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Variation of the Shubnikov–de Haas amplitudes with ionic scattering in silicon inversion layers

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The Shubnikov–de Haas effect has been measured in silicon inversion layers as a function of temperature, carrier density, and oxide charge density. The dependence on oxide charge density provides a critical test of the theory of ionic scattering in the Shubnikov–de Haas effect. The data are found to be in excellent agreement with theory. The temperature dependence permits the study of electron effective masses. We give a more detailed account of the dependence of the electron effective mass on ionic scattering and the effective mass extrapolated to zero ionic scattering.

INTRODUCTION

Oscillatory magnetoconductance, Shubnikov–de Haas effect, has been observed in many materials, and has been used primarily as a probe of the Fermi surface. The Shubnikov–de Haas (SdH) effect was first observed in silicon inversion layers of Fowler *et al.*,[1] and used to establish the two-dimensional nature of the electric subbands. Smith and Stiles[2] have analyzed the temperature dependence of the Shubnikov–de Haas oscillation amplitudes, and extracted the effective mass for the inversion-layer electrons over a wide range of carrier densities. Many papers then appeared that attempted to explain the observed dependence of the effective mass on carrier density in terms of many-body effects.[3-6] Experimental results have recently been shown to vary widely with experimental conditions,[7] hence bringing into question the validity of their interpretation. In addition, Trylski[8] has recently suggested that the effective mass derived from a SdH experiment will not be accurate as long as ionic scattering is important.

We have utilized metal-oxide-semiconductor (silicon) field-effect-transitor (MOSFET) samples, in which Na$^+$ ions were purposely introduced into the gate oxide to study the effects of changing the ionic scattering on the SdH amplitudes. Our motivation for doing this experiment was twofold. First, we wanted to look at the effects of ionic scattering on the effective masses determined from the SdH experiment, and to extrapolate the mass to the limit of zero ionic scattering. It is also interesting to examine directly the effect of the ionic scattering on the SdH amplitudes. The effect of scattering was first considered by Dingle.[9] The only experimental study that attempted to test this portion of the SdH theory was an alloying experiment reported by Shoenberg.[10] The experiment was consistent with the scattering theory of Dingle, although the experimental error was such that it was not possible to test the theory adequately. The present experiment is found to be in excellent agreement with the detailed predictions of the Dingle scattering theory.

THEORY

The theory of the SdH effect has a long history. The most recent and complete theories have been given by Adams and Holstein[11] and by Kubo *et al.*[12] The expression for the oscillatory part of the conductivity is given by[7]

$$A \propto \frac{2\pi k T \omega_c^2 \tau^2}{\hbar \omega_c (1+\omega_c^2\tau^2)^2} \, \frac{\cos(2\pi E_F/\hbar\omega_c + \phi)}{\sinh(2\pi^2 kT/\hbar\omega_c)} \times \exp\left(-\frac{\pi}{\omega_c \tau}\right), \tag{1}$$

where T is the temperature, k is Boltzmann's constant, $2\pi\hbar$ is Planck's constant, E_F is the Fermi energy, ω_c is the cyclotron frequency, ϕ is a phase factor, and τ is the scattering time, which gives the broadening of the Landau levels in the Dingle scattering term and in the preexponential. This expression only represents the leading term in a Fourier expansion and also ignores spin and valley splittings. Equation (1) predicts a particular dependence of amplitude on the scattering time, which we propose to test.

The scattering time due to ionic scattering associated with the conductivity mobility has been calculated by Stern and Howard,[13] and has been tested experimentally by Hartstein *et al.*[14] using the same samples as used in this study. Both of these results show that the inverse of the scattering time depends linearly on the concentration of ionic scatterers at the Si-SiO$_2$ interface:

$$1/\tau = BN_{\mathrm{ox}}, \tag{2}$$

where N_{ox} is the concentration of ionic scatterers and B is independent of N_{ox}. The scattering time for electrons associated with the dc conductivity is not necessarily the same as the scattering time associated with the scattering of electrons in Landau levels. However, we do expect both to be linearly dependent on the density of scatterers.

The expected dependence of the SdH amplitudes on the concentration of scatterers for the impurity scattering dominated regime is obtained by combining Eqs. (1) and (2). For simplicity, we have only taken the case where $\omega_c\tau \gg 1$. With this approximation, the amplitude becomes

$$A = A_0 N_{\mathrm{ox}}^2 \exp[-(\pi B/\omega_c) N_{\mathrm{ox}}] \;, \qquad (3)$$

where A_0 is proportional to the amplitude in the absence of ionic scattering.

EXPERIMENT

The samples used in this study have been described elsewhere.[15] The samples were MOSFET devices with an oxide thickness of 1000 Å. Na^+ ions were deliberately introduced into the gate oxide, and subsequently drifted to the Si-SiO_2 interface at elevated temperatures under the influence of an applied electric field. Since the number of Na^+ ions at the interface can be controlled in a single device, the scattering from these Coulomb centers can be readily investigated. In these experiments a known number of Na^+ ions was drifted to the interface, the sample was subsequently cooled to liquid-helium temperatures, and the SdH effect was measured as a function of both carrier concentration ($1.5\times10^{12}\ \mathrm{cm}^{-2} \leq N_s \leq 5\times10^{12}\mathrm{cm}^{-2}$) in the inversion layer and temperature ($4.2\ \mathrm{K} > T > 1.8\ \mathrm{K}$). The device was then raised in temperature and a different amount of Na^+ was drifted to the interface. The process was repeated for seven different Na^+ drifts. A final drift with N_{ox} the same as N_{ox} in the first drift was used to ensure the consistency of the experiment.

The SdH effect was measured in the conductivity, the transconductance (the derivative of conductivity with respect to gate voltage), and the derivative of the transconductance to ensure that any higher harmonics of the amplitude oscillation do not play a role. The best signal-to-noise ratio was obtained with the transconductance measurement, and will be presented here. The data were all taken at a magnetic field of $H = 38.8$ kG. The field was chosen so that the observed oscillations were sinusoidal in the gate voltage (also in E_F and N_s) over the entire range of Na^+ concentration, gate voltage, and temperature reported. A single magnetic field value was used, since it has been observed that the effective mass determined by SdH measurements in silicon inversion layers is dependent on magnetic field.[7]

RESULTS AND DISCUSSION

The amplitude of the SdH oscillations was obtained by interpolating between the peaks and valleys of the oscillations to obtain the desired carrier concentrations (gate voltages). Figure 1 shows a semilogarithmic plot of A/N_{ox}^2 as a function of Na^+ concentration, N_{ox}, for various temperatures and for a representative carrier concentration. The SdH theory [Eq. (3)] predicts the observed linear relationship. Although only representative curves are shown in Fig. 1 the linear relationship is found to fit the data over the entire range of temperature, carrier density, and Na^+ concentration investigated.

The data that have been obtained allow us to determine the dependence of the parameter B in Eqs. (2) and (3) on both temperature and carrier density. The results for the dependence on carrier density are shown in Fig. 2. We have found that B is essentially temperature independent. Figure 2 shows some dependence on carrier density. This dependence is most likely due to increased screening as the carrier density is increased. The fact that B is independent of temperature indicates that the ionic scattering is independent of temperature, as one might expect.

It is interesting to compare the values of B obtained from the SdH experiment with the values obtained from the dc mobility experiment.[14] B in the mobility experiment is both larger and shows

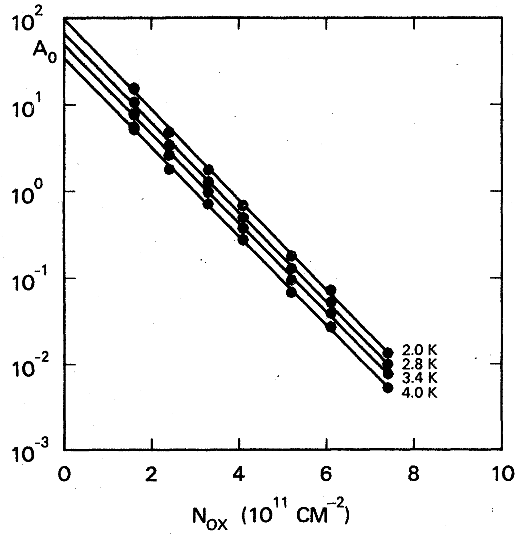

FIG. 1. Typical dependence of SdH amplitudes on oxide charge density. Curves are shown for $N_s = 3.5\times10^{12}$ cm^{-2} and the temperatures noted.

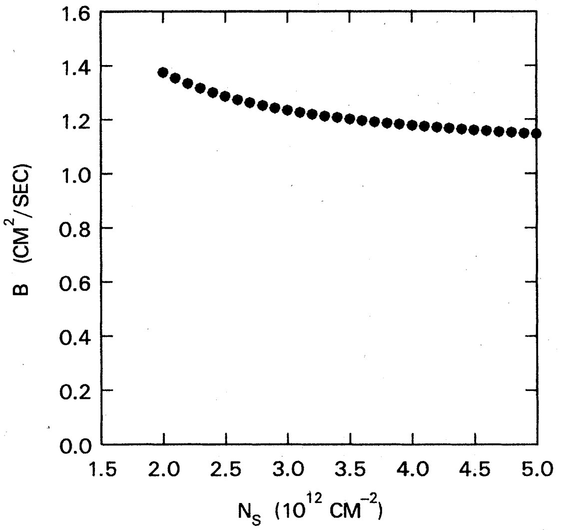

FIG. 2. Dependence of the scattering parameter B on carrier density. The data shown apply for all temperatures studied (1.8 K $\leq T \leq$ 4.2 K).

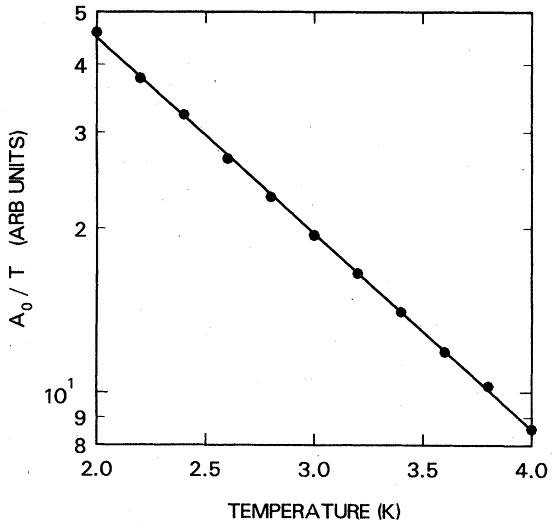

FIG. 3. Dependence of the SdH amplitude extrapolated to zero oxide charge (A_0) on temperature. Slope of plot is proportional to effective mass. Data shown are for $N_s = 3.5 \times 10^{12}$ cm^2.

more of a dependence on carrier density than B from the SdH experiment, a rather surprising result. However, this is consistent with our previously reported results[7] that scattering times obtained from SdH measurements are often larger than the scattering times taken from mobility measurements for both inversion and accumulation layers. Perhaps these more-detailed measurements of the scattering from ionized impurities can be used to shed more light on these differences.

The effective mass for the inversion-layer electrons can be determined from the temperature dependence of the SdH amplitudes, as indicated in Eq. (1). The temperature dependence is contained in the sinh term of that expression. For our data the sinh term can be approximated by an exponential, vastly simplifying the data analysis.

The data that have been obtained allow us to go one step further. Plots such as shown in Fig. 1 were used to obtain the SdH amplitudes (A_0) extrapolated to zero Na$^+$ concentration. This means that we have the SdH amplitudes in the absence of ionic scattering, and any possible difficulties in determining the mass when ionic scattering is important should disappear. To obtain the mass, the amplitudes are plotted as $\log(A/T)$ vs T. Figure 3 illustrates the procedure for A_0 and one particular carrier density. The slope of this plot is obtained from Eq. (1) by noting $\omega_c = eH/m^*c$, and is given by $2\pi^2 kcm^*/\hbar eH$ where c is the speed of light, e is the electronic charge, and m^* is the electron effective mass. This analysis was carried out for the extrapolated amplitudes A_0 and for the original amplitudes A.

The masses determined in this way are shown in Figs. 4 and 5 for the original amplitudes and for the extrapolated amplitudes, respectively. These effective mass results have been previously published.[7] The effective masses are seen to depend on Na$^+$ concentration. This is the type of result which was suggested by Trylski.[8] The mass determined from the extrapolated amplitudes can be considered as the mass in the absence of ionic scattering, and as such can be considered to be the best determination, at present, of the effective mass of electrons in silicon inversion layers. However, it remains an open question as to

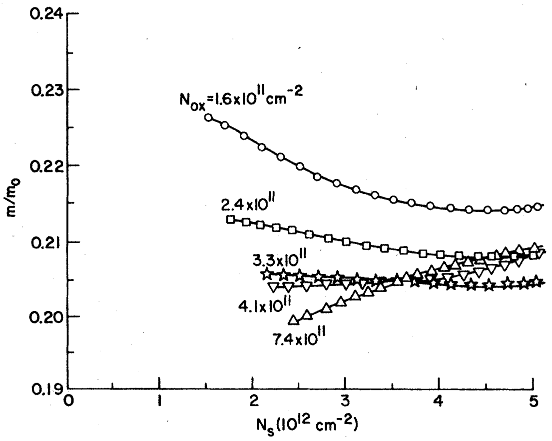

FIG. 4. Effective mass as a function of carrier concentration determined for different oxide charge densities.

whether other types of scattering, such as surface roughness scattering, would play a prominent role in determining the mass in this procedure.

It should be noted that the effective mass obtained by extrapolating the *amplitudes* is not the same as the effective mass obtained at low oxide charge or the mass obtained by extrapolating *masses* to zero oxide charge. Also, the theory gives no basis for a variation in the effective mass with oxide charge. These inconsistencies lead us to mistrust both the theory and the procedures used to obtain effective masses in inversion and accumulation layers.

SUMMARY

We have investigated the dependence of the SdH amplitudes on the concentration of ionic scatterers for inversion-layer electrons. The results are found to be in excellent agreement with the predictions of the Dingle scattering theory. The difference between the effectiveness of the ionic scattering in the SdH experiment, as opposed to the effectiveness in a conductivity experiment, has been noted. It is quite possible that these differences arise from the detailed microscopies of the scattering in each situation. One might expect that in a magnetic field the electrons would tend to be scattered into regions with lower than average density of scatterers, whereas in the conductivity case this would not be possible. The essential difference comes from the fact that in a magnetic field the electrons are already localized in Landau orbits.

We have obtained the effective mass for electrons in the inversion layer in the absence of ionic scattering. However, we have not shown whether or not the mass determination is independent of surface roughness scattering. This extrapolated mass, which has a value of approximately $0.21m_0$, differs from the bulk silicon mass of $0.19m_0$. Therefore, although we have demonstrated that scattering is important to the mass determination and have obtained a mass with the effects of ionic scattering removed, we can not be certain that even this is the mass that should be compared with theoretical calculations.

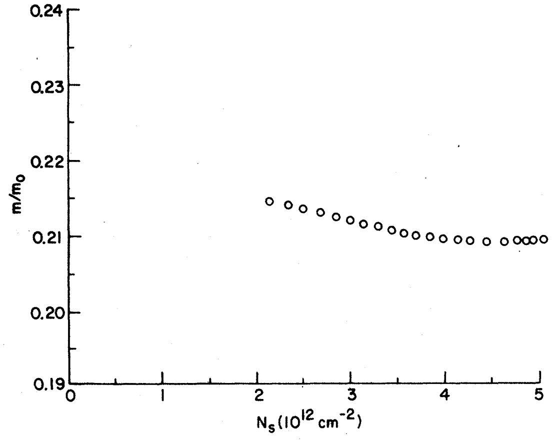

FIG. 5. Effective mass as a function of carrier concentration derived by extrapolating SdH amplitudes to zero oxide charge.

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