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Ultrasonic attenuation in copper and the temperature dependence of the nonlinearity parameter

M. Nandanpawar and S. Rajagopalan
Department of Physics, Nagpur University, Nagpur-440010, India
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Phonon-phonon interactions give rise to hypersonic losses whose estimation is possible using theories proposed by Akhieser and Mason. These estimations involve the nonlinearity parameter D which is a function of second- and third-order elastic moduli. In the present work, the temperature dependence of the nonlinearity parameter of copper has been evaluated in the temperature range 50–300 K, for longitudinal as well as shear wave propagation along the $\langle 100 \rangle$ axis, making use of the experimental results on the temperature variation of second- and third-order moduli. The nonlinearity parameter D_{long} for longitudinal waves is found to increase with temperature, while an opposite effect has been obtained for shear waves. Our estimations of ultrasonic attenuation in Cu at 150 MHz using the temperature variation of D_{long} are in good agreement with experimental results. The agreement is remarkably good at very low temperatures where the calculations of previous workers depart considerably from experiment.

INTRODUCTION

Phonon-phonon interactions play a very important role in acoustic attenuation; the acoustic losses at low temperatures indicate that the attenuation is caused by direct conversion of acoustic waves into heat. Klemens[1] suggests that the attenuation is due to direct interactions of the acoustic wave with individual phonons, when the product of the angular frequency ω of the acoustic wave and the relaxation time τ, is greater than unity. When $\omega\tau < 1$, direct interactions of the sound wave with phonons cannot be followed and other techniques have to be sought. Another method of conversion of acoustic energy into thermal energy was first proposed by Akhieser,[2] by taking into consideration the temperatures of the phonons propagating in individual longitudinal and shear modes.

ATTENUATION DUE TO PHONON VISCOSITY

The alterations in temperature cause a storage of thermal energy which are proportional to alterations in elastic moduli associated with the strain.

Mason,[3] by assuming that there is no dispersion in ultrasonic velocity, has shown that the associated stress

$$T_1 = \left(C^s_{11} + 3\sum_i E_i(\gamma^1_i)^2\right)S_1 + 3\sum_i E_i\gamma^1_i \ , \tag{1}$$

where C^s_{11} is the elastic modulus resulting for zero heat exchange between any of the modes. E_i is here the thermal energy associated with each direction and mode, and γ^1_i the corresponding Grüneisen number.

The term $3\sum_i E_i\gamma^1_i$ represents the stress required to keep the volume constant as the temperature varies. The term $3\sum E_i(\gamma^1_i)^2$ shows that there is an addition ΔC to the elastic modulus, as postulated by Akhieser. For a longitudinal mode, the increase in modulus resulting from the difference between adiabatic and isothermal conditions should be subtracted and, therefore,

$$\Delta C = 3\sum_i E_i(\gamma^1_i)^2 - \gamma^2 C_v T_0 \ , \tag{2}$$

where γ is the volume Grüneisen constant defined in terms of the temperature expansion coefficient α,

$$\gamma = 3\alpha B/C_v \ . \tag{3}$$

Here B is the bulk modulus and C_v the specific heat per unit volume of the material. The Grüneisen numbers γ^1_i can all be calculated for different directions in the crystal when all the third-order moduli have been measured.

Since the Akhieser effect is essentially a relaxation process, it results in an attenuation

$$A = \omega^2 \Delta C\tau / [2\rho V^3(1+\omega^2\tau^2)]\,(\text{Np/cm}) \ . \tag{4}$$

Here ρ is the density and τ is the relaxation time associated with the equilibration of the energy stored by the suddenly applied strain in the form of phonon temperatures. For shear waves, τ is determined by the thermal relaxation time τ_{th}

$$\tau_{\text{th}} = 3\text{K}/C_v \overline{V}^2 \ , \tag{5}$$

where K and $\overline{V}$ stand, respectively, for the thermal conductivity and Debye average velocity given in terms of the longitudinal and shear velocities V_l and V_s by

$$\overline{V}^3 = \tfrac{1}{3}(2/V_s^3 + 1/V_l^3) \ . \tag{6}$$

Experimental results bear out the fact that the relaxation time for longitudinal waves is about twice the thermal relaxation time.

The attenuation, then, can be written as

$$A = \frac{E_0(D/3)\omega^2\tau}{2\rho V^3(1+\omega^2\tau^2)} \text{ (Np/cm)} , \qquad (7)$$

where

$$D = \frac{3}{E_0}\left(3\sum_i E_i(\gamma_i^1)^2 - \gamma^2 C_v T_0\right), \qquad (8)$$

which is the "nonlinearity parameter" governing attenuation in the Akhieser region.

CALCULATION OF D

Equation (8) suggests that the nonlinearity parameter is temperature dependent. We have calculated the temperature variation of D for copper from the second-order[4] and third-order[5] moduli in the range 50–300 K. We have also taken into consideration the temperature dependence[6] of the Debye characteristic temperature Θ_D which is required in the estimation of the thermal-energy density E_0. The variation of specific heat and density has been taken from literature.[7] For shear waves, $\gamma = 0$ and γ_i^1 has to be replaced by γ_i^5.

Our calculations indicate that D is a monotonically increasing function of temperature for longitudinal waves, and a decreasing one for shear waves (Fig. 1). At higher temperatures, the variation in D is small. Table I gives a representative calculation leading to the estimation of D.

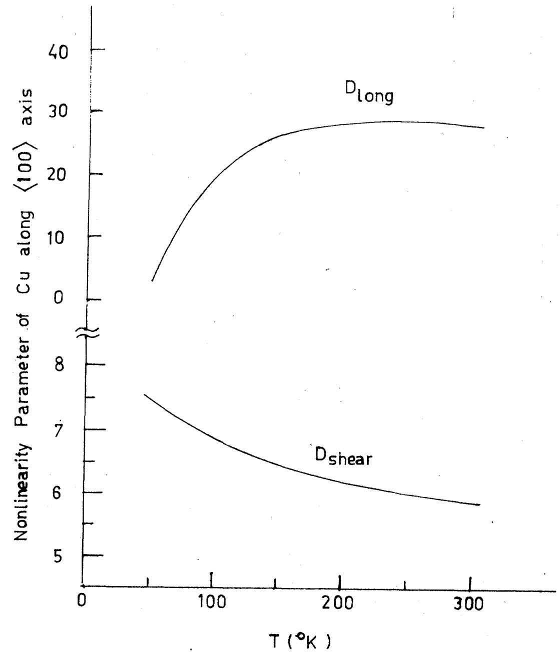

FIG. 1. Temperature variation of D_{long} and D_{shear} in copper for wave propagation along the ⟨100⟩ axis.

ULTRASONIC ATTENUATION

Mason and Rosenberg[8] have evaluated the temperature variation of thermoelastic attenuation, electron viscosity and phonon viscosity losses, employing an arbitrary value $D_{\text{long}} = 42.8$, in the temperature range 50–300 K for longitudinal waves along ⟨100⟩ axis at 150 MHz. The sum of these three effects is not in good agreement with

TABLE I. γ_i^1 along ⟨100⟩ axis and calculation of D of copper at 300 °K.

No.	No. of waves $= n$	$n\gamma_i^1$	$n(\gamma_i^1)^2$
1	1	2.953 95	8.725 82
2	2	6.321 93	19.983 39
3	2	4.326 74	9.360 34
4	2	4.944 95	12.226 27
5	2	0.180 12	0.016 22
6	2	2.558 29	3.272 42
7	2	7.604	28.910 41
8	2	4.944 95	12.226 27
9	4	11.732 45	34.412 59
10	4	−0.501 59	0.062 90
11	4	6.502 06	10.569 19
12	4	9.484 54	22.489 12
13	4	10.073 82	25.370 46
14	4	2.965 16	2.198 04

$$\sum n\gamma_i^1 = 74.5428 \qquad \sum n(\gamma_i^1)^2 = 189.823\,44$$

$$\gamma_{\text{av}} = \frac{\sum n\gamma_i^1}{39} = 1.9114$$

$$D_{\text{long}} = 3\left[\frac{3\sum n(\gamma_i^1)}{39} - \frac{\gamma_{\text{av}}^2 C_v T}{E_0}\right] = 27.80$$

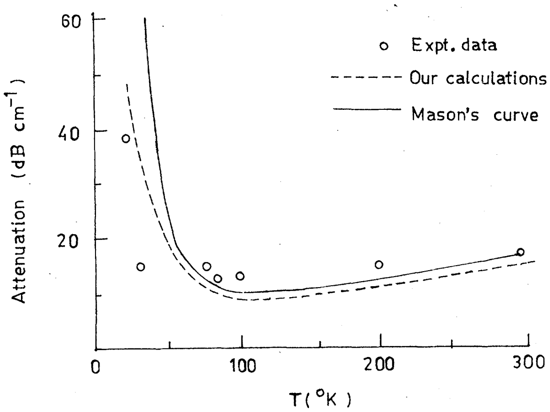

FIG. 2. Ultrasonic attenuation in copper for longitudinal wave propagation along the ⟨100⟩ axis at 150 MHz.

the experimental values at low temperatures, as indicated by the solid line in Fig. 2. Our calculation of the phonon-viscosity contribution using the estimated temperature dependence of D_{long} leads to values of total attenuation which have much closer agreement with the measured values even at low temperatures. The trend of our calculations (shown by the dashed line in Fig. 2) bears a resemblance to the experimental points, while the other curve shows an almost exponential increase. The discrepancy between experiment and the theoretical predictions of former workers in the low-temperature region is considerably reduced by our approach. Qualitatively, this explains the temperature-dependent behavior of D. One may also assess the behavior of D from a theoretical concept[9] utilizing the lattice thermal conductivity of Cu as obtained by White and Woods,[10] or by the Leibfried formula.[11,12]

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[1]P. G. Klemens in *Physical Acoustics*, Vol. IIIB, edited by W. P. Mason (Academic, New York, 1965).

[2]A. Akhieser, J. Phys. USSR 1, 277 (1939).

[3]W. P. Mason, in Ref. 1.

[4]W. C. Overton, Jr. and J. Gaffney, Phys. Rev. 98, 969 (1955).

[5]K. Salama and G. A. Alers, Phys. Rev. 161, 673 (1967)

[6]D. A. Waldorf, Phys. Chem. Solids 16, 90 (1960).

[7]*American Institute of Physics Handbook*, 3rd ed. (McGraw-Hill, New York, 1972), Chap. 4.

[8]W. P. Mason and A. Rosenberg, J. Acoust. Soc. Am. 45, 470 (1969).

[9]R. P. Singh and G. S. Verma, Phys. Rev. 171, 838 (1968).

[10]G. K. White and S. B. Woods, Philos. Mag. 45, 1343 (1954).

[11]G. Leibfried and E. Schlöman, Nachr. Akad. Wiss. Goettingen Math. Phys. Kl. 2 11, 71 (1954).

[12]S. K. Kor, U.S. Tandon, and G. Rai, Phys. Rev. B 6, 2195 (1972).