Dynamics as a substitute for replicas in systems with quenched random impurities

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Traces over random degrees of freedom may be performed *ab initio*, without replicas, provided one is using a dynamic description of the system. An effective Lagrangian is obtained after the random degrees of freedom are eliminated. An order parameter is then defined, à la Edwards and Anderson, in terms of the time-persistent part of the correlation function. Taking a system with random temperature as a model example, we characterize to all orders the equation of state in the ordered phase. It is found that this equation contains odd powers of the order parameter that destabilize the simplest solutions. These terms suggest that the $t^{-1/2}$ decay law, found by Ma and Rudnick for the correlation function, is restricted to a region close to the presumed transition point. The formalism is particularly suitable for treating dynamics in the ordered phase.

INTRODUCTION

Investigations of the properties of random systems (random magnetic fields, temperatures, couplings or bonds) are awkward because *physical* quantities must be averaged over the random degrees of freedom.[1] This difficulty is often circumvented and with success, by using the replica method.[2-4] that yields physical results in the limit $n \to 0$. However, this procedure involves an unphysical order parameter and also leads to certain analytic difficulties at low temperatures. Other authors have therefore employed direct term-by-term expansions of physical averages of static[5] and dynamic[6] quantities. This second approach can be performed more systematically by using the Lagrangian formulation of dynamics introduced by Martin, Siggia, and Rose (MSR).[7] Indeed, the trace over random degrees of freedom can then be performed *ab initio* leaving a Lagrangian form where all random degrees of freedom have been eliminated by integration. The removal of closed loops, which is achieved in the replica method by giving them vanishing weight n, comes about in the present method by the impossibility to build closed loops since they cannot be constructed from retarded propagators alone.

The effective Lagrangian without randomness may be conveniently employed to derive the critical behavior near the ferromagnetic transition. At first glance, the procedure may seem like a step backward for phase transitions of the spin-glass type.[8] At such transitions the commonly used[2,9] order parameter involves distinguishing an average $Q \simeq \langle\langle\phi\rangle\langle\phi\rangle\rangle$, over randomness and over field variables, while our Lagrangian contains only field variables. That this is not a step backward, but can be rather turned into an advantage, is due to the possibility of characterizing the ordered phase physically as Edwards and Anderson did in terms of a long- (infinite) range correlation in time. Specifically, we define the averaged order parameter as the persistent time part of the correlation function

$$Q(x-x') = \langle\phi(x,t)\phi(x',t')\rangle\big|_{|t-t'|\to\infty} ,$$

where ϕ is the (continuous) spin field. For $x=x'$, Q is the order parameter used by Ma and Rudnick.[10] The source H that couples to Q is then proportional to $\delta(\omega)$, where ω is the Fourier conjugate of $t-t'$.

To obtain the "equation of state" in the ordered phase one expresses H as a functional of the correlation function, *an equation that splits into two distinct equations* when the correlation function acquires a time-persistent part Q. One of them governs the averaged order parameter Q (yielding the equation of state in the ordered phase) and the other gives the time-decaying part of the correlation. For simplicity, in this paper we carry out the analysis for a system with a random temperature. We find that the equation of state for this system contains destabilizing odd powers of Q which suggest that the $t^{-1/2}$ decaying law of Ma and Rudnick[10] is limited to a region close to the transition temperature. The same kind of destabilizing term is also found in the random-bond Ising model. It also was present in the replica Hamiltonian of Harris *et al.*[9] The method we shall discuss can be used with an order parameter that depends on time center of mass $t+t'$ (and the space variable $x+x'$) to treat dynamics in the ordered phase. This extension will be discussed elsewhere.

This paper is organized as follows: In Sec. I we derive the generating functional (i.e., the effective Lagrangian) for a system with quenched random degrees of freedom including a random easy axis. In Sec. II, we show how to treat the ordered phase of a (nonrandom) ferromagnetic system by the MSR technique. In Sec. III, we introduce the time-persistent order parameter Q and express the source H in terms of the correlation functions. In Sec. IV, we derive the equation of state for Q and analyze and discuss its structure.

I. GENERATING FUNCTIONAL AND EFFECTIVE LAGRANGIANS

Consider the Hamiltonian

$$H=\int d^dx\,\{\tfrac{1}{2}\phi(x)(r_0-\nabla^2)\phi(x)+u_0[\phi^4(x)/4!]-h(x)\phi(x)\}+H_R \tag{1}$$

whose random part

$$H_R=\int d^dx\,\{r_R(x)[\tfrac{1}{2}\phi^2(x)]+u_R(x)\times[\phi^4(x)/4!]-h_R(x)\phi(x)\} \tag{2}$$

contains random variables $r_R(x), u_R(x), h_R(x)$ governed by probability laws which may, for example, be expressed by their cumulants. Assume also that $\phi(x,t)$ obeys a stochastic equation of motion. For convenience, let it be of the Landau-Ginzburg type[10,11]

$$\frac{\partial\phi(t)}{\partial t}=-\Gamma_0[\delta H/\delta\phi(t)]+\zeta(t) \tag{3}$$

with the usual noise correlations

$$\langle\zeta\rangle=0\ , \tag{4}$$

$$\langle\zeta(x,t)\zeta(x',t')\rangle=2\Gamma_0\delta(t-t')\delta(x-x') \tag{5}$$

that ensure an equilibrium governed by (1). For the MSR generating functional $\hat{Z}_R(l)$ we have[12–14]

$$\hat{Z}_R(l)=\int \mathfrak{D}\phi\,\mathfrak{D}\hat{\phi}\,J\{\phi\}\exp\left(L\{\phi,\hat{\phi}\}+\int d^dx\,dt\,l(x,t)\phi(x,t)\right)\ , \tag{6}$$

$$L\{\phi,\hat{\phi}\}=\int d^dx\,dt\left[i\hat{\phi}(x,t)\left(-\Gamma_0^{-1}\frac{\partial\phi(x,t)}{\partial t}+(r_0-\nabla^2)\phi(x,t)+\frac{u_0}{3!}\phi^3(x,t)-h\right)-\Gamma_0^{-1}\hat{\phi}^2(x,t)\right]+L_R\ , \tag{7}$$

$$L_R\{\phi,\hat{\phi}\}=\int d^dx\,dt\left[i\hat{\phi}(x,t)\left(r_R(x)\phi(x,t)+u_R(x)\frac{\phi^3(x,t)}{3!}-h_R(x)\right)\right]\ . \tag{8}$$

Here $i\hat{\phi}(x,t)$ is a field conjugate to $\phi(x,t)$ and we have taken traces over the noise. The Taylor expansion coefficients in l are the correlation functions of the field ϕ. The Jacobian[14] $J\{\phi\}$ ensures that $\hat{Z}_R(l)$ satisfies the normalization condition

$$\hat{Z}_R(0)=1\ . \tag{9}$$

In particular $\hat{Z}_R(0)$ is independent of the random variables. The only effect of $J\{\phi\}$ is to subtract the self-contraction $\langle\hat{\phi}(x,t)\phi(x,t)\rangle$ wherever it occurs. What is left is a perturbation expansion whose propagator $\langle\hat{\phi}(x,t)\phi(x',t')\rangle$ is *retarded* which thus constrains to zero all *closed* loops built with it.[15] It is immediately seen that, in the absence of the source l, and because of the nature of the $\phi,\hat{\phi}$ couplings, there exists necessarily one such closed loop in each connected diagram contributing to $\hat{Z}_R(l=0)$; hence, the normalization (9). We may take advantage of this property of the generating functional to average $\hat{Z}(l)/\hat{Z}(0)$ over the random degrees of freedom. For simplicity we shall assume that the only nonvanishing cumulants in the probability law for the random degrees of freedom are given by (white noise)

$$\begin{aligned}\langle r_R(x)r_R(x')\rangle&=\Delta_r\delta(x-x')\ ,\\ \langle u_R(x)u_R(x')\rangle&=\Delta_u\delta(x-x')\ ,\\ \langle h_R(x)h_R(x')\rangle&=\Delta_h\delta(x-x')\ .\end{aligned} \tag{10}$$

We then obtain, after all traces are taken,

$$\hat{Z}(l)=\int\mathfrak{D}\phi\,\mathfrak{D}\hat{\phi}\,J\{\phi\}\times\exp\left(L_e\{\phi\hat{\phi}\}+\int d^dx\,dt\,l(x,t)\phi(x,t)\right)\ , \tag{11}$$

$$L_e\{\phi\hat{\phi}\}=L\{\phi\hat{\phi}\}+\frac{1}{2}\int d^dx\left[\Delta_r\left(\int dt\,i\hat{\phi}(x,t)\phi(x,t)\right)^2+\Delta_u\left(\int dt\,i\hat{\phi}(x,t)\phi^3(x,t)\right)^2+\Delta h\left(\int dt\,i\hat{\phi}(x,t)\right)^2\right]\ . \tag{12}$$

This is the effective generating functional. It contains new quadratic terms[16] in $\hat{\phi}$ that are nonlocal in time [Fig. 1(a)]. In the standard replica trick, the $n=0$ limit eliminates closed loop (loops that appear when one averages Z instead of $\ln Z$) leaving only contributions with no closed loops for correlation functions (or with one single loop for $\langle \ln Z \rangle$). Here the *same result* is obtained by a $(\int \hat{\phi}\phi\, dt)^2$ interaction [Fig. 1(b)] and the Deker-Haake theorem.

The effective Lagrangian (12) may be analyzed in the vicinity of the ferromagnetic transitions by the same techniques that have proven efficient in studying critical statics[17] and dynamics.[14,18,19] At the price of introducing a renormalization constant associated with the new vertex $(\phi\hat{\phi})^2$ (and the corresponding Wilson function) one may then reproduce the results obtained via the Wilson iterative method for statics[4,20,21] and dynamics.[6] The forms (11) and (12) are also quite handy for computing instanton contributions.

Extension to fields $\hat{\phi}$ with n components is trivial. In the presence of a random easy axis $\vec{a}_R(x)$, i.e., of an extra term δH_R in (1)

$$\delta H_R = \frac{1}{2}\int d^dx \left[(\vec{a}_R\cdot\vec{\phi})^2 - \frac{1}{n}(\vec{a}_R)^2(\vec{\phi})^2 \right] \tag{13}$$

the result is slightly more complicated,

$$\delta L_e \simeq -\frac{1}{2}\int d^dx\, \mathrm{Tr}\ln A \ , \tag{14}$$

$$A_{jk} = \delta_{jk}\left(\frac{1}{\Delta a} - \frac{2}{n}\int dt [i\hat{\vec{\phi}}(t)\cdot\vec{\phi}(t)] \right. + \int dt[\, i\hat{\phi}_j(t)\phi_k(t) \left. + i\hat{\phi}_k(t)\phi_j(t)] \right) \ . \tag{15}$$

Expanding the log in powers of $\int dt\, i\hat{\phi}\phi$ one then relies on dimensional counting to keep only quadratic terms (the linear term vanishes) in the critical region, i.e.,

$$\delta L_e \simeq -\Delta_a^2 \left[\sum_{jk} \left(\frac{1}{2}\int dt (i\hat{\phi}_j\phi_k + i\hat{\phi}_k\phi_j) \right)^2 - \frac{1}{n}\left(\int dt (i\hat{\vec{\phi}}\cdot\vec{\phi}) \right)^2 \right] \ . \tag{16}$$

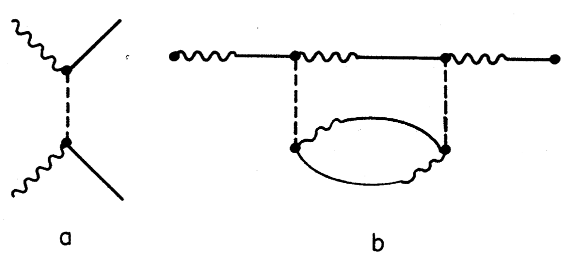


FIG. 1. (a) Interactions arising from random temperature degrees of freedom $\simeq \Delta_r$. The dotted line only carries zero frequency. (b) Closed loop (built with Δ_r interactions) contribution to response function.

II. ORDERED PHASE: NONRANDOM SYSTEMS

Let us first examine how the MSR functional (6) can be used for systems which have no random degrees of freedom in the ordered phase. In order to work with the observable $\langle \phi \rangle \equiv m$ we perform a Legendre transform on $\hat{Z}(l,h)$ or rather on

$$\hat{W}(l,h) \equiv \ln \hat{Z}(l,h) \ . \tag{17}$$

We define a pseudo-free-energy $\hat{\Gamma}$

$$\hat{W}(l,h) + \hat{\Gamma}(m,\hat{m}) = lm + h\hat{m} \ , \tag{18}$$

$$m = \frac{\partial \hat{W}}{\partial l} \ , \tag{19}$$

$$\hat{m} = \frac{\partial \hat{W}}{\partial h} \ . \tag{20}$$

It is easy to see that $\hat{\Gamma}$ is given by the sum of all one-particle-irreducible (1-PI) diagrams or, more precisely, all 1-PI diagrams constructed with the (unperturbed) propagators for the response and correlation functions

$$\langle i\hat{\phi}\phi \rangle_0 \equiv R_0(k,\omega) = \frac{\Gamma_0}{-i\omega + \Gamma_0(k^2+r_0)} \ , \tag{21}$$

$$\langle \phi\phi \rangle_0 \equiv C_0(k,\omega) = \frac{2\Gamma_0}{|-i\omega + \Gamma_0(k^2+r_0)|^2} \ . \tag{22}$$

We may write

$$\hat{\Gamma}(m,\hat{m}) = L\{m,\hat{m}\} + \cdots \ , \tag{23}$$

where the dots stand for terms with one loop or more. Here we only have displayed the zero loop contribution of $\hat{\Gamma}$, namely $L(m,\hat{m})$. Note further that (18) is stationary with respect to changes of $m,\hat{m}$ (at l,h fixed). The stationarity equations are

$$h(x,t) = \left[\Gamma_0^{-1}\frac{\partial}{\partial t} + (r_0 - \nabla^2) \right] m(x,t) + \frac{u_0}{3!}m^3(x,t) + \Gamma_0\hat{m}(x,t) + \cdots \ , \tag{24}$$

$$l(x,t) = \left[\Gamma_0^{-1}\frac{\partial}{\partial t} + (r_0 - \nabla^2) \right] \hat{m}(x,t) + \frac{u_0}{2!}\hat{m}(x,t)m^2(x,t) + \cdots \ , \tag{25}$$

where, once more, only the zero loop contributions are exhibited. We see immediately that $\hat{m}=0$ unless the source is present (i.e., $l\neq 0$) and that (24) becomes the equation of state when $l=0$. From (24) we may also obtain the free energy (as shown in the Appendix). Note finally that if we were given any of the three equations (23)–(25) we could infer the Lagrangian L governing the dynamics [and generating (23)–(25)].

III. ORDERED PHASE: RANDOM SYSTEMS

The contributions arising from the random terms $L_e - L$ introduce couplings that are completely *delocalized in time.* Consider the trivial system in which the only nonvanishing coupling is Δ_h. We then have

$$C(k,\omega)=\frac{2\Gamma_0}{|-i\omega+\Gamma_0(r_0+k^2)|^2}+\frac{\delta(\omega)}{(r_0+k^2)^2}\Delta_h \;, \tag{26}$$

that is,

$$C(k,t)=\frac{1}{r_0+k^2}\exp[-|t|\Gamma_0(r_0+k^2)]+\frac{\Delta_h}{(r_0+k^2)^2}\;. \tag{27}$$

The second term is generated by the (delocalized) *source* $\Delta_h\delta(\omega)$, the first results from the (local) source $1/\Gamma_0$.

This type of persistent-time correlation, survives when the coupling u_0 is switched on. It is obviously due to the fact that the eliminated magnetic field h_R is random in space but persists with the same strength and orientation for all times. If, instead, there is a random temperature fluctuation around r_0, the persistent-time correlation may only set in below some critical value r_{0f}.

In the spirit of Edwards and Anderson[2] we define the order parameter as the persistent part of the correlation function, i.e., the part of

$$\begin{aligned}\langle(\phi-\langle\phi\rangle)(\phi-\langle\phi\rangle)\rangle &\equiv C(k,\omega)\\ &\equiv \tilde{C}(k,\omega)+Q(k)\delta(\omega)\end{aligned} \tag{28}$$

which is proportional to $\delta(\omega)$. Here averages are taken over all fields (ϕ, $\hat{\phi}$, noise and random fields). $Q(k)$ is an *averaged* order parameter (the analog of m). As a field [the analog of $\phi(k,\omega)$] we shall need $\Phi_{K\Omega}(k)$ where K and Ω are the Fourier conjugate of the center-of-mass variables.

Below the ferromagnetic transition temperature (when there is one) the full correlation function $\langle\phi\phi\rangle$ acquires a disconnected part which is persistent in time and space. This piece, which is present whenever h is present, is *not* incorporated in Q.

In order to obtain an equation for C it is useful to introduce sources $\frac{1}{2}L\phi\phi+\frac{1}{2}H\hat{\phi}\hat{\phi}$ and perform a further Legendre transform

$$\hat{\Gamma}=\mathrm{tr}[\ln\underline{G}-\tfrac{1}{2}(\underline{G}^0)^{-1}\underline{G}]+K\{m,\hat{m};\underline{G}\}\;, \tag{29}$$

where $\underline{G}$ is a 2×2 matrix

$$\underline{G}=\begin{bmatrix}C & R\\ R^* & \hat{C}\end{bmatrix} \tag{30}$$

and the function

$$\hat{C}=\langle\hat{\phi}\hat{\phi}\rangle \tag{31}$$

vanishes when the sources L and l vanish. The free inverse propagator is given by

$$(\underline{G}^0)^{-1}=\begin{bmatrix}L & i\omega+\Gamma_0(r_0+k^2)\\ -i\omega+\Gamma_0(r_0+k^2) & H+\Delta_h\delta(\omega)+r_0\end{bmatrix} \tag{32}$$

and the functional K has the property that it does not break into two disjoint pieces when two lines are cut (no self-energy parts). Otherwise K contains all diagrams built with the vertices u_0, Δ_r, $\Delta_u,m,\hat{m}$ connected by C, R, or $\hat{C}$ lines.

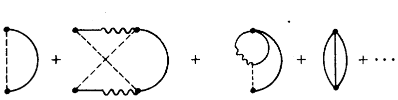
Note that $\hat{W}$ is now stationary under variations of C, R, and $\hat{C}$ or m and $\hat{m}$. The stationarity equation for $\hat{C}$ yields

$$\Gamma_0+H+\Delta_h\delta(\omega)=\frac{C}{|R|^2-C\hat{C}}+\frac{\delta K}{\delta\hat{C}}\;. \tag{33}$$

In the absence of the sources L and l, $\hat{C}=\hat{m}=0$, and we have

$$\Gamma_0+\Delta_h\delta(\omega)=\frac{C(k,\omega)}{|R(k,\omega)|^2}+\frac{\delta K}{\delta\hat{C}}\{C,R,R^*,\hat{C}\}|_{\hat{C}=0}\;, \tag{34}$$

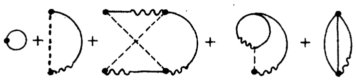
where the last term is the "mass operator" associated with a correlation function

$$\frac{\delta K}{\delta\hat{C}}\equiv-\Sigma_{\hat{\phi}\hat{\phi}}=$$

(35)

For simplicity, we have only written the terms to two loop order which are present when $m=0$ (and when there is no random coupling, i.e., $\Delta_u=0$). The full lines stand for correlations (C), the mixed lines for responses (R), the dotted lines for Δ_r interactions. Together with (34) and (35) we also have an equation for R

$$R^{-1}(k,\omega)=-\frac{i\omega}{\Gamma_0}+r_0+k^2+\frac{\delta K}{\delta R^*}\{C,R,R^*,\hat{C}\}|_{\hat{C}=0}\;, \tag{36}$$

where the last term is the usual mass operator

$$\frac{\delta K}{\delta R^*} \equiv -\Sigma_{\hat\phi\hat\phi} =$$  $$+\cdots \quad (37)$$

The response R and correlation C are determined by (34)–(37) and related by the fluctuation-dissipation theorem. Under appropriate circumstances (e.g., when $\Delta_h \neq 0$, or in a certain range of temperature) Eqs. (34) and (35) may contain both a $\delta(\omega)$ part and a part that is nonlocalized in frequency. That is, Eq. (34) may *split into two distinct equations.* We separate them out by writing (34) with $C(k,\omega)$ given by Eq. (28). The new equation for the averaged order parameter $Q(k)$ is obtained by collecting the $\delta(\omega)$ contributions to (34) which we now discuss.

IV. EQUATION OF STATE IN THE ORDERED PHASE

Let us first make two remarks on the graphical expansion of Eqs. (34) and (37).

(i) The graphs are built with local vertices (u_0) and time delocalized vertices (Δ_r). The "random" dotted lines always carry zero frequency (i.e., there is frequency conservation at each end of the Δ_r coupling). Thus if a correlation line carries a $\delta(\omega)$, it will be preserved by Δ_r vertices.

(ii) If there are no "random" interaction, Δ_r, then $\Sigma_{\phi\hat\phi}$ would be built from one single tree of R lines each of which ends in a noise field ζ. Pairs of noise fields coalesce to give C lines (those pairings are obtained automatically when the trace over the noise field is taken). Likewise $\Sigma_{\hat\phi\hat\phi}$ is built as a *pair* of trees. The coalescence of a pair of noise fields gives rise to two types of C lines corresponding to *intertree* or *intratree* connections. Distinguishing between those two types of correlation lines we may write

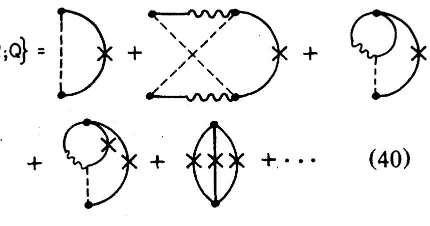
$$\Sigma_{\hat\phi\hat\phi} \equiv \Sigma_{\hat\phi\hat\phi}\{C,R;C_e\} \ , \quad (38)$$

where the last argument stands for the *intertree* correlation functions. Topologically the intertree C_e lines may be defined as the minimum set of correlation lines that must be cut to separate $\Sigma_{\hat\phi\hat\phi}(\omega)$ into two pieces. They are also the only set of lines that may carry the external frequency ω, and they are thus the set of lines that must carry the $Q(k)\delta(\omega)$ part of $C(k,\omega)$ in order to contribute to the $\delta(\omega)$ part of (38).

We then have

$$\Delta_h\delta(\omega) = \frac{Q(k)\delta(\omega)}{|R(k)|^2} - \Sigma_{\hat\phi\hat\phi}\{C;R;Q\}\delta(\omega) \ , \quad (39)$$

where the portion of $\Sigma_{\hat\phi\hat\phi}$ with all intertree correlations replaced by Q's corresponds to the Σ^d of Ma and Rudnick. It falls apart into two pieces, if all the random Δ_r lines *and* all intertree Q lines are opened (the Q lines are drawn as full lines bearing one cross)

$$\Sigma^d_{\hat\phi\hat\phi} \equiv \Sigma_{\hat\phi\hat\phi}\{C;R;Q\} =$$  $$+\cdots \quad (40)$$

Equations (39) and (40) are the *equation of state* for the phase with persistent time correlation. In Eq. (40) the Q^2 term is made of one intratree and one intertree Q, whereas the Q^3 term is all intertree. This last term is opposite in sign to the Q^2 term (u_0 is repulsive but Δ_r is attractive). Therefore, unless higher-order terms damp it out, the Q^3 term acts as a destabilizing term.

In the presence of the source field Δ_h (random magnetic field) there always is a nonzero Q solution of (39) and (40). In zero field ($\Delta_h = 0$) a nonzero solution may exist below some critical temperature T_f defined by the existence of a *zero eigenvalue* for the matrix

$$\delta\Delta_h(k)/\delta Q(l) \ . \quad (41)$$

To lowest order this condition reduces to the well-known condition for r_{0f}

$$1 = \Delta_r \int \frac{d^dk}{(r_{0f}+k^2)^2} \ . \quad (42)$$

The equations of state (39) and (40) describe the ordered phase and its order parameter below T_f. The companion equations

$$\Gamma_0 = \tilde C(k,\omega)/|R(k,\omega)|^2 - \Sigma^c_{\hat\phi\hat\phi} \ , \quad (43)$$

$$\Sigma^c_{\hat\phi\hat\phi} \equiv \Sigma_{\hat\phi\hat\phi}\{C;R;C\} - \Sigma^d_{\hat\phi\hat\phi}\delta(\omega) \ , \quad (44)$$

where $\Sigma_{\hat\phi\hat\phi}$ is given by (35) and $\Sigma^d_{\hat\phi\hat\phi}$ by (34), give $\tilde C$, the time-dependent (nonpersistent) part of the correlation function, the only part that is related to R via the fluctuation-dissipation theorem

$$\tilde C(k,\omega) = (2/\omega)\,\mathrm{Im}R(k,\omega) \ . \quad (45)$$

It is easily seen that if $\Sigma_{\hat\phi\hat\phi}\{C;R;Q\}$ were *linear* in its intertree Q dependence (linear in the Q exhibited argument) the Ma-Rudnick theorem[10] would be exact and would lead to $\tilde C(\omega) \simeq \omega^{-1/2}$ near zero frequency. This is certainly true at T_f and nearby where the destabilizing term in (40) is irrelevant. However, when the value of Q is not small, the cubic term becomes more important, cutting off the $\omega^{-1/2}$ divergence. The destabilizing terms and their effects require more careful investigation. Do higher-order terms damp them out? If not, and they are genuinely present, where do they drive the ordered phase, and with what characteristic time?

The formalism outlined here allows for the detailed investigation of these questions. It also enables us to study the dynamics of the order parameter. Indeed by letting the sources $L\phi\phi + H\hat{\phi}\hat{\phi}$ depend on the center of mass in space and time, we generate an average order parameter $Q_{K,\Omega}(k)\delta(\omega)$ (similar to $m_{k,\omega}$ in the ferromagnetic case). By inspecting the resulting (time-dependent) equation of state one may infer the form of the Lagrangian of the composite field variables $\Phi_{K,\Omega}(k)$ which describe the dynamics of the order parameter. We will examine this question elsewhere.

Finally let us note that the same treatment may be applied to the random bond spin model by (i) transforming to continuous spin variables (Stratonovich transform); (ii) guessing a stochastic equation of motion for them. In its simplest form it may be taken as a purely relaxational equation for the ferro and antiferro combinations of the spin variables, with the appropriate noise fluctuation to ensure a proper equilibrium limit; and (iii) expanding the hyperbolic tangent of the local field variables, and averaging the resulting MSR Lagrangian over bonds.

In this case we also find that the equation of state contains cubic attractive terms. This fact may not be too surprising since an attractive coupling appears in the replica Hamiltonian of Harris, Lubensky, and Chen[9] in the $n=0$ limit.

Note added in proof. Global considerations show that the destabilizing terms of the Ising model (or models with fixed length fields) are damped out for large fields in contrast with, e.g., the above continuous ϕ^4 model where the destabilization appears to be genuine.

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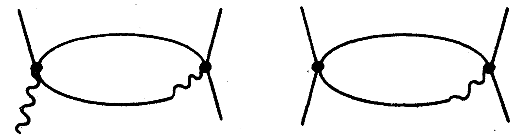

FIG. 2. (a) $\hat{m}m^3$ contribution to $\hat{\Gamma}$ (weight $\frac{1}{2}$). (b) Corresponding m^4 contribution to Γ (weight $\frac{1}{8}$).

APPENDIX: FREE ENERGY IN THE MSR FORMALISM

The standard free energy is defined as

$$W(h) + \Gamma(m) = hm \quad , \tag{A1}$$

where

$$\delta W/\delta h = m \quad , \tag{A2}$$

i.e.,

$$\delta\Gamma/\delta m = h \quad . \tag{A3}$$

Equation (24) taken for $l=0$, $\hat{m}=0$ gives $h\{m\}$ where $m(x,t)$ is not necessarily time independent. By integrating (24) one may define

$$\Gamma\{m\} = \int_0^m \delta m h\{m\} + \Gamma\{m=0\} \quad , \tag{A4}$$

that is,

$$\Gamma\{m\} = \int dt\, d^dx \left[\frac{1}{2} m(x,t) \left(\Gamma_0^{-1}\frac{\partial}{\partial t} + (r_0 - \nabla^2)\right) m(x,t) + \frac{u_0}{4!} m^4(x,t)\right] + \cdots + \Gamma\{m=0\} \quad , \tag{A5}$$

where we have only exhibited the zero loop term. The dots stand for the 1-PI graphs built with MSR rules where one $\hat{m}$ is replaced by one m (and then $\hat{m}$ set equal to zero) with the appropriate weight due to the functional integration. An example is given in Fig. 2. Finally $\Gamma\{m=0\}$ are the 1-PI graphs containing no $\hat{m}$, obtained directly from the statics. The time-dependent free energy (A5) enjoys the usual stationarity property under changes of $m(x,t)$.

The form (A5) may also be retrieved by direct resummations of

$$W = W(u_0=0) + \int_0^{u_0} du_0 \langle d^dx\, dt \frac{\phi^4(x,t)}{4!}\rangle \quad , \tag{A6}$$

where the average is computed with the weight e^L. It is easily shown by effecting the frequency integrals that (A6) itself reduces to $-\ln \mathrm{tr} e^{-H}$ in the $m=0$ disordered phase.

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