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Acoustoelectric interaction of surface phonons in semiconductors: Isotropic approximation

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The acoustoelectric interaction of surface phonons with conduction electrons in piezoelectric semiconductors is investigated assuming elastic isotropy. The electronic states near a semiconductor surface are properly determined taking into account the existence of the surface space-charge layer and the potential barrier associated with it. Employing the electron wave functions obtained in this manner, the acoustoelectric interaction is specified and it is applied to the study of the amplification of the surface phonons. For an n-type semiconductor with bulk-electron concentration of 10^{16} cm^{-3}, we find that the frequency dependence of the amplification rate has two bumps in the frequency range from 1 to 10 GHz. This result will be interpreted in terms of the depth dependence of the electron wave functions and the electric potential produced by the surface phonons.

I. INTRODUCTION

Since a surface acoustic wave (SAW) travels along a solid surface localizing its amplitude within a wavelength or so from the surface, the SAW or the surface phonon (quantum of the SAW) will play an important role in a number of surface phenomena. A considerable amount of works has been devoted in recent years to the problem of attenuation of the SAW in order to understand the properties of a solid near the surface.[1]

Assuming elastic isotropy, we have developed theoretically the interaction of the surface phonon with conduction electrons in a piezoelectric semiconductor and have applied it to the study of the amplification characteristics of SAW's of GHz frequencies.[2] Because of their distinctive nature as stated above, the propagation characteristics of the SAW are affected deeply by the properties of the solid in the vicinity of its surface. As for the acoustoelectric interaction of the SAW in a semiconductor, it might be highly sensitive to the electrons which may travel within a wavelength off the semiconductor surface. In an n-type (p-type) semiconductor, the surface-electron depletion (accumulation) layers are generally formed at its free surfaces,[3] therefore their existence will have a critical effect upon the interaction if the thickness of the surface depletion (accumulation) layer begins to compete with the wavelength of the SAW.

In the treatment of the amplification of the SAW given in Ref. 2, we used the approximation that the conduction electrons are distributed uniformly from the bulk to the surface of the n-type piezoelectric semiconductor. This can be justified in the case that the thickness of the electron-depletion layer is much smaller than the wavelength of the SAW. However, this approximation may be also applicable to slightly wider cases if we remember that the thickness of the depletion layer can be made narrower by applying an electric field perpendicular to the corresponding surface or by making the surface region of the semiconductor as a layered structure of epitaxial films such that donor impurities are more highly doped in the layers close to the surface.

The purpose of the present paper is to investigate the acoustoelectric interaction of the SAW with the electrons in the n-type piezoelectric semiconductor in the situation in which the thickness of the surface-depletion layer becomes of the same order of magnitude as the wavelength of the acoustic field. We shall try to solve this problem by making use of quantum mechanics. One of the reasons for employing quantum theory is that the electronic states near the semiconductor surface can be determined by solving the Schrödinger equation, once the potential in the depletion layer is found. Another reason is that the frequencies in which we are interested are 1–10 GHz and at 1-GHz frequency the condition $ql \gtrsim 1$ is attained, where q and l are the wave number of the SAW and the mean free path of the electrons, respectively. However, for those frequencies $\omega\tau$ (ω and τ are the angular frequency of the SAW and the relaxation time of the electrons, respectively) is still less than unity, the effect of the finite relaxation time of the electrons might be important.

Now, we shall remark on some basic assumptions made in this work. First: the elastic isotropy of the medium. That is, we approximate the semiconductor to be isotropic in its elastic properties and regard the Rayleigh wave as a well-defined surface mode of the acoustic waves that may exist at the free flat surface of a nonpiezoelectric semiconductor of infinite extent in the thickness and width directions. We assume furthermore that piezoelectric coupling between the acoustic field of the Rayleigh wave and the

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electromagnetic field in the semiconductor acts only as a small perturbation, and the Rayleigh wave remains a well-defined mode even when the piezoelectric coupling is turned on. Here, it should be noted that the piezoelectric coupling is the dominant mode of the acoustoelectric interaction for phonons of frequencies less than 10 GHz.

The latter assumption may be justified for such semiconductors as GaAs and InSb which have small electromechanical coupling coefficients, e.g., $K^2 = 3.6\times 10^{-3}$ and 0.73×10^{-3}, respectively,[4] which should be compared with unity. On the other hand, unlike the case of noncrystalline (amorphous) semiconductors the real crystalline semiconductors have to some extent elastic anisotropy. For instance, the anisotropy ratios [defined as $2c_{44}/(c_{11}-c_{12})$] for GaAs and InSb of zinc-blende structure are 1.80 and 1.99, respectively.[4] The effects, produced by the presence of the anisotropy on the SAW theory are considerably complicated and extensive computer calculations are required to incorporate them.[5] In this paper, we wish only to survey the general characteristics of the interaction and to obtain some qualitative results which may be observed in experimental investigations. Accordingly, we assume elastic isotropy as a first approximation and isotropically approximated elastic constants[6] will be used in the numerical calculations. Hence, in the following, the SAW is to be understood as the Rayleigh wave and the surface phonon as the quantum of the Rayleigh wave.

In Sec. II, we solve the Poisson and the Schrödinger equations under some approximations in order to specify the electronic states in the surface depletion layer of the n-type semiconductor. In Sec. III, we derive in some detail the electric potential produced by the SAW and quantize it in terms of the phonon variables. Then, the acoustoelectric interaction of the surface phonon with the electrons is derived using the wave functions of the electrons obtained in Sec. II. The formula for the attenuation (or amplification) rate of the surface phonon is presented in Sec. IV in the Born approximation. Section V is devoted to the numerical example of the amplification rate for n-type GaAs at $T=77$ K and for 1–10-GHz phonon frequencies. A discussion of the results is given in Sec. VI with several remarks on the possible effects we may have neglected in our calculations. Finally, we shall briefly remark that the same kind of amplification characteristics would be obtained also in the case of $ql<1$, if a suitable condition were satisfied.

II. ELECTRONIC STATES

In order to specify the interaction of the surface phonon with the electrons, we must know the electronic states near the semiconductor surface in the absence of acoustic disturbance. The electronic states in the vicinity of the semiconductor surface are different from those of the bulk region owing to the presence of a space-charge region and the potential barrier associated with it. This space-charge region at a free surface of a semi-infinite conducting solid may be produced by an electric field outside the solid or by the presence of a localized charge layer at the surface due to surface states.[3] From the viewpoint of the acoustoelectric interaction, we are mainly interested in the n-type semiconductor and we shall consider its energy-band configuration near the surface.

In the absence of surface states, the energy bands of a semiconductor continue straight up to the surface, provided there is no external field. When acceptorlike surface states are introduced below the Fermi level, they will not be in equilibrium with the energy bands as long as they remain unoccupied. Since these states are empty and below the Fermi level, some of the electrons in the conduction band fall into them. In this process, the surface becomes negatively charged while a positive space-charge layer, i.e., a surface-depletion layer, is formed adjacent to the surface. Consequently, the energy bands at the surface bend upwards with respect to the Fermi level.

The surface-charge density and the shape of the potential barrier associated with it are determined by solving the Poisson equation under appropriate boundary conditions, and the electronic states are determined by solving the Schrödinger equation with the potential obtained in this manner. Now, we put the plan of solving these equations into effect.

The motion of electrons in a semiconductor can be characterized in the effective-mass approximation by the following wave function:

$$\Phi_{\vec{k},\kappa}(\vec{r}) = e^{i\vec{k}\cdot\vec{x}}\psi_\kappa(z) , \tag{1}$$

where $\vec{r}=(\vec{x},z)=(x,y,z)$ and $\vec{k}=(k_x,k_y)$. We have chosen the Cartesian coordinates such that the semiconductor occupies the half space $z>0$ with its surface $z=0$ parallel to the x-y plane. The wave function $\psi_\kappa(z)$ satisfies the Schrödinger equation

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}+V(z)\right)\psi_\kappa(z)=\epsilon_\kappa\psi_\kappa(z) , \tag{2}$$

where m is the electron effective mass (assuming a spherical constant-energy surface) and the energies of the electrons are

$$E_{\vec{k},\kappa}=\epsilon_\kappa+\hbar^2\vec{k}^2/2m . \tag{3}$$

The potential $V(z)$ in the conduction band is obtained by solving the Poisson equation. One has

$$\frac{d^2V}{dz^2} = -\frac{4\pi e^2}{\epsilon_0}[n(z) - n_0]\Theta(l_0 - z), \quad (4)$$

where ϵ_0 is the static dielectric constant and n_0 is the bulk electron concentration which is assumed to be equal to the concentration of ionized donors uniformly distributed in the semiconductor from the surface to the bulk. The hole and the acceptor concentrations have been neglected. $\Theta(z)$ denotes the unit step function, i.e., $\Theta(z)=1$ for $z \geq 0$ and 0 otherwise. l_0 is a parameter which represents the thickness of the depletion layer. The local electron concentration $n(z)$ is defined as follows assuming that the wave function $\psi_\kappa(z)$ is normalized:

$$n(z) = \sum_{\vec{k},\kappa} |\psi_\kappa(z)|^2 f_{\vec{k},\kappa}, \quad (5)$$

where $f_{\vec{k},\kappa} = f(E_{\vec{k},\kappa})$ is the electron-distribution function which should be replaced by the Boltzmann distribution for a nondegenerate semiconductor. It should be noted that in the absence of the space-charge layer at the semiconductor surface $n(z)$ is equal to n_0 irrespective of z.

Thus, in order to obtain the correct wave functions of the electrons near the semiconductor surface, we must solve Eqs. (2) and (4) simultaneously with (5) under appropriate boundary conditions. They should be solved numerically in general cases, but unlike the case of electrons in the inversion layer where one may consider that only one subband is occupied by the electrons,[7] the situation may not be so simple. However, if we confine ourselves to the qualitative investigation of the effect of the surface-depletion layer on the interaction of the surface phonon with the electrons, we may solve Eqs. (2)–(4), and (5) approximately as follows:

In the zeroth-order approximation, the wave function $\psi_\kappa(z)$ is assumed to be

$$\psi_\kappa(z) = (2/L)^{1/2}\sin[\kappa(z - l_0)]\Theta(z - l_0), \quad (6)$$

where L is the thickness of the semiconductor in the z direction. This is equivalent to assume the electron concentration as

$$n(z) = n_0\Theta(z - l_0), \quad (7)$$

that is, there are no electrons in the depletion layer.[8]

Next, making use of Eq. (7), let us integrate the Poisson equation (4) with the boundary conditions

$$V = \frac{dV}{dz} = 0 \quad \text{at } z = l_0,$$

$$V = e\phi_s \text{ and } \frac{dV}{dz} = -\frac{4\pi e^2}{\epsilon_0}N_s \quad \text{at } z = 0, \quad (8)$$

where $e\phi_s$ is the band bending at the free surface and N_s is the density of the charged-surface states. The solution gives the well-known potential barrier of the Schottky type (see Fig. 1),

$$V = (2\pi e^2 n_0/\epsilon_0)(l_0 - z)^2\Theta(l_0 - z). \quad (9)$$

The width of the depletion layer l_0 depends on ϕ_s and n_0 as

$$l_0 = (\epsilon_0\phi_s/2\pi e n_0)^{1/2}. \quad (10)$$

Also, we have a relation

$$N_s = l_0 n_0. \quad (11)$$

Now, with the potential given by Eq. (9) we try to solve the Schrödinger equation (2). For our purpose, we need to know the electron wave functions over the region where the electronic motion is forbidden classically as well as over the region accessible to classical motion. Unfortunately, Eq. (2) can not be solved exactly and one of the possible ways to find a solution is to employ the WKBJ approximation. As is well known, this semiclassical approximation is applicable to the wave function in the range where the potential energy changes so slowly that momentum of an electron

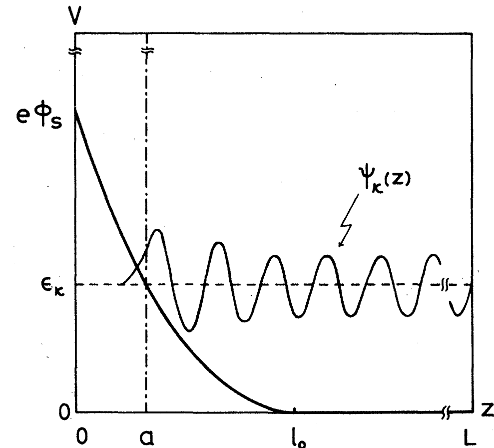

FIG. 1. Schematic drawing of the energy configuration of the conduction band near an n-type semiconductor surface. $e\phi_s$ denote the band bending at the surface and l_0 the thickness of the depletion layer. $z=a$ is the turning point of the classical motion of an electron with energy ϵ_κ. Also schematically illustrated is the wave function of the electron with energy ϵ_κ.

is sensibly constant over several wavelengths. Therefore, this method fails near the turning point of the classical motion, at which the kinetic energy of the electron is 0 and its wavelength is infinite. Hence, in the neighborhood of the turning point we must look for a solution of the Schrödinger equation regarding the potential, for instance, as a linear function of the coordinate.

If the potential is slowly varying and has only one turning point, such as Eq. (9) considered here, a fairly good approximate solution of the Schrödinger equation is known which has the asymptotic form of the WKBJ approximation.[9] Such a solution is

$$\psi_\kappa(z)=\begin{cases} A_\kappa\left(\dfrac{\xi(z)}{p(z)}\right)^{1/2}\,[J_{1/3}(\xi)+J_{-1/3}(\xi)] & \text{for } z\geq a\\ A_\kappa\left(\dfrac{\eta(z)}{p(z)}\right)^{1/2}\,[I_{1/3}(\eta)-I_{-1/3}(\eta)] & \text{for } z<a, \end{cases} \tag{12}$$

with

$$p(z)=[2m|\epsilon_\kappa-V(z)|]^{1/2}/\hbar,$$

$$\xi(z)=\int_a^z p(z')dz' \quad \text{for } z\geq a,$$

and

$$\eta(z)=\int_z^a p(z')dz' \quad \text{for } z<a,$$

where a is the classical turning point defined by $\epsilon_\kappa=V(a)$ (see Fig. 1). J_ν and I_ν are the Bessel functions of real and imaginary arguments, respectively, and A_κ is the normalization factor defined by $A_\kappa=(\pi\kappa/3L)^{1/2}$ which is obtained by neglecting the contribution to the normalization integral of the exponentially decreasing part of the wave function.

Here, we note that for a semi-infinite semiconductor ($L\gg l_0$), the quantization condition of the WKBJ approximation tells us that the wave number κ takes the quasicontinuous values $\kappa=2\pi n/L$, $n=1,2,\ldots,\infty$, and the energy eigenvalue is given by $\epsilon_\kappa=\hbar^2\kappa^2/2m$. ξ and η are explicitly integrated to be

$$\xi(z)=\begin{cases} \dfrac{\epsilon_\kappa}{\hbar\omega}\,[\sin^{-1}t+t(1-t^2)^{1/2}+\tfrac{1}{2}\pi] & \text{for } a\leq z<l_0,\\ \dfrac{\epsilon_\kappa}{\hbar\omega}\,(2t+\tfrac{1}{2}\pi) & \text{for } z\geq l_0, \end{cases} \tag{13}$$

and

$$\eta(z)=\frac{\epsilon_\kappa}{\hbar\omega}[\ln|t+(t^2-1)^{1/2}|-t(t^2-1)^{1/2}], \tag{14}$$

where $\omega^2=4\pi e^2 n_0/m\epsilon_0$ and $t=(m\omega^2/2\epsilon_\kappa)^{1/2}(z-l_0)$. For large values of ξ and η, $\psi_\kappa(z)$ tends to the solution of the WKBJ approximation, e.g.,

$$\psi_\kappa(z)=\left(\frac{2\kappa}{p(z)L}\right)^{1/2}\cos[\xi(z)-\pi/4]$$

$$\text{for } z\geq a \text{ and } \xi\gg 1, \tag{15}$$

and

$$\psi_\kappa(z)=\left(\frac{\kappa}{2p(z)L}\right)^{1/2}e^{-\eta(z)} \quad \text{for } z<a \text{ and } \eta\gg 1, \tag{16}$$

respectively.

The z dependence of the electron concentration calculated from the wave functions (12) is shown in Fig. 2. In this approximation, the electrons have a finite distribution in the surface depletion layer, contrary to the zeroth-order approximation. If we continue to solve the Poisson equation numerically with the aid of this local electron concentration and then solve the Schrödinger equation and so forth, we shall ultimately arrive at the self-consistent solution of the electron wave function. The electron density calculated in terms of the self-consistent solution will have a more slightly extended distribution into the depletion layer than the one illustrated here. However, for the reason stated before, further calculations will be furthered by the approximate solution given by Eq. (12).

Finally, we introduce the field operator $\Psi(\vec{r})$ of

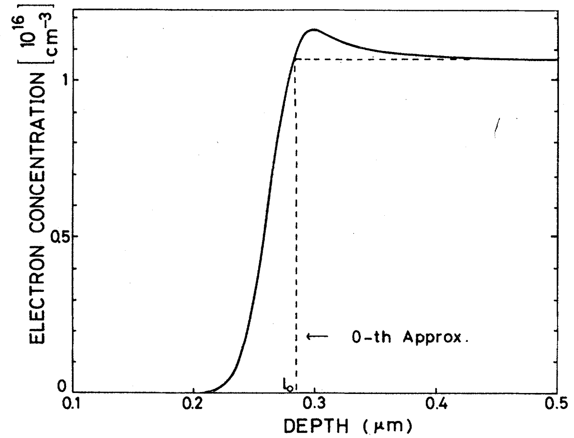

FIG. 2. Local electron concentration calculated from the wave function (12) with $n_0=1.07\times10^{16}$ cm^{-3}, $e\phi_s=0.59$ eV, $\epsilon_0=12.9$, $l_0=0.283$ μm, $m=0.07m_0$, and $T=77$ K. The dashed line represents the electron concentration adopted as zeroth-order approximation.

the electrons as follows:

$$\Psi(\vec{r}) = \frac{1}{\sqrt{S}} \sum_{\vec{k},\kappa} b_{\vec{k},\kappa} \psi_\kappa(z) e^{i\vec{k}\cdot\vec{x}} , \tag{17}$$

where $b_{\vec{k},\kappa}$ and its Hermitian conjugate $b^\dagger_{\vec{k},\kappa}$ are annihilation and creation operators of the electrons which satisfy commutation relations of the Fermi type, and S is the surface area.

III. INTERACTION

In the piezoelectric semiconductor, the conduction electrons interact with the surface phonon through the deformation potential which is proportional to the dilatation caused by the acoustic field.[10] They also interact with the phonon through piezoelectricity. Although at very high acoustic-wave frequencies the deformation potential becomes the dominant mode of the acoustoelectric interaction even for the piezoelectric semiconductor, it can be neglected compared with the piezoelectric coupling in the frequency region from 1 to 10 GHz to be considered here.

Now, let us derive the acoustoelectric potential produced by the surface phonon through piezoelectricity and then specify the interaction of the surface phonon with the electrons in the semiconductor. Analogous to the case of bulk phonons,[11] it is assumed that the interaction is only appreciable if the induced electric field is longitudinal.

We start from the fundamental equations for the acoustic and the electric fields. If the electrostatic approximation is valid, the electric field E_i is derivable from a scalar potential ϕ,

$$E_i = -\frac{\partial \phi}{\partial r_i} . \tag{18}$$

Within insulating crystals, Gauss law is expressed as

$$\frac{\partial D_i}{\partial r_i} = 0 , \tag{19}$$

where D_i represents the electric displacement and the summation convention is applied. The modification of Eq. (19) needed for conducting crystals such as the semiconductors with high mobility considered in this paper is equivalent to incorporate the electronic screening effects.[12] This will be carried out in Sec. IV by introducing a dielectric function.

The piezoelectric equations of state for the acoustic and the electromagnetic fields are as follows:

$$D_i = 4\pi e_{ikl} S_{kl} + \epsilon_{ik} E_k , \tag{20}$$

$$\sigma_{ij} = c^E_{ijkl} S_{kl} - e_{kij} E_k , \tag{21}$$

where $\{e_{ikl}\}$, $\{S_{kl}\}$, $\{\epsilon_{ik}\}$, $\{\sigma_{ij}\}$, and $\{c^E_{ijkl}\}$ are the piezoelectric, the strain, the dielectric, the stress, and the elastic-stiffness (at constant electric field) tensors, respectively. Taking the divergence of Eq. (20) and comparing with Eqs. (18) and (19), we get the relation

$$\Delta \phi = \frac{4\pi}{\epsilon_0} e_{ijk} \frac{\partial S_{jk}}{\partial r_i} , \tag{22}$$

where dielectric isotropy ($\epsilon_{ij} = \epsilon_0 \delta_{ij}$) is assumed. The quantized version of this equation is used to obtain the electric potential $\tilde{\varphi}$ produced by the surface phonon in the semiconductor. Equation (21) together with the force equations for the particle displacements gives after elimination of the electric field the piezoelectric stiffening, that is, the velocity of the acoustic field becomes large owing to the presence of piezoelectricity. This modification of the sound velocity, however, is neglected in the present work because it is small for the material considered here, e.g., $\Delta c/c \simeq K^2/2 = 1.8 \times 10^{-3}$ for GaAs.

The strain tensor is written explicitly in terms of the displacement vector $\vec{u}$ of the medium, as $S_{ij} = \frac{1}{2}(\partial u_i/\partial r_j + \partial u_j/\partial r_i)$, hence, if we put

$$u_i(\vec{r}) = a_i e^{-\beta q z} e^{i\vec{q}\cdot\vec{x}} \quad i = x, y, \text{ and } z , \tag{23}$$

and

$$\phi(\vec{r}) = a_0 e^{-\beta q z} e^{i\vec{q}\cdot\vec{x}} \tag{24}$$

with $q = |\vec{q}| = (q_x^2 + q_y^2)^{1/2}$, a_0 is expressed as a linear combination of a_i as follows:

$$a_0 = \frac{4\pi}{\epsilon_0(1-\beta^2)q^2} \sum_{j=(x,y,z)} a_j \left(\sum_{l=(x,y)} [e_{ljl} q_l^2 + i\beta(e_{ljz} + e_{zjl}) q q_l] - e_{zjz}\beta^2 q^2 + (e_{xjy} + e_{yjx}) q_x q_y \right) , \tag{25}$$

For the crystals with zinc-blende structure ($e_{14} = e_{25} = e_{36} = e_p$ and other components vanish), Eq. (25) yields

$$a_0 = \frac{4\pi e_p}{\epsilon_0(1-\beta^2)q^2} [q_x q_y a_z + i\beta q (q_x a_y + q_y a_x)] . \tag{26}$$

Since we have assumed elastic isotropy and piezoelectricity as a small perturbation on the acoustic field, for wave functions of the SAW we can use those of the Rayleigh wave. Explicit expressions of them are

$$u_{\vec{q}}^{j}(\vec{r})=i\,\frac{q_j}{q}\left(\frac{q}{J}\right)^{1/2}\left[e^{-\gamma q z}-\frac{2\gamma\sigma}{1+\sigma^2}\,e^{-\sigma q z}\right]e^{i\vec{q}\cdot\vec{x}},\quad j=x,y,\qquad(27)$$

and

$$u_{\vec{q}}^{z}(\vec{r})=-\gamma\left(\frac{q}{J}\right)^{1/2}\left[e^{-\gamma q z}-\frac{2}{1+\sigma^2}e^{-\sigma q z}\right]e^{i\vec{q}\cdot\vec{x}},$$

where the wave vector $\vec{q}$ is related to the angular frequency ω and the velocity c of the Rayleigh wave by $\omega=c|\vec{q}|=cq$, and γ, σ, and J are constants determined by the velocities of the longitudinal sound wave c_l and the transverse sound wave c_t as $\gamma^2=1-(c/c_l)^2$, $\sigma^2=1-(c/c_t)^2$ and $J=(\gamma-\sigma)-(\gamma-\sigma+2\gamma\sigma^2)/2\gamma\sigma^2$.

With the aid of Eq. (27), the following electric potential is produced by the acoustic vibration of the SAW traveling with wave vector $\vec{q}$:

$$\varphi_{\vec{q}}(\vec{r})=\frac{4\pi e_p}{\epsilon_0}\left(\frac{q}{J}\right)^{1/2}\frac{q_x q_y}{q^2}\times\gamma\left(-\frac{3}{1-\gamma^2}\,e^{-\gamma q z}+\frac{2(1+2\sigma^2)}{1-\sigma^4}\,e^{-\sigma q z}\right)e^{i\vec{q}\cdot\vec{x}}\equiv\hat{\varphi}_{\vec{q}}(z)e^{i\vec{q}\cdot\vec{x}}.\qquad(28)$$

In the phonon picture, the displacement vector of the SAW can be expanded as

$$\vec{u}(\vec{r})=\sum_{\vec{q}}\left(\frac{\hbar}{2\rho\omega S}\right)^{1/2}a_{\vec{q}}\vec{u}_{\vec{q}}(\vec{r})+\text{H.c.},\qquad(29)$$

where ρ is the mass density of the medium and $a_{\vec{q}}$ and its Hermitian conjugate $a_{\vec{q}}^{\dagger}$ are annihilation and creation operators of the surface phonon satisfying commutation relations of the Bose type. Making use of the expansion (28), we find that the electric potential is quantized as follows:

$$\tilde{\varphi}(\vec{r})=\sum_{\vec{q}}\left(\frac{\hbar}{2\rho\omega S}\right)^{1/2}a_{\vec{q}}\varphi_{\vec{q}}(\vec{r})+\text{H.c.}\qquad(30)$$

Taking these results into account, the interaction Hamiltonian of the surface phonon with the electrons can be written

$$H_I=-e\int\Psi^{\dagger}(\vec{r})\tilde{\varphi}(\vec{r})\Psi(\vec{r})d\vec{r}=\frac{4\pi e_p e}{\epsilon_0\sqrt{S}}\sum_{\vec{k},\vec{q}}\sum_{\kappa,\kappa'}b^{\dagger}_{\vec{k}+\vec{q},\kappa'}b_{\vec{k},\kappa}\times a_{\vec{q}}\Xi_{\kappa\kappa'}(\vec{q})+\text{H.c.},\qquad(31)$$

with

$$\Xi_{\kappa\kappa'}(\vec{q})=-\left(\frac{\hbar\gamma^2}{2\rho cJ}\right)^{1/2}\frac{q_x q_y}{q^2}\int_0^{\infty}\psi^{*}_{\kappa'}(z)\left(-\frac{3}{1-\gamma^2}\,e^{-\gamma q z}+\frac{2(1+2\sigma^2)}{1-\sigma^4}\,e^{-\sigma q z}\right)\psi_{\kappa}(z)dz.\qquad(32)$$

From this expression, one can see that the surface phonon traveling in the [110] or its equivalent directions on the (001) plane couples to the electrons most strongly in the isotropic semiconductor of the zinc-blende crystal structure with piezoelectricity. On the other hand, the surface phonon with its wave vector parallel to the [100] direction (or its equivalent directions) decouples from the electrons.

IV. ATTENUATION RATE OF SURFACE PHONON

We wish to discuss the amplification (or attenuation) of the surface phonon due to its emission or absorption by the electrons coupled through H_I. A detailed calculation of these processes requires the solution of an integral equation, because for the phonon frequencies of 1–10 GHz in which we are interested, ql becomes unity or larger than unity but not very large, and then the effect of the finite relaxation time of the electrons might be important.

However, since we shall be primarily interested in the frequency dependence of the amplification rate, which may be strongly sensitive to the existence of the surface-electron depletion layer or electronic states near the semi-conductor surface, and since we are not interested in anything more than its qualitative characteristics, it will suffice to calculate the amplification rate of the surface phonon in the lowest order of perturbation theory.

According to the golden rule, the width Γ of the surface phonon will be

$$\Gamma=\frac{2\pi}{\hbar}\sum_f|\langle f|H_I|i\rangle|^2\delta(E_f-E_i),\qquad(33)$$

which becomes, upon applying it to the case at hand,

$$\Gamma(\vec{q})=\frac{2\pi}{\hbar}\left(\frac{4\pi e_p e}{\epsilon_0\epsilon(q)}\right)^2N_q\sum_{\vec{k}}\sum_{\kappa,\kappa'}(f_{\vec{k},\kappa}-f_{\vec{k}+\vec{q},\kappa'})\times|\Xi_{\kappa\kappa'}(\vec{q})|^2\times\delta(E_{\vec{k}+\vec{q},\kappa'}-E_{\vec{k},\kappa}-\hbar\omega),\qquad(34)$$

where $N(f)$ is the occupation number of the surface phonon (the electrons) and it should be replaced by

the Planck (the Fermi) distribution function. In this expression, we have introduced the static dielectric function $\epsilon(q) \equiv 1 + (q_s/q)^2$ which stands for the electronic-screening effect, where q_s is the reciprocal of the screening length.

If the attenuation rate α of the surface phonon is defined as Γ divided by the phonon flux $N_q c$, then it follows after slight manipulations,

$$\alpha = -\frac{m\omega}{2\pi\hbar^2 c}\left(\frac{4\pi e_p e}{\epsilon_0\epsilon(q)}\right)^2 \sum_{KK'} |\Xi_{KK'}(\vec{q})|^2 \int dE\, \frac{df(E+\epsilon_K)}{dE}\left(\frac{2\hbar^2 q^2}{m}E - (\epsilon_{K'} - \epsilon_K + \frac{\hbar^2 q^2}{2m} - \hbar\omega)^2\right)^{-1/2}, \quad (35)$$

where we have used the fact that the energy $\hbar\omega$ of the surface phonon is much smaller than the characteristic energy of the electrons and have kept terms up to $O(\hbar\omega)$ in the expansion of the electron distribution functions in terms of $\hbar\omega$. The integration over E should be performed in the region where the argument of the square root is non-negative.

Now, we easily see that the α defined by Eq. (35) is positive. This tells us that the surface phonon is attenuated by the interaction with the electrons. In the presence of an applied dc electric field, however, the situation is considerably modified. In this case, a displaced distribution must be used for the electrons. If the electrons have a drift velocity v, by the applied field, in the direction parallel to the wave vector $\vec{q}$ of the surface phonon, the effect of the displaced-electron distribution is equivalent to replace the frequency ω of the phonon by $-\omega x$ in Eq. (35), where x is the so-called drift parameter defined by $x = v/c - 1$. Therefore, the attenuation rate α changes its sign from positive to negative as the electron drift velocity v exceeds c and amplification of the surface phonon is attained.

V. NUMERICAL RESULT

As a numerical example, n-type GaAs is considered at $T = 77$ K with a bulk-electron mobility $\mu = 1.71 \times 10^4$ cm^2/V sec. The electron effective mass is $m = 0.07m_0$, where m_0 is the mass of the free electron. The velocity of the surface phonon, $c = 2.79 \times 10^5$ cm/sec, has been calculated using the values of elastic constants obtained from the anisotropic ones by employing the isotropic approximation[6] and with density $\rho = 5.32$ g/cm^3. The dielectric constant is taken as $\epsilon_0 = 12.9$ and the piezoelectric constant as $e_p = 4.71 \times 10^4$ esu/cm^2. The value of the mobility cited above corresponds to the value of the bulk-electron concentration $n_0 = 1.07 \times 10^{16}$ cm^{-3}. Combining with the band bending $e\phi_s = 0.59$ eV, the thickness of the depletion layer is computed to be $l_0 = 0.283$ μm.

The electronic screening will be considered in the Debye approximation in which the difference between electronic motions parallel and perpendicular to the surface is neglected. However, in this approximation there still remains the problem of how to estimate the Debye screening length l_D. For the present purpose of a qualitative investigation, we have calculated l_D in terms of the bulk-electron concentration and found $l_D^{-1} = q_s = 4.75 \times 10^5$/cm. If we consider the fact that the electrons which develop into the depletion layer also interact with the surface phonon, this value for the reciprocal of the screening length should be slightly overestimated. We shall return in Sec. VI to the question of how the correct screening length changes our results quantitatively.

The calculated frequency dependence of the amplification rate of the surface phonon is shown in Fig. 3 for a drift parameter $x = 10$ and $q_x = q_y = q/\sqrt{2}$. The solid and the dot-dash curves are those obtained based on the electron wave func-

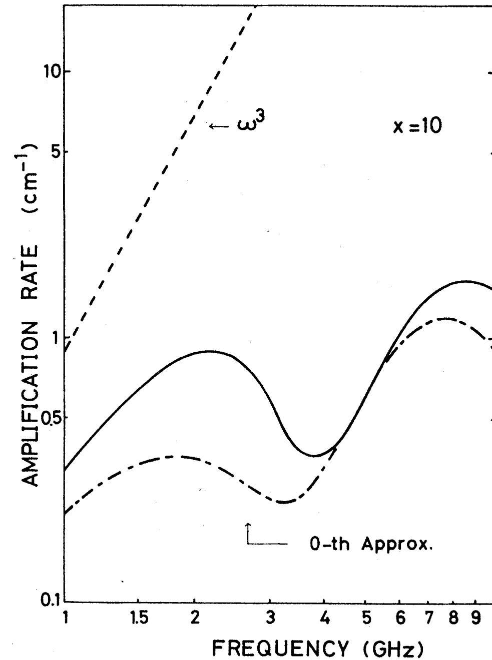

FIG. 3. Frequency dependence of the amplification rate for a drift parameter $x = 10$. The solid curve and the dot-dash curve are amplification rates calculated from the electron wave functions Eqs. (12) and (6), respectively. The dashed line represents the amplification rate proportional to ω^3 which is obtained assuming no depletion layer.

tions Eqs. (12) and (6), respectively. We have also shown by the dashed curve the amplification rate evaluated assuming no surface depletion layer. The frequency dependence of the latter is expressed as ω^3 for frequencies corresponding to wave numbers satisfying $q \ll q_s$. Referring to this, the existence of the depletion layer at the semiconductor surface turns out to change the frequency dependence drastically when the wavelength of the surface phonon becomes the same order of magnitude as the width of the depletion layer. It should be noted that the wavelength is 3.58 μm at 1-GHz frequency of the surface phonon. For a 0.283-μm thickness of the depletion layer, a remarkable change in the frequency dependence of the amplification rate can be seen in the range from 1 to 10 GHz, that is, two bumps appear in this frequency region and the amplification rate decreases rapidly at higher frequencies. Furthermore, if we compare two curves which reflect the existence of the depletion layer, we find that the electrons distributed in the depletion layer act to shift the frequencies to higher values at which two local maxima and a minimum of the amplification are reached, as well as to make the overall magnitude of the amplification rate larger.

These notable characteristics of the frequency dependence of the amplification rate brought by the existence of the surface depletion layer may be understood qualitatively with the aid of the electric potential produced by the surface phonon. In Fig. 4, we have illustrated the electric potentials induced piezoelectrically in the isotropic medium. Roughly speaking, the frequency dependence of the amplification rate is determined by the product of q^4, which comes from the electronic screening assuming $q \ll q_s$, and the square of the overlapping

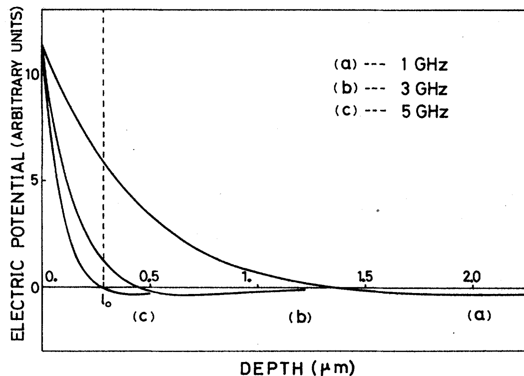

FIG. 4. The depth dependence of the electric potentials which are produced by the surface phonon in the isotropic semiconductor are drawn for 1-, 3-, and 5-GHz frequencies. The moduli of the potentials decrease exponentially at large distances away from the surface.

integral of the electron wave functions and the electric potential produced by the surface phonon, i.e.,

$$q^4 \left| \int_0^\infty \psi^*_{\kappa'}(z) \hat{\varphi}_{\vec{q}}(z) \psi_\kappa(z)\, dz \right|^2 .$$

For the electron wave functions, we shall employ those of the zeroth-order approximation, for simplicity. In this case, there are no electrons for $z < l_0$. The dominant term of the above expression is the one for $\kappa = \kappa'$. Since

$$|\psi_\kappa(z)|^2 = \theta(z - l_0)[1 - \cos 2\kappa(z - l_0)]/L ,$$

and the second term in the square brackets oscillates with z, we neglect the latter against unity. Then, the above expression is reduced to

$$q^4 \left| \int_{l_0}^\infty \hat{\varphi}_{\vec{q}}(z) dz \right|^2 .$$

In practice it must be oversimplified, but we can still understand the essential point of the problem in terms of this expression as follows.

First, let us consider what will happen when we increase the frequencies continuously from 1 GHz. With the help of Fig. 4, we see that the square of the integral decreases with increasing frequencies up to a frequency of about 3 GHz at which it becomes zero. In this process, owing to the presence of the factor q^4, the amplification rate will grow with frequency at much less than 3 GHz. However, it will begin to decrease at some frequency and tend to zero corresponding to the vanishing of the integral. The correct amplification, of course, never vanishes and has a local minimum at a certain frequency around 3 GHz, since we must sum up all contributions arising from the integrals for $\kappa \neq \kappa'$. If the frequencies grow higher, the modulus of the integral begins to increase and then the amplification rate also does. Because the potential diminishes exponentially with increasing frequencies and at very high frequencies this effect will surpass the rising of the amplification due to the factor q^4, the amplification rate of the surface phonon will finally fall off exponentially. It should be remarked that the sign change of the electric potential induced by the SAW is associated with the fact that the particle motion of the Rayleigh-type SAW is backward directed at a depth closer to the free surface, but forward directed at greater depth.

VI. DISCUSSION

In this paper we have investigated the acoustoelectric interaction of the surface phonon in the

isotropic semiconductor by solving for the motions of the electrons near the surface. Applying the interaction to the study of the amplification rate of the surface phonon in the GHz region, we found that the existence of the surface depletion layer in the n-type semiconductor has a crucial effect on its frequency dependence when the thickness of the depletion layer becomes the same order of magnitude as the wavelength of the surface phonon. The calculated wave functions of the electrons are approximate ones, but they should describe qualitatively the correct characteristics of the genuine wave functions.

In addition to the approximation made concerning the electron wave functions, we made several simplifications in order to derive the results, that is, we neglected the finite relaxation time of the electrons and used an approximate electron screening length. We shall now comment briefly on these points and finally discuss the anisotropy which would be important in practical applications of the theory to real semiconductors.

A. Relaxation time of electrons

For the value of the mobility given in Sec. V, ql becomes unity at about 1-GHz frequency of the surface phonon, so that the effect of the finite relaxation time or the broadening of the energies of the electrons may contribute somewhat to the interaction of surface phonons of the frequencies considered in this paper. Generally speaking, this effect may modify to some extent the magnitude of the amplification rate and the frequency dependence, especially at lower frequencies near 1 GHz. However, the qualitative characteristics of the amplification rate, that is, the appearance of two local maxima in the frequency range from 1 to 10 GHz, will not be changed by incorporating the finite relaxation time of the electrons because their existence is substantially due to the depth dependence of the electron wave functions and not to the detailed structure of their energy levels.

B. Screening effect

In the numerical example, the electronic screening is incorporated by following the Debye approximation. There we have estimated the screening length simply in terms of the bulk-electron concentration, although this may not be justified quantitatively. Strictly speaking, the screening length should be evaluated by taking into account the local variation of the electronic density, which can be obtained from Eq. (5) combined with Eq. (12). In the surface depletion layer, the electron concentration is small, and so the screening effect would be expressed by an effective screening length slightly larger than the one we have used, that is, the screening may be less effective. However, the local electron concentration decreases rapidly going from $z = l_0$ into the depletion layer, so that even if we use the correct screening length, it will only act to increase the overall magnitude of the interaction by some modest amount.

C. Lower-frequency phonon

In this paper, we have developed the acoustoelectric interaction along a microscopic quantum-mechanical approach assuming the relaxation time of the electrons to be infinite. Hence, strictly speaking, the results are valid in the high-frequency regime satisfying $ql \gg 1$. However, the main characteristics caused by the effect of the surface depletion layer on the frequency dependence of the amplification rate will also be present for a surface phonon of low frequency, that is $ql < 1$, provided an appropriate condition is satisfied. According to the discussion given in Sec. III, the appearance of the two bumps seems to occur when the wavelength of the surface phonon changes exceeding the thickness of the depletion layer. Therefore, for the semiconductor of lower bulk-electron concentration, or of thicker depletion layer, we may find the same kind of amplification characteristics at lower frequencies of the surface phonon if the corresponding wavelengths become of the same order of magnitude as the thickness of the depletion layer.

D. Elastic anisotropy

As was mentioned in Sec. I, the real crystalline semiconductors have elastic anisotropy. The main characteristics of the SAW introduced by the presence of the anisotropy are: (i) the particle motion may include three independent orthogonal components, against just two in isotropic media; (ii) the SAW velocity depends upon the direction of propagation; and (iii) the decay constants may be complex instead of real numbers and the particle displacements decay oscillating away from the surface.

As for the first point, it is generally shown that the SAW traveling in a direction parallel to a crystal axis of a cubic crystal. e.g., in the [100] direction on the (001) plane, has no displacement component normal to the sagittal plane. So, in this case, the wave has the form of a pure Rayleigh wave. When the direction of propagation is rotated away from the crystal axis, the displacement normal to the sagittal plane grows gradually. Hence, as far as the SAW traveling in a direction rotated by a small angle from the crystal axis is concerned, the isotropic approximation may be

justified.

The second point is not so relevant to the isotropic assumption and its effect can be taken into account by calculating the sound velocity in terms of the isotropically approximated elastic constants.

From the viewpoint of the acoustoelectric interaction of the surface phonons in piezoelectric semiconductors the last item may be the most serious characteristic introduced by the anisotropy confuting the isotropic approximation. The electric potential produced by the SAW in real semiconductors will also manifest itself as an oscillatory decay away from the surface, having the same behavior as the particle displacements. Since the main characteristics of the amplification rate which we have obtained depend essentially upon the spatial decay profile of the potential as well as on the local density of the electrons near the surface, this oscillatory damping might modify our results unless the oscillation is not so prominent.

More detailed and quantitative discussions about all these problems are outside of the scope of the present paper and we will come to them in a future publication.

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[8]For electrons obeying nondegenerate statistics, the explicit calculation based on Eq. (6) gives the local electron concentration

$$n(z)=n_0\theta(z-l_0)\{1-\exp[-2m(z-l_0)^2/\hbar^2\beta]\}.$$

This result differs from Eq. (7) in the region

$$0\leq z-l_0\lesssim(\hbar^2\beta/2m)^{1/2}=2.10\times10^{-16}(m_0/m)^{1/2}T\ (\text{cm}),$$

where m_0 is the free-electron mass. At $T=77$ K and for $m=0.07m_0$, the right-hand side is 1.03×10^{-3} μm and can be neglected in comparison with l_0 which is $O(10^{-1}\ \mu\text{m})$.

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