Ultrasonic attenuation in superconducting zinc*

Sushil Auluck
Department of Physics, Punjab Agricultural University, Ludhiana-141004, India
(Received 5 October 1976)

The differences in the Zn ultrasonic attenuation data of different workers are analyzed. The superconducting energy gaps deduced from our analysis of the ultrasonic-attenuation data of Cleavelin and Marshall are consistent with the gaps deduced from the knowledge of the Fermi surface and the electron-phonon mass enhancement factor.

The measurement of the superconducting energy gap by ultrasonic attenuation has recently picked up a lot of momentum. The ultrasonic attenuation data provide a useful method for measuring the energy-gap anisotropy as well as detecting multiple energy gaps. Extensive experimental studies of the ultrasonic attenuation in zinc have been performed by Cleavelin and Marshall[1] (hereafter CM) and by Dobbs, Lea, and Peck[2] (hereafter DLP). There is a disagreement between these two sets of data. We present a resolution of this disagreement and analyze the CM data in the light of the known Fermi surface and the electron-phonon mass enhancement factor.[3]

The point of difference in the two sets of data is that DLP obtain a smaller anisotropy in the energy gap $\Delta_{\vec{k}}$ compared to the CM data. This can easily be traced to the sample purity. The work of Markowitz and Kadanoff[4] demonstrates that impurities wipe out the anisotropy in $\Delta_{\vec{k}}$ (or any electronic property, for that matter) because they are random and they destroy the symmetry. Thus we expect a less pure sample to have a smaller anisotropy in $\Delta_{\vec{k}}$ compared to a more pure sample. The sample purity is ascertained from the resistivity ratio R or the mean free path Λ. For the samples used by DLP R is 2000 and for the CM samples R can be estimated from Λ. Using $\Lambda \simeq 0.020R$,[2] we obtain $R \simeq 10\,000$ for the CM samples. Thus the CM samples are more pure compared to the samples used by DLP. Hence we expect more anisotropy in $\Delta_{\vec{k}}$ for the CM data. This is in agreement with the data, according to which $\langle a^2 \rangle = 0.025 \pm 0.001$ for DLP and $\langle a^2 \rangle = 0.040$ for CM data. Thus we suggest that whenever a comparison is made of the anisotropy in $\Delta_{\vec{k}}$ obtained by different workers or methods, it is of some importance to specify the sample purity.

As a result of the selective coupling of the electrons with the ultrasonic waves, this method provides an excellent way of measuring anisotropy in $\Delta_{\vec{k}}$. From the conservation of momentum and energy it follows that the electrons which interact with the sound waves are confined to an "effective zone." At high frequencies ($q\Lambda >> 1$), the sound waves interact with the electrons moving perpendicular to $\vec{q}$ (the sound-wave propagation direction). Morse[5] showed that for free electrons in the limit $q\Lambda >> 1$,

$$\cos\theta = V_S/V_F ,$$

where V_S is the velocity of sound, V_F is the Fermi velocity, and θ is the angle between V_S and the electron velocity V_F. Since $V_S/V_F \simeq 10^{-3}$, $\theta \simeq 90°$. This equation has been generalized by Leibowitz[6]:

$$\cos\theta \simeq 1/q\Lambda + V_S/V_F .$$

The larger the value of $q\Lambda$, the more selective is the ultrasonic data. For $q\Lambda \simeq 1$ we lose completely the selectivity of the process and all the electrons on the Fermi surface contribute to the attenuation. We are thus left with a thermal selection rule. At low temperatures the attenuation in the superconducting state will be dominated by the smaller $\Delta_{\vec{k}}$ while at higher temperatures the larger $\Delta_{\vec{k}}$ contributes to the attenuation. This is exactly what happens on the low-temperature specific-heat data. Hence at $q\Lambda \simeq 1$ we expect[6] the ultrasonic attenuation data to be similar to the low-temperature specific-heat data.

In the following, we demonstrate these ideas to the CM data for zinc. We do not consider the DLP data since in their samples the anisotropy in $\Delta_{\vec{k}}$ is greatly influenced (reduced) by impurities. It might appear at first sight that their data should be of interest for comparison with the specific-heat data because of the more complete ($q\Lambda < 1$) smearing of the energy-gap anisotropy compared to the CM data and that their attenuation $\alpha(T)$ at lower T should suggest the smaller gap. The appearance of the smaller gap manifests itself around $T_c/T \gtrsim 4$. The DLP data do not go to this temperature range. We note that the energy gaps deduced by DLP lie inside the range of the gaps deduced by CM, reinforcing our impurity-based analysis. Without any excess to the data it is not possible to explain more quantitatively the differences in the two sets of data.

The range of the $q\Lambda$ values for the CM data is 0.13–2.69 except for $\vec{q}$ along [0001]. We see that for $\vec{q}$ in the basal plane and for $q\Lambda \lesssim 1$ the data can be analyzed in terms of two distinct energy gaps, the smaller gap showing up at lower temperatures. This is in accordance with the specific-heat data.[7] Thus for $q\Lambda \lesssim 1$, the attenuation data shows a temperature dependence similar to the specific-heat behavior. The specific-heat data showed a smaller gap at $T_c/T \gtrsim 4$, which was assigned to the cap. The acoustic data are suggestive of two gaps whose ratio ranges from 1.21 to 1.33 (mean 1.27). The close agreement between these and our $\Delta_l/\Delta_m = 1.31$ suggests that these gaps are probably due to the lens and monster. Thus these two data taken together suggest the presence of three distinct energy gaps in zinc. This lends credence to our earlier theoretical analysis.[3] The reason for not seeing the third gap in the acoustic data may be that the lowest temperature attained was not sufficiently low to show its contribution.

We note that for $q\Lambda > 1$ only one gap is observed. We now identify the energy gaps deduced for the $q\Lambda > 1$ limit with distinct and single sheets of the Fermi surface selected for each orientation of $\vec{q}$. For $\vec{q}$ along [0001] CM find that their data show $q\Lambda > 6.8$ for all magnetic field orientations and for all frequencies investigated.[8] In this case the electrons moving perpendicular to $\vec{q}$ contribute to the attenuation. Plots of the partial density of states $dN(0)$ of electrons with velocity component $|V_{k\perp}|$ perpendicular to [0001] have been given earlier.[3] The monster gives the dominant contribution to α since it has a large $dN(0)$ for $|V_{k\perp}| \simeq 0$. For $\vec{q}$ along [0001] the attenuation data can be analyzed in terms of a single energy gap and this we assign to the monster. Thus $2\Delta_m = 3.0 k_B T_c$. This is in agreement with the gap deduced from the microwave absorption measurements by Evans *et al.*[9] for most of the Fermi surface (except near the c axis). For $\vec{q}$ in the basal plane, the mesh of points used to generate the $dN(0)$ plots is not sufficiently dense to obtain reliable results for $dN(0)$. Hence we resort to the "effective zone" argument.[10] For $\vec{q}$ along $[10\bar{1}0]$ the attenuation will again be dominated by the monster while for $\vec{q}$ along $[11\bar{2}0]$ the major contribution to the attenuation (80%) comes from the lens.[2] We therefore identify the energy gaps deduced (for $q\Lambda > 1$) for $\vec{q}$ along [1010] as that arising from the monster, i.e., $2\Delta_m = 2.80 k_B T_c$, and for $\vec{q}$ along $[11\bar{2}0]$ as arising from the lens, i.e., $2\Delta_l = 3.8 k_B T_c$. This Δ_m is in agreement with the Δ_m deduced from the $\vec{q}$ along [0001] data and the Δ_l is in agreement with the gap deduced from the microwave absorption measurements by Evans *et al.*[9] for the Fermi surface near the c axis (i.e., lens). Thus for the $q\Lambda > 1$ data we obtain 1.27 and 1.37 (mean 1.32) for the ratio Δ_l/Δ_m. This is in agreement with our earlier theoretical calculations[3] and with the analysis of the $q\Lambda \lesssim 1$ data.

Thus we see that we can understand the difference in the attenuation data on zinc of CM and DLP, in a simple manner based on sample purity. The assignment of the energy gaps is facilitated with a knowledge of the Fermi surface and the electron-phonon mass enhancement factors. The CM data suggests the existence of two distinct energy gaps which can be assigned to the lens and monster sheets of the Fermi surface. The ratio Δ_l/Δ_m for $q\Lambda \lesssim 1$ and for $q\Lambda > 1$ are in agreement with one another and with our earlier theoretical predictions.[3] The low-temperature specific-heat data suggest the existence of a third and smaller energy gap which has been assigned to the cap.[7] Thus these two data suggest the existence of three distinct energy gaps for zinc, one for each sheet of the Fermi surface, and the ratio of these gaps is in agreement with our earlier theoretical calculations.

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*Work supported by a grant from the UGC, India.

[1]C. R. Cleavelin and B. J. Marshall, Phys. Rev. B 10, 1902 (1974).

[2]E. R. Dobbs, M. J. Lea, and D. R. Peck, Proc. R. Soc. A 334, 379 (1973).

[3]R. W. Stark and L. M. Falicov, Phys. Rev. Lett. 19, 795 (1967); S. Auluck, J. Low Temp. Phys. 12, 601 (1973); R. W. Stark and S. Auluck (unpublished). The Fermi surface of zinc consists of three main sheets: first, band hole sheet centered at H (cap); second, band multiply connected hole sheet centered at Γ (monster); and third, band electron sheet centered at Γ (lens). Based on the calculation of the electron-phonon mass enhancement factors we suggested three distinct energy gaps (one for each sheet of the Fermi surface) for zinc in the ratio 2.30:1.76:1.00. The largest gap Δ_l is assigned to the lens, the smallest gap Δ_c is assigned to the cap, and the intermediate gap Δ_m to the monster. We take two views: (i) The energy gap $\Delta_{\vec{k}}$ is anisotropic and varies by a factor of 2.30 over the Fermi surface. In this case it is meaningful to talk in terms of mean-square anisotropy $\langle a^2 \rangle$. This is about 0.020 for our model for zinc. This approach is useful when analyzing bulk properties like effect of alloying on T_c and the temperature dependence of the critical field $H_c(T)$. Here $\langle a^2 \rangle$ is obtained by fitting the data to a theroetical formula. (ii) Zinc has three distinct energy gaps (one for each sheet of the Fermi surface) whose ratios are $\Delta_l/\Delta_m = 1.31$ and $\Delta_l/\Delta_c = 2.30$. This approach is useful when a selection process is in operation as in temperature dependence of specific heat, microwave absorption, and ultrasonic attenuation.

[4]D. Markowitz and L. P. Kadanoff, Phys. Rev. 131, 563 (1963).

[5]R. W. Morse, Prog. Cryog. 1, 211 (1959).

[6]J. R. Leibowitz, Phys. Rev. 133, A84 (1964).

[7]R. W. Stark and S. Auluck, in *Anisotropy Effects in Superconductors*, edited by H. W. Weber (Plenum, New York, 1976).

[8]However, in their Table I they quote values of $q\Lambda$ in the

range 180–1010. CM have determined their Λ from fitting their data to Pippard's expression for α. Since Λ is a fitting parameter it is not possible to ascertain the physical significance of $q\Lambda$ so obtained. There seems to be no simple reason to account for such large $q\Lambda$ and the apparent anisotropy of this $q\Lambda$ from that of other $\vec{q}$ directions. Moreover, such large anisotropy in $q\Lambda$ cannot be explained by either q anisotropy of the Λ anisotropy given in their Fig. 3.

[9]J. B. Evans, M. P. Garfunkel, and D. A. Hays, Phys. Rev. B 1, 3629 (1970).

[10]See, M. J. Lea, J. D. Llewellwyn, D. R. Peck, and E. R. Dobbs, Proc. R. Soc. A 334, 357 (1973) for the effective zones corresponding to different $\vec{q}$ directions.