained from our previous results by setting Λ_1 and Λ_e equal to zero and by replacing $h + \epsilon/2\pi$ and $\gamma+\beta$ with h and γ, respectively. The interesting point to note is that the nonlinear conductivity is not present for this polarization.

For the phonon echo problem, the boundary conditions in the time domain are well specified, whereas the spatial boundary conditions are more difficult to formulate. Hence, in order to keep our discussion as simple as possible, we shall concentrate on the situation where the amplitudes of the waves are only dependent on the time. For $t < \tau$, only the sound wave generated by the first pulse is present in the crystal; this wave is described by the solution to the equation

$$\frac{\partial u_1}{\partial t} = (\alpha_1 - \alpha_l)v_s u_1. \tag{2.33}$$

For $t > \tau$, both the echo and the initial sound wave are present, and one must obtain the solution to the coupled-mode equations (2.32a) and (2.32b). The solutions in the two time domains are then assumed to join continuously at $t = \tau$. The results of this calculation are given by

$$u_1(t) = u_{10}\exp[-v_s(\alpha_l - \alpha_1)t] \times \exp[\tfrac{1}{2}v_s(\alpha_e - \alpha_1)(t-\tau)] \times\{\cosh[\tfrac{1}{2}v_s N(t-\tau)] + \frac{\alpha_1 - \alpha_e}{N}\sinh[\tfrac{1}{2}v_s N(t-\tau)]\}, \tag{2.34a}$$

and

$$u_e(t) = -\frac{2\eta_e \mathcal{E}_{M0}}{N^*} u_{10}^* \exp[-v_s(\alpha_l - \alpha_1)t] \times \exp[\tfrac{1}{2}v_s(\alpha_e - \alpha_1)(t-\tau)] \sinh[\tfrac{1}{2}v_s N(t-\tau)] , \tag{2.34b}$$

where

$$N = [(\alpha_1 - \alpha_e)^2 + 4\eta_1\eta_e^* |\mathcal{E}_{M0}|^2]^{1/2} . \tag{2.34c}$$

From the above expressions, one finds that the condition for net amplification of the forward-traveling wave and overall growth of the echo wave with time is given by

$$|\mathrm{Re}N| > 2\alpha_l - (\alpha_e + \alpha_1) . \tag{2.35}$$

Let $\alpha_T \equiv 2\alpha_l - (\alpha_e + \alpha_1)$. For $\alpha_T < 0$, the above condition is immediately satisfied. We shall find in Sec. III that such a situation can arise in a low-mobility semiconductor such as CdS. For $\alpha_T > 0$, on the other hand, which is the situation for a material like GaAs, we can define a gain ratio R

$$R = |\mathrm{Re}N| / \alpha_T , \tag{2.36}$$

where values of $R > 1$ correspond to acoustical gain. Numerical estimates of the magnitude of this quantity will be presented in Sec. III. The interesting thing to note is that when $R > 1$ both the forward-traveling wave and the echo wave are amplified. This is a consequence of the coupled nature of our solutions. It contrasts with the situation in the absence of the microwave field, where the attenuation (amplification) of a sound wave depends upon its direction of propagation in the dc field.

The threshold microwave field $|\mathcal{E}_{M0}|_{\mathrm{th}}$ satisfies the condition $|\mathrm{Re}N| = \alpha_T$. Solving for $|\mathcal{E}_{M0}|_{\mathrm{th}}$, one obtains

$$|\mathcal{E}_{M0}|^2_{\mathrm{th}} = \tfrac{1}{2}\alpha_T^2 (\mathrm{Im}\eta_1\eta_e^*)^{-2} \times \left\{ -\mathrm{Re}\eta_1\eta_e^* + \left[|\eta_1\eta_e^*|^2 - \left(\frac{\alpha_1 - \alpha_e}{\alpha_T}\right)^2 \times (\mathrm{Im}\eta_1\eta_e^*)^2 \right]^{1/2} \right\}.$$

It is evident that no threshold field is required for $(\alpha_1 - \alpha_e)^2 > \alpha_T^2$. Although this situation cannot arise in the absence of a dc field, where $\alpha_1 = \alpha_e$, we shall find that the condition can be satisfied for $E_{\mathrm{dc}} \neq 0$.

III. NUMERICAL RESULTS

We shall now determine the magnitude of the gain ratio R for various values of the frequency ω, the carrier concentration n_0, and the dc electric field E_{dc}. The calculation will be performed for the parameters characteristic of n-type GaAs and n-type CdS. We must somehow estimate the values of the nonlinear coefficients d, h, and γ, because these parameters have not yet been measured at microwave frequencies in these materials. This can be accomplished by following the procedure of McMahon[9] and of Thompson and Quate[1] which makes use of an extension of Miller's rule. According to this rule, it is assumed that when the free energy is expressed as a function of the electric polarization rather than the electric field, then the nonlinear coefficients which result are relatively insensitive to changes in the frequency and show little variation from one material to another. As was mentioned previously, the coefficients d and h are related to the electro-optical and photoelastic coefficients at optical frequencies. Furthermore, the nonlinear elastic constants for quartz have been determined.[10] Hence, by using Miller's rule, we can obtain estimates for d and h from tabulated values of the electro-optical and photoelastic coefficients and an estimate for γ from the data on quartz. Expressions for the linear and nonlinear conductivity tensors can be obtained from the results of Sec. II, while values for α_l can be estimated from the data of Palik and Bray.[8] It therefore becomes possible to estimate the magnitude of the gain ratio. We must stress, however, that our treatment neglects the tensor character of the coefficients. Therefore, our numerical results can only be taken to yield order-of-magnitude estimates. Nevertheless, the calculation should serve to give us some indication of the relative importance of the various nonlinear coefficients.

Following McMahon, we find that d and h are given by the expressions

$$d = \frac{1}{2\pi}(\epsilon_M - 1)^2 \left(\frac{n^2}{n^2-1}\right)^2 r , \tag{3.1a}$$

$$h = \frac{1}{4\pi}(\epsilon_M - 1)^2 \left(\frac{n^2}{n^2-1}\right)^2 p , \tag{3.1b}$$

where r is the electro-optical coefficient, p is the photoelastic coefficient, n is the index of refraction at optical frequencies, and ϵ_M is the dielectric constant at microwave frequencies (the static dielectric constant). For GaAs, some relevant parameters are

$$n = 3.34 , \quad \epsilon_M = 12 , \quad \rho = 5.34 \text{ g/cm}^3 , \quad \mu = 8000 \text{ cm}^2/\text{V sec} , \quad m = 0.072\, m_0 = 6.56 \times 10^{-29} \text{ g} , \tag{3.2}$$

where m is the effective mass of the electrons and μ is their mobility. From data on the electro-optical and photoeleastic effects, we find that[11]

$$r = 1.6 \times 10^{-12}\ \mathrm{m/V}\ , \quad p = 2.25 \times 10^{-1}\ , \tag{3.3}$$

where the value for p corresponds to longitudinal waves traveling with a velocity of 5.15×10^5 cm/sec. Expressions (3.1a) and (3.1b) can now be used to obtain

$$d = 1.12 \times 10^{-6}\ \mathrm{cm^2/esu} \tag{3.4}$$

and

$$h = 2.61\ . \tag{3.5}$$

Let us now determine a value for γ. We can consider expression (2.6c) as a series expansion in the field quantities E and S. Then the expansion coefficient γ is given by

$$\gamma = 2\frac{\partial C'}{\partial E}\ . \tag{3.6}$$

We can express this in terms of the polarization P as

$$\gamma = 2\frac{\partial C'}{\partial P}\,\chi\ , \tag{3.7}$$

where $\chi = \epsilon_M - 1$ is the electric susceptibility at microwave frequencies. Using Miller's rule, we can write

$$\frac{\partial C'}{\partial P} = \left(\frac{\partial C'}{\partial P}\right)_Q = \left(\frac{\partial C'}{\partial E}\right)_Q \frac{1}{\chi_Q}\ ,$$

in order to obtain

$$\gamma = 2\left(\frac{\partial C'}{\partial E}\right)_Q \frac{\chi}{\chi_Q}\ , \tag{3.8}$$

where the subscript refers to quartz. Following Thompson and Quate, we shall take

$$\left(\frac{\partial C'}{\partial E}\right)_Q = 4\mathrm{N/V\,m}\ , \tag{3.9a}$$

$$\chi_Q = 3.5\ . \tag{3.9b}$$

It is then found that

$$\gamma = 7.54 \times 10^6\ \mathrm{esu/cm^2}\ . \tag{3.10}$$

We also need an estimate of the piezoelectric coefficient β. We shall assume that the electromechanical coupling constant has the value[12]

$$4\pi\beta^2/\epsilon C = 3.7 \times 10^{-3}\ . \tag{3.11}$$

Since $C = \rho v_s^2$, this gives

$$\beta = 7.07 \times 10^4\ \mathrm{esu/cm^2}\ . \tag{3.12}$$

The relevant parameters have been listed in Table I, where we have also included the parameters for CdS. We see that for reasonable dc field strengths

$$(4\pi d/\epsilon)E_{\mathrm{dc}} \ll 1\ , \quad hE_{\mathrm{dc}} \ll \beta\ , \quad (\gamma+\beta)E_{\mathrm{dc}} \ll C\ , \tag{3.13}$$

which greatly simplifies the problem. By neglecting these terms, we find that only the conductivity tensors are modified by the dc electric field; that is, the dc field has no effect in insulating GaAs or CdS. This is to be expected in a linear theory,

TABLE I. Physical parameters for n-GaAs and n-CdS.

Parameter	n-GaAs (77 K)	n-CdS (77 K)
$d(\mathrm{cm^2 esu^{-1}})$	1.12×10^{-6}	3.80×10^{-6}
h	2.61	1.32
$\gamma(\mathrm{esu\,cm^{-2}})$	7.54×10^6	6.17×10^6
$\beta(\mathrm{esu\,cm^{-2}})$	7.07×10^4	1.68×10^5
ϵ	12	10
$\rho(\mathrm{g\,cm^{-3}})$	5.34	4.82
$\mu(\mathrm{cm^2\,V^{-1}\,sec^{-1}})$	8000	300
m	$0.072m_0$	$0.20m_0$
$v_s(\mathrm{cm\,sec^{-1}})$	5.15×10^5	4.30×10^5
$v_0(\mathrm{cm\,sec^{-1}})$	1.80×10^7	1.08×10^7

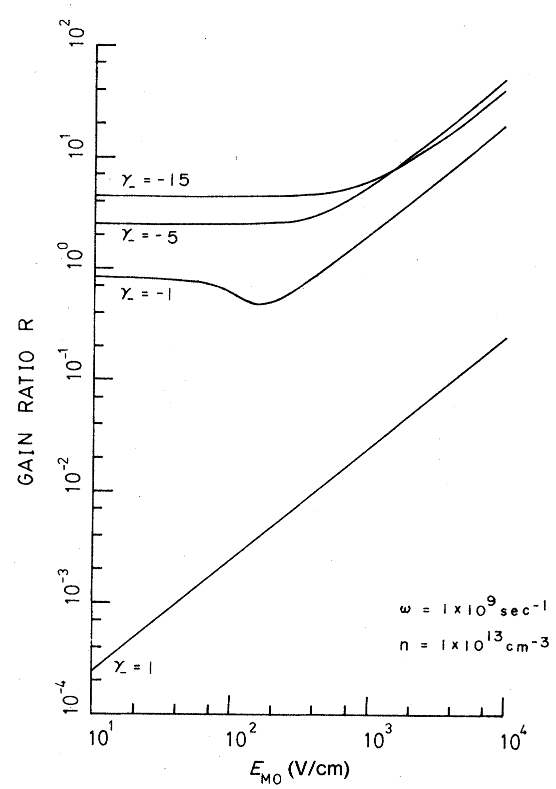

FIG. 1. Gain ratio R shown as a function of the microwave electric field $\mathcal{E}_{M0}$ for various values of the dc electric field and for $\omega = 1 \times 10^9$ rad/sec and $n = 1 \times 10^{13}$ cm^{-3}. The parameters are those characteristic of n-type GaAs. For this figure, the microwave field is polarized along the direction of propagation of the acoustic waves: $\vec{\mathrm{E}}_M = \hat{z}\,E_M$.

where the dc field affects the motion of the electrons and therefore the conductivity. In a nonlinear theory, however, the dc field can be coupled to the acoustic waves even in the absence of carriers, thereby modifying the equilibrium properties of the solid. The question naturally arises, therefore, as to whether there are materials for which the above-mentioned nonlinear terms are not negligible. The expressions given above for d, h, and γ indicate that such a situation should exist in materials with large static dielectric constants, such as ferroelectrics, and preliminary calculations using the parameters characteristic of $BaTiO_3$ have verified this. In insulating $BaTiO_3$, for example, the electric field dependence of the dielectric constant can be appreciable. Additional complications arise in such materials, however, and consequently we shall deal with them in a forthcoming paper.

By making the above approximations, we find that Γ_1 can be written

$$\Gamma_1 = \frac{\gamma_- + i(\omega_\epsilon/\omega + \omega/\omega_D)}{\gamma_- + i\omega/\omega_D}, \tag{3.14}$$

where $\omega_\epsilon = 4\pi\sigma_0/\epsilon$ is the dielectric relaxation frequency and $\gamma_- = 1 - v_d/v_s$. It follows that one can write

$$\alpha_1 = \frac{-(2\pi\beta^2/\epsilon C)(\omega_\epsilon/v_s)\gamma_-}{\gamma_-^2 + (\omega_\epsilon/\omega + \omega/\omega_D)^2}. \tag{3.15}$$

In the linear theory, the electric field dependence of the sign of α_1 leads to the possibility of changing an acoustic loss into an acoustic gain. The expressions for Γ_e and α_e are obtained by replacing γ_- with $\gamma_+ = 1 + v_d/v_s$. For future reference, we note that the maximum of α_1 with respect to γ_- occurs at a value of $\gamma_M = -(\omega_\epsilon/\omega + \omega/\omega_D)$, while the frequency of maximum gain is given by $\omega_M = (\omega_\epsilon\omega_D)^{1/2}$. We also note that for GaAs, $\omega_\epsilon = 1.21 \times 10^{-3} n_0$, while for CdS, $\omega_\epsilon = 5.43 \times 10^{-5} n_0$.

The calculations were performed for carrier concentrations in the range of $10^{13}-10^{16}$ cm^{-3}. If one makes a rough estimate of the electronic mean free path in GaAs at a temperature of 77 °K, it is found that $l \sim 6 \times 10^{-6}$ cm. This restricts our calculations to frequencies ω in the range of values $< 7 \times 10^{10}$ sec^{-1}. Values of α_l can be obtained from

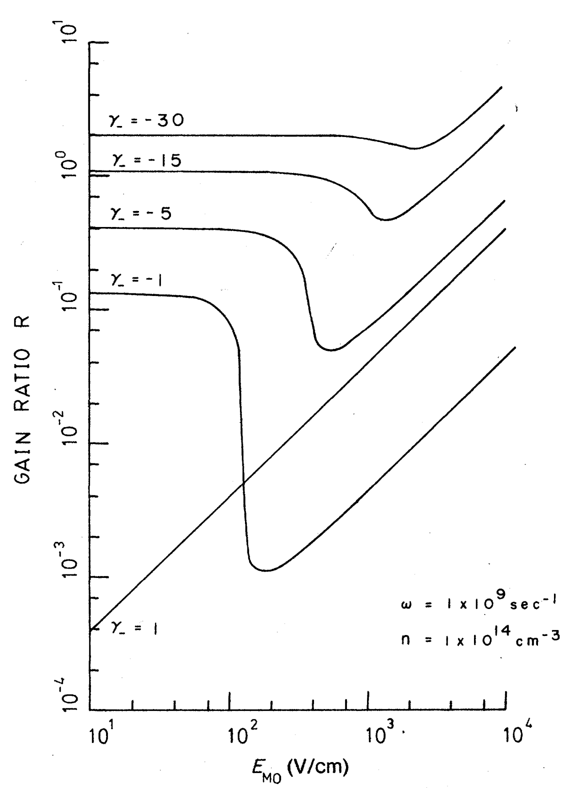

FIG. 2. Gain ratio R shown as a function of the microwave electric field $\mathcal{E}_{M0}$ in n-type GaAs for various values of the dc electric field and for $\omega = 1 \times 10^9$ rad/sec, $n = 1 \times 10^{14}$ cm^{-3}, and $\vec{E}_M = \hat{z}\,E_M$.

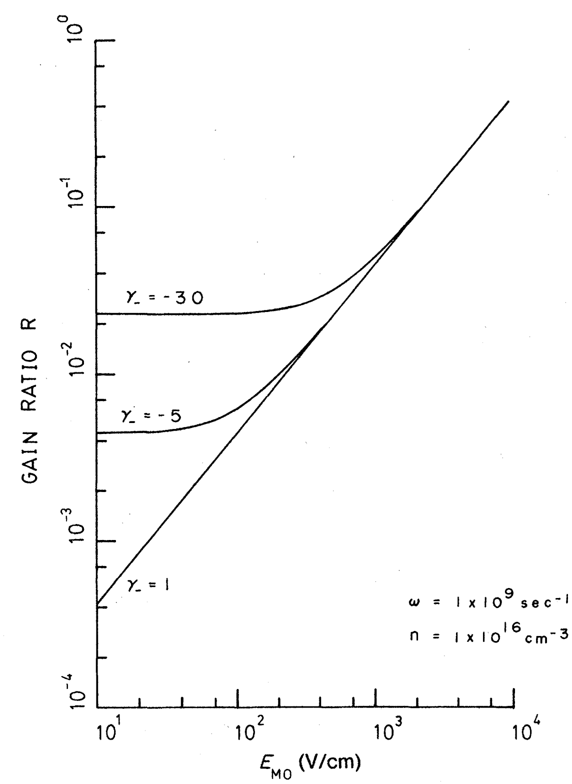

FIG. 3. Gain ratio R shown as a function of the microwave electric field $\mathcal{E}_{M0}$ in n-type GaAs for various values of the dc electric field and for $\omega = 1 \times 10^9$ rad/sec, $n = 1 \times 10^{16}$ cm^{-3}, and $\vec{E}_M = \hat{z}\,E_M$.

the paper by Palik and Bray.[8] For longitudinal waves at a frequency ω of 10^{10} sec^{-1}, one finds from their data that $\alpha_l = 8.0$ cm^{-1}. Their data also suggests a variation of α_l with frequency given by $\alpha_l \sim \omega^{1.3}$. We shall assume that this relation is valid over the frequency range with which we are concerned. Expression (2.36) can then be used to compute the value of the gain ratio R.

In Figs. 1–6, we have plotted the gain ratio R as a function of the microwave electric field $\mathcal{E}_{M0}$ in GaAs for various values of the dc electric field, the frequency and the carrier concentration. Here $\vec{E}_M$ is polarized in the z direction. These graphs illustrate a number of interesting points. For low values of $\mathcal{E}_{M0}$, the value of R is essentially independent of the magnitude of $\mathcal{E}_{M0}$, while for large values of $\mathcal{E}_{M0}$, the dependence of R on $\mathcal{E}_{M0}$ approaches that found in the absence of the dc field (a linear dependence). In the absence of the dc field, amplification can only occur for very large values of the microwave field. In fact, the threshold field in this case does not differ greatly from its value in the absence of charge carriers, where R is directly proportional to $\mathcal{E}_{M0}$. To be more specific, for the insulating material we have $R = |\eta \mathcal{E}_{M0}| / \alpha_l$, where $\eta = \eta_1 = \eta_e$. Typical values of the quantity $|\eta| / \alpha_l$ are listed in Table II. The application of the dc electric field, however, greatly alters the characteristics of the problem. In this case, it is evident that acoustic gain can occur at relatively low microwave field strengths and that there are certain combinations of ω, n_0, and

TABLE II. Values of $|\eta| / \alpha_l$ for insulating GaAs.

ω ($\mathrm{rad\,sec}^{-1}$)	$\lvert\eta\rvert / \alpha_l$ ($\mathrm{cm\,V}^{-1}$)
1×10^9	4.57×10^{-5}
5×10^9	2.81×10^{-5}
1×10^{10}	2.28×10^{-5}
5×10^{10}	1.41×10^{-5}
1×10^{11}	1.14×10^{-5}
5×10^{11}	7.07×10^{-6}
1×10^{12}	5.73×10^{-6}

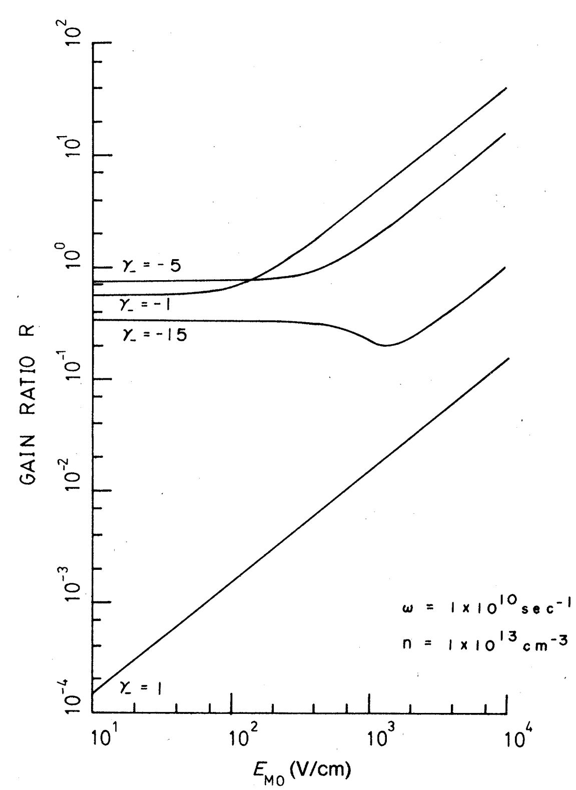

FIG. 4. Gain ratio R shown as a function of the microwave electric field $\mathcal{E}_{M0}$ in n-type GaAs for various values of the dc electric field and for $\omega = 1 \times 10^{10}$ rad/sec, $n = 1 \times 10^{13}$ cm^{-3}, and $\vec{E}_M = \hat{z} E_M$.

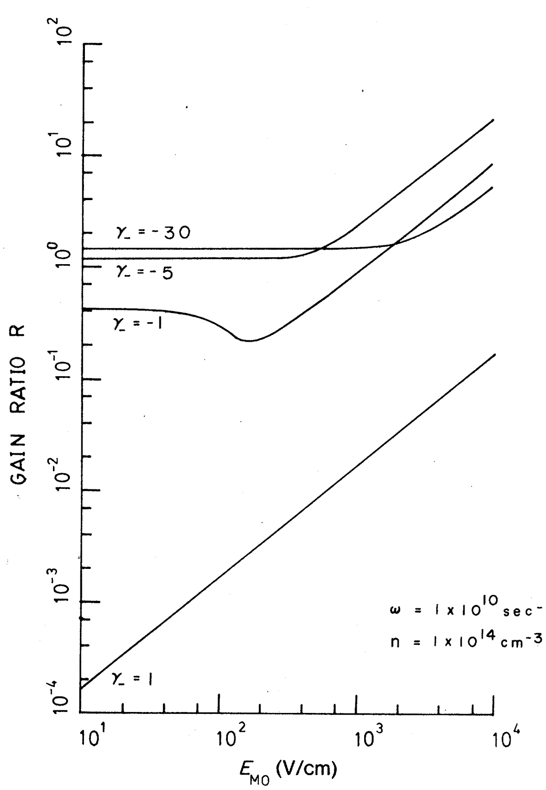

FIG. 5. Gain ratio R shown as a function of the microwave electric field $\mathcal{E}_{M0}$ in n-type GaAs for various values of the dc electric field and for $\omega = 1 \times 10^{10}$ rad/sec, $n = 1 \times 10^{14}$ cm^{-3}, and $\vec{E}_M = \hat{z} E_M$.

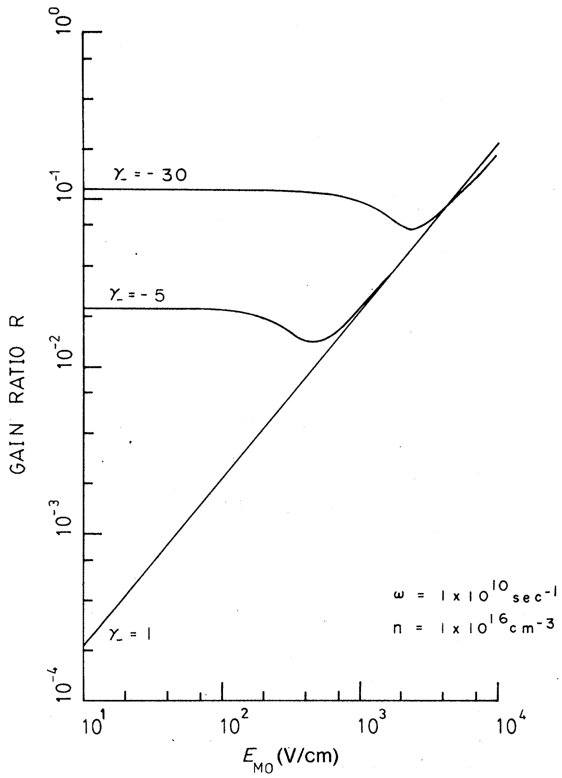

FIG. 6. Gain ratio R shown as a function of the microwave electric field $\mathcal{E}_{M0}$ in n-type GaAs for various values of the dc electric field and for $\omega = 1\times10^{10}$ rad/sec, $n = 1\times10^{16}$ cm^{-3}, and $\vec{E}_M = \hat{z}E_M$.

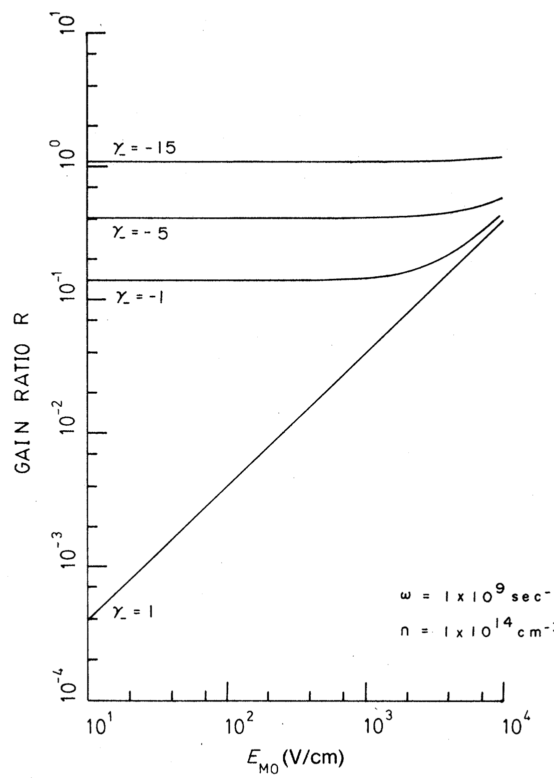

FIG. 7. Gain ratio R shown as a function of the microwave electric field $\mathcal{E}_{M0}$ in n-type GaAs for various values of the dc electric field and for $\omega = 1\times10^{9}$ rad/sec and $n = 1\times10^{14}$ cm^{-3}. For this figure, the microwave field is polarized transverse to the direction of propagation of the acoustic waves: $\vec{E}_M = \hat{y}E_M$.

γ_- for which no threshold field is required. This suggests that the presence of both the dc field and the microwave field can enhance the amplification of the waves. Under certain conditions, however, the amplification can be suppressed; that is, the value of R decreases with increasing $\mathcal{E}_{M0}$. Our calculations for the polarization $\vec{E}_M = \hat{y}E_M$, shown in Figs. 7–9, indicate that this behavior results from the nonlinear conductivity. This can be seen analytically as follows. The minimum of R with respect to $\mathcal{E}_{M0}$ can be shown to depend on the sign of $\mathrm{Re}(\eta_1\eta_e^*)$. For $\mathrm{Re}(\eta_1\eta_e^*)>0$, the minimum occurs at $|\mathcal{E}_{M0}|=0$, while for $\mathrm{Re}(\eta_1\eta_e^*)<0$, it occurs at the value of $|\mathcal{E}_{M0}|$ given by

$$|\mathcal{E}_{M0}|^2 = -(\alpha_1-\alpha_e)^2\,\mathrm{Re}(\eta_1\eta_e^*)/2|\eta_1\eta_e|^2 . \tag{3.16}$$

Substitution of the above value for $|\mathcal{E}_{M0}|$ into N^2 yields

$$N^2 = (\alpha_1-\alpha_e)^2(1-\eta_1\eta_e^*/\eta_1^*\eta_e) . \tag{3.17}$$

For GaAs, it turns out that $\eta_1 \approx -\eta_e^*$. Thus, the condition $\mathrm{Re}(\eta_1\eta_e^*)<0$ implies that $\mathrm{Re}(\eta_1^2)>0$ or $|\mathrm{Re}\eta_1| > |\mathrm{Im}\eta_1|$, a relation which cannot be satisfied when the nonlinear conductivity vanishes. There exists, therefore, an interplay between the contributions from the linear and nonlinear currents and the effect of the microwave field is, under certain conditions, to bring about a reduction in the nonlinear amplification of the acoustic wave.

It is of interest to consider what determines the value of γ_- for which a threshold field is no longer required. For low values of $\mathcal{E}_{M0}$, the magnitude of R is essentially determined by the value of $\alpha_1-\alpha_e$. When $(\alpha_1-\alpha_e)^2>\alpha_T^2$, a threshold field is not required. We have mentioned above that α_1 and α_e are increasing functions of ω up to the frequency of maximum gain $\omega_M=(\omega_\epsilon\omega_D)^{1/2}$ and increasing functions of γ_- up to the value $\gamma_M = -(\omega_\epsilon/\omega + \omega/\omega_D)$. It follows that for frequencies in the range $\omega<\omega_M$, the value of γ_- required to satisfy the condition $(\alpha_1-\alpha_e)^2>\alpha_T^2$ decreases as ω is increased. For frequencies in the range $\omega>\omega_M$, α_1 and α_e decrease in value as

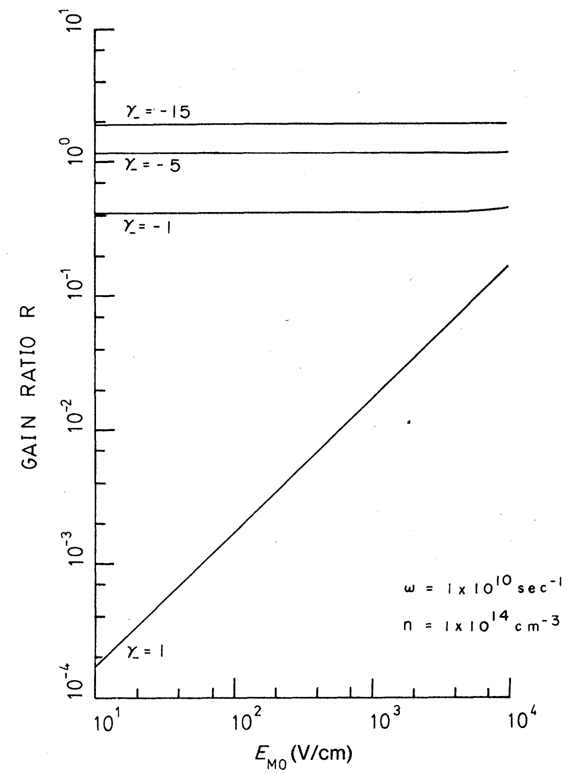

FIG. 8. Gain ratio R shown as a function of the microwave electric field $\mathcal{E}_{M0}$ in n-type GaAs for various values of the dc electric field and for $\omega = 1\times10^{10}$ rad/sec, $n = 1\times10^{14}$ cm^{-3}, and $\vec{E}_M = \hat{y}E_M$.

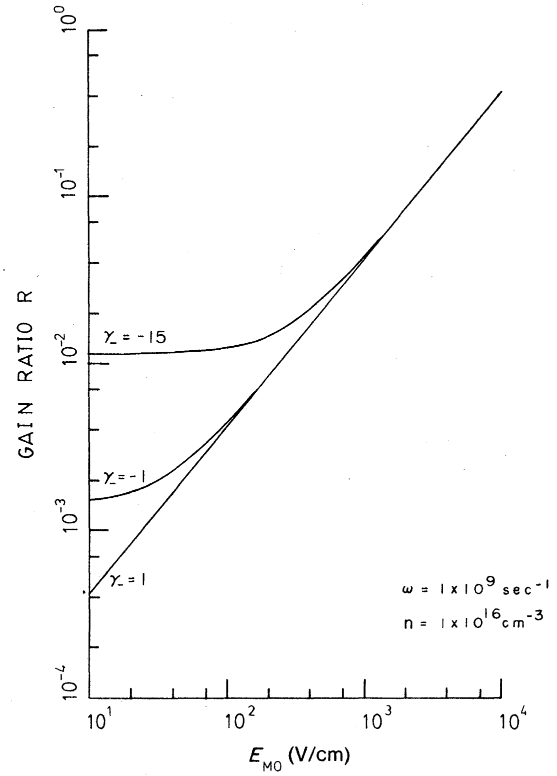

FIG. 9. Gain ratio R shown as a function of the microwave electric field $\mathcal{E}_{M0}$ in n-type GaAs for various values of the dc electric field and for $\omega = 1\times10^{9}$ rad/sec, $n = 1\times10^{16}$ cm^{-3}, and $\vec{E}_M = \hat{y}E_M$.

ω increases; thus for large frequencies, it will not be possible to satisfy the condition for zero threshold field. We also note that for a fixed frequency and carrier concentration the value of R increases as a function of γ_- up to the value of $\gamma_M = -(\omega_\epsilon/\omega + \omega/\omega_D)$. Figures 1–6 verify these results.

In CdS, we find that the lower value of the mobility gives rise to the possibility of having the linear electronic gain exceed the lattice loss; that is $\alpha_T < 0$. This leads to an additional mechanism for amplifying the waves. Once again, increasing the microwave field strength can enhance the process.

As a final note, let us consider the effect of the dc field on the amplitude of the echo wave. For $E_{dc} = 0$, we have $|\eta_e\mathcal{E}_{M0}/N^*| = |\eta_e/\eta_1|^{1/2} \approx 1$, which is independent of the microwave field. For $E_{dc} \neq 0$, on the otherhand, we find that $|\eta_e\mathcal{E}_{M0}/N^*|$ is small compared to unity for low microwave field strengths and approaches unity for increasing values of the microwave field. Let us restrict our attention to low values of $\mathcal{E}_{M0}$, in which case the acoustic waves are attenuated in the absence of the dc field. For this situation, it is evident that the effect of the dc field is to suppress the generation of the echo wave during its initial period of growth. As we have seen, however, the exponential growth of the wave can be enhanced in the presence of the dc field and thus the wave grows rapidly with time. The initial suppression of the wave is therefore negligible compared to the amplification which occurs with increasing t.

IV. CONCLUSION

The results of our calculations indicate that there can be a significant enhancement of the phonon echo process in the presence of a dc electric field in semiconductors. In particular, the calculations suggest that amplification of the forward-traveling wave and generation of the echo wave can be attained at much lower microwave field strengths in the presence of a dc field than in its absence. In those cases where a threshold field is not required, the microwave field is needed only to couple the acoustic waves. Our calculations have also shown that the effect of increasing the microwave field strength is either to enhance further the amplification of the waves, or to suppress this amplification. The latter situation has been attributed to the nonlinear currents. Thus, the nature of the amplification process is dependent upon the nonlinear contributions to the current density.

A more complete treatment of this problem would require a discussion of how the amplitude of the microwave field varies as a result of the interaction. As was mentioned in the text, a description of this effect must make use of the full set of Maxwell's equations. It is evident, however, that the lowest-order correction to the microwave field would be proportional to the product of the amplitudes u_1 and u_e, and it is therefore a higher-order effect. We intend to discuss this aspect of the problem at a later date, along with a theoretical description of the phonon echo process in ferroelectrics, where many of the terms that were neglected in this paper cannot be ignored. It is also evident that in this paper we have considered the situation in which the wave vectors are perfectly phase matched. If, however, the wave vector of the photon is included in our analysis, then we may also have to consider the effects of dispersion. Finally, it may be interesting to extend the analysis to high mobility semiconductors such a n-InSb, where the electron mean free path is much greater than an acoustic wavelength ($ql \gg 1$). In this case, the conductivities would have to be derived using a quantum Liouville equation approach.

It was brought to the attention of the authors after this paper was submitted for publication that some of the results presented in this paper were anticipated by R. B. Thompson[13] in an unpublished portion of his Ph.D. thesis, Stanford University, 1971.

ACKNOWLEDGMENTS

The authors would like to acknowledge the partial support provided by a grant from the NSF Material Research Laboratory Program, Grant No. DMR 76-24466. This work was submitted by one of us (J. E. E.) in partial fulfillment of the requirements for the Ph.D. degree in the Dept. of Physics at Illinois Institute of Technology, Chicago, Ill. 60616.

---

[1]R. B. Thompson and C. F. Quate, J. Appl. Phys. 42, 907 (1971).
[2]Ch. Frénois *et al.*, Solid State Commun. 11, 327 (1972); J. Joffrin and A. Levelut, Phys. Rev. Lett. 29, 1325 (1972); A. Billmann *et al.*, J. Phys. 34, 453 (1973); Ch. Frénois *et al.*, *ibid.* 34, 747 (1973).
[3]N. S. Shiren and T. G. Kazyaka, Phys. Rev. Lett. 28, 1304 (1972). N. S. Shiren, *et al.*, *ibid.* 31, 819 (1973). R. L. Melcher and N. S. Shiren, *ibid.* 34, 731 (1975).
[4]I. Bajak, Wave Electron. 3, 51 (1977).
[5]The reader is referred specifically to the papers by Thompson and Quate (Ref. 1) and by Bajak (Ref. 4). The papers by Shiren and his co-workers (Ref. 3) contain discussions of possible physical mechanisms in semiconducting materials which could explain their experimental results.
[6]L. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1960), Chap. 2.
[7]H. N. Spector, Phys. Rev. B 6, 2409 (1972).
[8]For a discussion of lattice attenuation see the paper by E. D. Palik and R. Bray, Phys. Rev. B 3, 3302 (1971).
[9]D. H. McMahon, J. Acoust. Soc. Amer. 44, 1007 (1968).
[10]K. Hrŭska and V. Kazda, Czech. J. Phys. B 18, 500 (1968).
[11]The reader is referred to the following sources: A. Yariv, *Quantum Electronics*, 2nd ed. (Wiley, New York, 1975); *Handbook of Lasers* (CRC, Cleveland, 1971); *American Institute of Physics Handbook*, 3rd ed. (McGraw-Hill, New York, 1972).
[12]D. L. Spears, Phys. Rev. B 2, 1931 (1970).
[13]R. B. Thompson, Ph.D. thesis (Stanford University, 1971) (unpublished).