

Electrical resistivity of antiferromagnetic chromium near the Néel temperature

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Precision electrical-resistivity measurements around the Néel temperature, T_N , of three samples of polycrystalline chromium with different thermal histories are reported. The critical exponent of $d\rho/dT$ below T_N is found to be 0.65 ± 0.05 over temperature ranges that are sample dependent. Thus, the Suezaki-Mori theory for the resistivity of an antiferromagnet predicting the dominant behavior below T_N is confirmed. However, above T_N , our data cannot be fit into any meaningful power law.

I. INTRODUCTION

In recent years considerable effort has been spent on the measurement of equilibrium and non-equilibrium properties of solids in the immediate vicinity of second-order critical points. Much of this work has been stimulated by the success of the scaling-law approach in predicting the critical exponents. It is expected that the critical exponents and certain other parameters will be universal in the sense that they are identical for apparently vastly differing systems, provided the physical dimensionality d and the number of degrees of freedom η of the order parameter are the same, and provided the interactions have a sufficiently short range. The effect of critical fluctuations on the electrical resistivity in a variety of ferromagnets can be understood by taking into consideration both the short-range and long-range correlations as well as the details of the Fermi surface. The situation is less clear in the case of resistive anomalies at the Néel temperature T_N of antiferromagnets. The various theories proposed are in substantial disagreement in many details. Among the existing few experimental studies, results on chromium especially are inconsistent indicating possible sample- or system-dependent characteristics. The role of critical fluctuations in antiferromagnets was first studied by Suezaki and Mori¹ and later, but independently, by Geldart and Richard.² The former authors concluded that energy gaps in the electron dispersion relation reflect the long-range order below T_N , while the long-range correlations dominate throughout the paramagnetic region assuming the scaling relations. Thus, according to these investigators, the contribution to $d\rho/dT$ due to the critical scattering above the Néel temperature is given by

$$\frac{d\rho}{dT} = -C\epsilon^{-(\alpha+\gamma-1)}, \quad T > T_N \quad (1)$$

with $\epsilon = (T - T_N)/T_N$, where C is a constant and α and γ are the critical indices of the specific heat and the magnetic susceptibility, respectively. For a Heisenberg antiferromagnet, $\alpha = 0$ and $\gamma = \frac{2}{3}$. Thus $d\rho/dT$ varies as $\epsilon^{-1/3}$. More extensive calculations by Takada³ confirm these results. Below T_N a term equivalent to Eq. (1) should also be present in $d\rho/dT$. However, the band gap arising from the superzone effects leading to a sharp increase in resistivity must play an important role. According to Suezaki and Mori, the dominant term in $d\rho/dT$ below T_N and close to it, therefore, is of the form

$$\frac{d\rho}{dT} = -C\epsilon^{(\beta-1)}, \quad T < T_N \quad (2)$$

where $\epsilon = (T_N - T)/T_N$, C is a constant, and β is the critical exponent of the order parameter related to α and γ by the usual scaling relation $\alpha + 2\beta + \gamma = 2$. Equation (2) assumes that the number of effective conduction electrons n_{eff} is reduced proportionally to the increased order parameter Δ . If instead, $n_{\text{eff}} \sim -\Delta^2$, as assumed by Takada, both terms in $d\rho/dT$ below T_N have the same exponent corresponding to that of Eq. (1).

More recent theoretical work predicting the critical indices for the electrical resistivity of an antiferromagnet include the following. Alexander and co-workers⁴ find that there should be a temperature region (close to the critical point), where $d\rho/dT$ due to short-range correlations is proportional to the specific heat, thus confirming earlier results for antiferromagnets by Richard,⁵ and Kasuya and Kondo.⁶ They further state that for large ϵ one should have $d\rho/dT \sim \epsilon^{-1/2}$, and that values of the exponent in between the specific-heat

exponent and $-\frac{1}{2}$ are characteristic for a transitional region and do not have any physical significance. Fenton⁷ develops a theory for $T > T_N$ and finds that λ is of the order of 0.1 in $d\rho/dT \sim \epsilon^{-\lambda}$. Ausloos⁸ reconsiders the temperature dependence of the number of conduction electrons due to long-range spin fluctuations above as well as below T_N to obtain an expression of the form

$$\frac{1}{\rho} \frac{d\rho}{dT} = C_{\pm} \epsilon^{-(\alpha_{\pm} + \gamma_{\pm} - 1)}, \quad (3)$$

where “+” refers to the region $T > T_N$ and “-” to $T < T_N$. Here the constant C_{\pm} incorporates the fluctuation and the gap effects. The first experimental work on the critical indices of polycrystalline chromium near T_N was carried out by Amitin and Kovalevskaya,⁹ who found approximately logarithmic behavior of $d\rho/dT$ below T_N . However, above T_N they were unable to fit the experimental data of $d\rho/dT$ to any meaningful power law. The second experimental study of ρ , also on polycrystalline chromium, from the viewpoint of critical indices, was done by Meaden and Sze,¹⁰ who obtained $d^2\rho/dT^2 \approx \epsilon^{-(\lambda+1)}$, with $\lambda = 0.34 \pm 0.02$, between 0.1 K and 1.0 K above T_N . Thus they concluded that the localized spin model of Suezaki and Mori could also be applied to the SDW anti-ferromagnet. Unfortunately, this finding is not substantiated by other more recent investigators¹¹⁻¹³ on polycrystalline chromium and chromium single crystals in either single- \vec{Q} state or multi- \vec{Q} states, where \vec{Q} is the wave vector of the spin-density wave. Specifically, Akiba and Mitsui,^{12,13} using single crystals with well-defined \vec{Q} states, find that $d\rho/dT$ satisfies the general equation

$$\frac{d\rho}{dT} = -A_{\pm} \epsilon^{-\lambda_{\pm}} + B. \quad (4)$$

Below T_N they find that $\lambda_{-} \approx 0.5$ and above T_N that $\lambda_{+} \approx 1.5$. They also concluded that the indices are almost isotropic with respect to the \vec{Q} state. Their measurements on chromium in a multi- \vec{Q} state give $\lambda_{+} = 0.8$ and $\lambda_{-} = 0.3$ for the cooling run and $\lambda_{-} = 0.4$ for the heating run. They also state that the heating-run data did not fit a power law above T_N . According to Salamon, Simons, and Garnier,¹¹ polycrystalline chromium obeys Eq. (4) below T_N with $\lambda_{-} = 0.17 \pm 0.02$. Furthermore, they state that according to their data, $\lambda_{+} \neq \frac{1}{3}$ as obtained by Meaden and Sze.¹⁰ Muir and Ström-Olsen¹⁴ analyze their measurements on single-crystal chromium below T_N with the somewhat arbitrary assumption that B in Eq. (4) is equal to the phonon slope at low (~ 125 K) temperatures and find $\lambda_{-} = 0.43$ both for single- and multi- \vec{Q} states over more than two decades.

In short, the results obtained so far on the critical behavior of chromium appear to be mutually inconsistent. Often this is attributable to very severe problems which make it impossible to obtain sufficient homogeneous samples for measurements very near T_C . Two other reasons are important for this observation. One stems from the influence of strains on the detailed behavior of the electrical resistance in the neighborhood of a phase transition. For example, in thin (10 μm) dysprosium single crystals we have observed¹⁵ that pressing copper potential edges onto the sample surface is sufficient to change the well-known first-order nature of the ferroantiferromagnetic transition into a second-order behavior. It might therefore be conceived that various different thermal histories and strain conditions have contributed to the large range of exponents reported for the resistance of chromium in the critical region. Practically, the number of parameters to fit a single set of data precludes an unambiguous determination of the parameter of greatest interest, the critical exponent. Thus, a further reason could be the various methods of analysis employed in different studies. Published work can sometimes be criticized on the basis that a large variety of different “power laws” can be fitted over sufficiently restricted temperature intervals to a smoothly curved experimental plot. Details on whether a different choice of T_N would fit equally well another power law over a different temperature range are often missing. Moreover, obtaining several successive derivatives may result in a considerable loss of the accuracy of the data and, unless carefully checked, spurious effects can arise.

We thus feel that careful work on the critical resistivity of chromium is highly desirable. In the present work we aim at avoiding the fallacies just discussed. Thus, three samples from the same chromium stock but with different thermal histories have been investigated. We give details of the numerical methods and obtain power laws that reproduce the original data very satisfactorily.

II. EXPERIMENTAL CONSIDERATIONS AND RESULTS

Three samples labeled chromium (1), chromium (2), and chromium (3) were sparkcut from an arc-melted ingot prepared from high-purity chromium stock (Chromally Corporation).¹⁶ After cutting, the samples were etched in HCl, placed in silica tubes, flushed with high-purity helium gas, evacuated to about 0.1 Torr and encapsulated and heat treated as summarized in Table I. Following these heat treatments the samples were reduced

TABLE I. Heat treatment of Cr samples.

Sample	Approximate size	Temperature (K)	Time (h)	Cooling
Chromium (1)	$(0.1 \times 1) \text{ mm}^2 \times 20 \text{ mm}$	1250	100	Water quenched
Chromium (2)	$(1 \times 1) \text{ mm}^2 \times 20 \text{ mm}$	1250	24	Furnace cooled in 12 h
Chromium (3)	$(1 \times 1) \text{ mm}^2 \times 15 \text{ mm}$	1250	100	Furnace cooled in 24 h

to the size suitable for our electric measurement system by further etching in HCl.

The facility for electrical measurements is based on a conventional four-probe dc measuring technique and a Guildline (model 9970) comparator bridge. It has been recently described in some detail elsewhere.¹⁷ Under favorable conditions (resistances $\sim 1 \Omega$) a resolution in resistance of 1 part in 10^7 is possible. In the present case, however the resolution was only about 2 parts in 10^5 . This is due to the fact that sometimes we worked with rather thick samples (Table I) (hence, low in resistance) in order to reduce the sensitivity to strains introduced while making contacts for the potential leads. The chromium samples were mounted on a copper block which also contained a platinum resistance thermometer. Copper potential contacts were carefully pressed into the sample surface. The block was placed in a Dewar and was surrounded by helium gas. The Dewar was enclosed into another Dewar. A wire-wound resistor of the length of the inner Dewar provided a uniform and controllable heating of the copper block, and hence the sample. All measurements were done with increasing temperatures using the

heating rate of about 1 K/h. The chromium (3) sample was thermally cycled before the measurements, while chromium (1) and chromium (2) were cooled only once through the phase transition. The results are shown in Fig. 1, where each resistance has been normalized to its value at T_N . It is clear that the detailed behavior in the critical region is influenced by the thermal history of the sample. In particular, the water-quenched sample [chromium (1)], which supposedly is more strained than the other samples, shows a more flat and spread out transition region than the other samples. One measure of the condition of the sample at the transition may be the normalized difference between the maximum and minimum resistance for each sample, $\Delta\rho/\rho$. In unannealed condition (Ref. 16) this quantity is 0.8%. In chromium (1) its value is 0.5%, in chromium (2) it is 1.1%, and in chromium (3) it is 1.2%. Another related quantity, the maximum of $-(1/R)(dR/dT)$ for each sample, shows a similar trend, as shown in Table II. These numerical values signify that we have successfully diminished the strain effects on the transition for chromium (2) and chromium (3) samples. Our

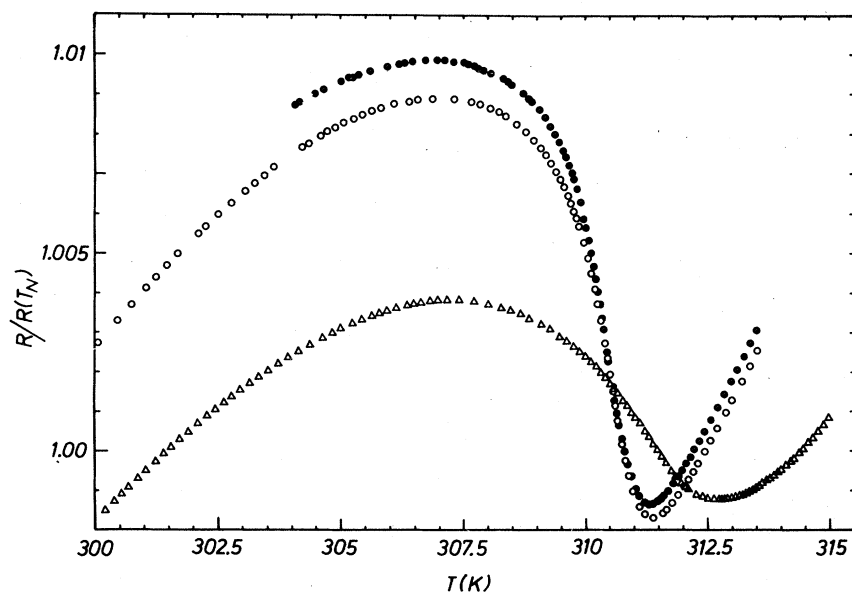


FIG. 1. Resistance vs temperature for three chromium samples. Δ is Cr (1) \circ is Cr (2) and \bullet is Cr (3). All resistance values have been normalized at T_N to the value of 1.

TABLE II. Results to fit $(d\rho/dT)/\rho(T_N) = -A\epsilon^{-\lambda} + B$ below T_N .

Sample	λ	$10^3 B$ (K ⁻¹)	$10^4 A$ (K ⁻¹)	T_N (K)	Temperature range of fit, $\epsilon_{\min} - \epsilon_{\max}$	rms deviation	Observed quantities		
							$10^3 B$ (K ⁻¹)	$10^3 \left(\frac{-1}{\rho} \frac{d\rho}{dT} \right)_{\max}$ (K ⁻¹)	Temperature at max slope (K)
Cr (2)	0.63	2.56	1.53	310.79	$1.7 \times 10^{-3} - 2.7 \times 10^{-2}$	3×10^{-5}	2.47	8.2	310.6
Cr (3)	0.67	2.19	1.13	310.77	$1.5 \times 10^{-3} - 2.1 \times 10^{-2}$	4×10^{-5}	2.27	9.9	310.5
Cr (1)	0.68	Limited power law			$1.2 \times 10^{-2} - 3 \times 10^{-2}$		2.40	1.9	311.5

figures for $\Delta\rho/\rho$ are comparable to the best found in the literature. The largest value of $\Delta\rho/\rho$ (~2%) ever obtained for chromium is that for a single-crystal single Q state^{13,14} when measured with the electric field parallel to the Q vector.

III. ANALYSIS

A. Method

A major problem in experimental studies of critical phenomena is the determination of the critical temperature. The exponent of a power law obtained over a limited temperature range can vary drastically with small changes in T_N . Furthermore, in chromium the minima in ρ and $d\rho/dT$ are separated by only about 1 K, which probably has contributed additional difficulties to the data analysis. The older method¹⁸ of identifying the ρ minimum with T_N has been shown to be wrong.^{1,8} Recently, in the absence of other independent methods of determining T_N , the most plausible approach is identification of T_N as the

temperature at which $d\rho/dT$ is a minimum.^{1,11-13} This approach is justified, for example in dysprosium, where¹⁷ a least-squares analysis with T_N as one of the parameters gives a value of T_N which is almost identical to that obtained from the minimum in $d\rho/dT$.

In our first attempt¹⁹ to analyze the data of Fig. 1 we therefore simply took T_N to be the temperature at the minimum in $d\rho/dT$. Several different power laws were then obtained for different samples which led us to a conclusion that the critical indices in chromium are sensitive to a variety of different parameters such as heat treatments, strains, etc. As will be demonstrated below, such a conclusion is, however, much too coarse. Using a more detailed analysis, it is possible to make more precise statements. This analysis is now described. First, we make no initial assumption of T_N and treat it as a variable parameter. This approach is equivalent to assuming that there is a power law of the form of Eq. (4) and T_N is then determined as the temperature giving the best fit to the selected power law. Second, in

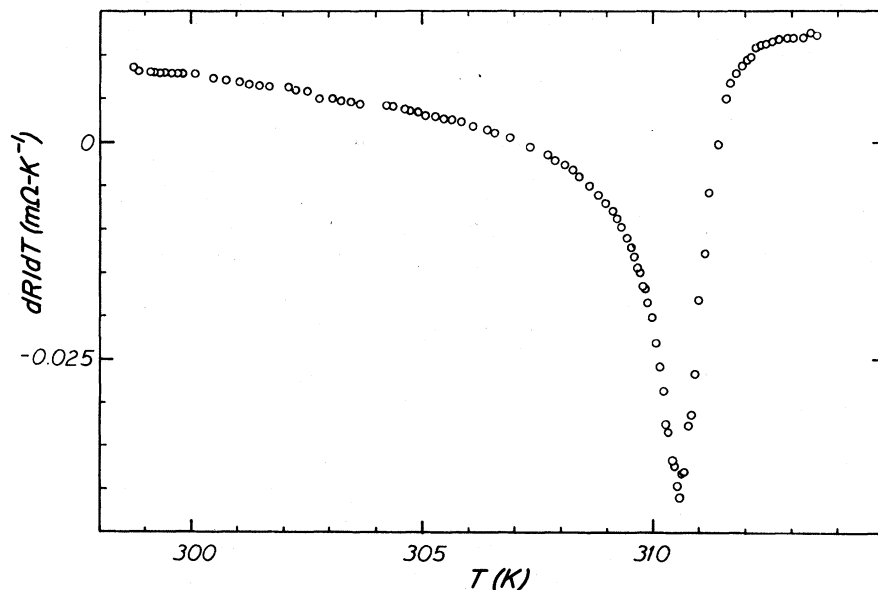


FIG. 2. Typical dR/dT vs T curve [Cr (2)].

view of the decreased accuracy of the second derivatives of the data, we have fitted the first derivatives to Eq. (4). This implies that the linear term B is not eliminated. Rather than assuming at the outset that it is equal to the linear term well above T_N , as was done by Akiba and Mitsui,¹³ we allow B also to be a freely varying parameter.

The first-temperature derivatives of the resistance were calculated by using spline functions of order 2, a standard ALGOL procedure available at our computer center. This method resembles that of a sliding third-degree polynomial fit, but has the additional advantage that the derivative of each fitted curve section is continuously matched at the end points to the neighboring intervals. Small errors in one measured point are magnified and spread through the data on taking the derivative numerically. To diminish these effects, some points were adjusted to a smooth curve through the bulk of the data. This correction was always below 3×10^{-5} of the measured resistance. We checked particularly that this smoothing did not violate the observed behavior as described below.

A typical plot of the first-temperature derivative of the resistance as a function of temperature is shown in Fig. 2.

B. Data below T_N

In the analysis of the data below T_N we first constructed plots of $\log|d\rho/dT - B|$ vs $\log\epsilon$ for a large range of values of B and T_N . From these plots it was concluded that it is not possible to fit the data below about $\epsilon_{\min} \approx 1.5 \times 10^{-3}$ to a straight line, and that the value of ϵ_{\min} is not very sensitive to the variation of T_N or B . Similarly, there is for chromium (2) a value of $\epsilon_{\max} \approx 2.7 \times 10^{-2}$ beyond which the data deviate from a straight line for any choice of T_N and B . [For chromium (3) the measurements were extended to only about $\epsilon \approx 2 \times 10^{-2}$.] The best fit of about 50 data points to Eq. (4) in this temperature range was then constructed by a least-squares analysis with λ_- , A_- , B , and T_N as parameters. This analysis is very similar to that used before for the determination of the critical behavior of dysprosium.¹⁷ In this case the minimization was done for the resistance data. The critical indices of chromium samples were obtained using the values of the first derivatives calculated from the measured ρ data. Figure 3 illustrates the least-squares procedure for the chromium (2) sample. The sum of the squared deviations Z between Eq. (4) and the first derivatives are plotted versus T_N . Each broken curve is associated with a constant B with A_- and λ_- chosen so that Z is minimized for given

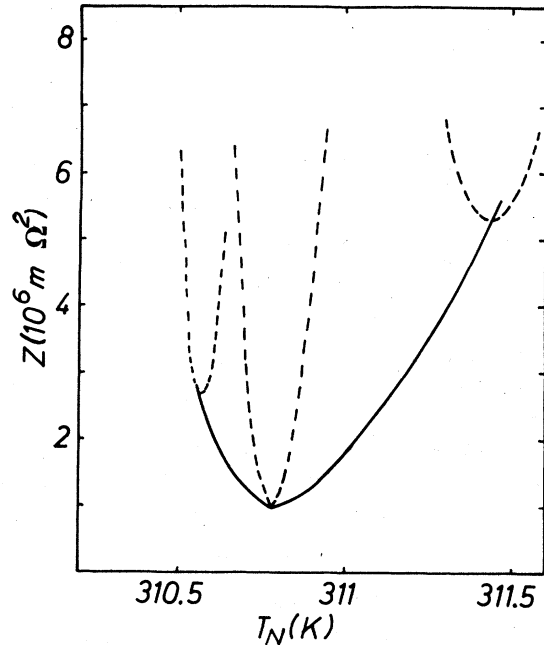


FIG. 3. Illustration of procedures for chromium (2) data in order to find the best fit for Eq. (4). B values on the dashed curves are from left to right 0.020, 0.013, and 0.008 m Ω /K. The data in this figure are not normalized to the resistance at T_N of about 5 m Ω . Further details are described in the text.

T_N . The full curve is the locus of minima of the broken curves. The minimum of this curve gives the parameters for the best fit of data to Eq. (4). An increase in Z by 15% for a fixed value of B , corresponding to about one standard deviation, changes λ_- by only 1%.

Identical data analysis was also done for chromium (3). Characteristic values for all three chromium samples are collected in Table II. In order to compare conveniently the different samples, the resistance in each case has been divided by its value at the Néel temperature. From Table II we observe that the calculated values of B for chromium (2) and chromium (3), obtained from fitting the data below T_N , are close to those obtained from the linear slope above T_N . The quantity B represents the phonon-scattering term which would not be expected to change much through the transition. The fact that this is confirmed by our results gives our calculations a high degree of consistency.

Figure 4 illustrates the best power-law fits for chromium (2) and (3). The range of validity is given in Table II. Over this temperature region, corresponding to about a factor of 15 in ϵ , the data are well described by straight lines. In particular, it is observed that the value of the exponent λ_- ,

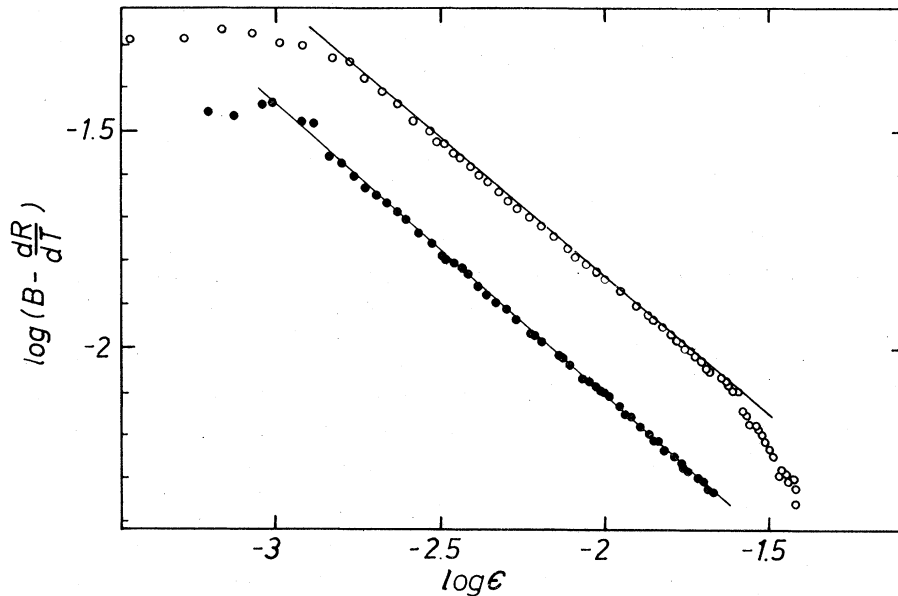


FIG. 4. $\log(B - d\rho/dT)$ vs $\log\epsilon$ for Cr (2) (open circles) and Cr (3) (filled circles) below T_N .

equal to the slope of these lines, is rather independent of the different thermal history of the two samples.

In contrast, it was not possible to fit a power law of the form of Eq. (4) to the data for chromium (1) over the same temperature range. We plotted $\log|d\rho/dT - B|$ vs $\log\epsilon$ for a range of T_N 's (from about 311 to 312 K) and for several values of B below as well as above the observed phonon value. No straight line could be fitted to these plots. Apparently this sample is too strained to be characterized by one single power law. Different regions of the sample may have different values

of T_N and, if the width of this distribution of T_N 's is considerable, it is clear that the functional relation given by Eq. (4) cannot be satisfied over a temperature region as close to T_N as for the samples with more well-defined transitions (Fig. 4). If our assumption that the failure to fit a power law to chromium (1) is due to a wide range of distributions of T_N 's is correct, one would expect the influence of such a distribution to be much reduced at temperatures far away from T_N . This is observed. In Fig. 5 we show the data for chromium (1), where B was taken to be the phonon value above T_N in accordance with the results

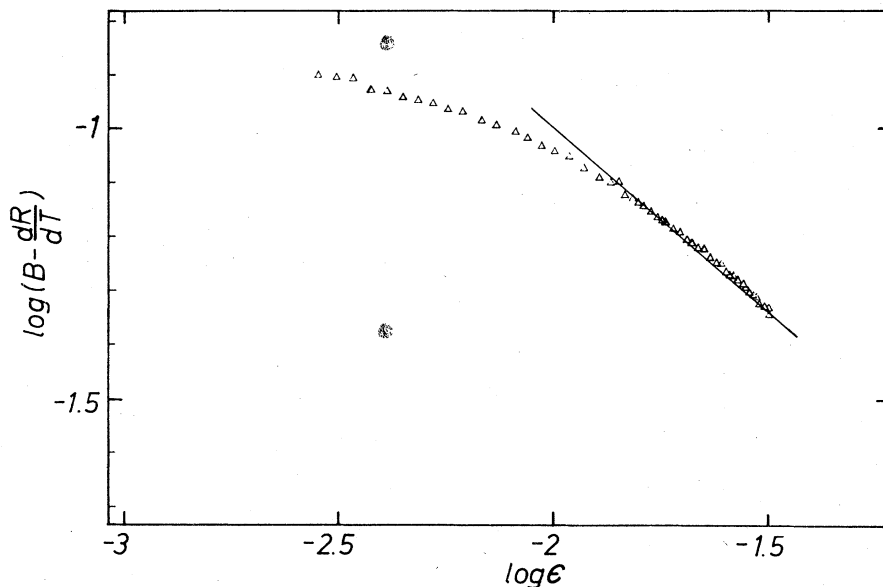


FIG. 5. $\log(B - d\rho/dT)$ vs $\log\epsilon$ for Cr (1) below T_N .

for the other samples and T_N was close to the temperature where $-d\rho/dT$ has a maximum (Table II). The slope of the straight line fitted to the high ϵ end of the data is -0.68 , thus confirming our previously obtained λ .

Finally, we checked our analysis for chromium (2) and chromium (3) to convince ourselves that no spurious effects have occurred in the data handling. The power laws (Fig. 4) were integrated and, with the integration constant as an adjustable parameter, the original data (Fig. 1) were reproduced almost within experimental error over the entire range of the fit.

C. Data above T_N

We were unable to fit a power law to our data above T_N for any of our chromium samples. A typical example of an attempted plot of $\log|d\rho/dT - B|$ vs $\log\epsilon$ for chromium (3) is shown in Fig. 6. Here B is equal to the observed linear term at high temperatures and T_N is taken from the analysis below T_N . Obviously, Fig. 6 does not give a straight-line fit. It should be pointed out that the failure to obtain a power law above T_N does not necessarily imply the nonexistence of such a power law even in the temperature region studied. With our experimental resolution of 2 parts in 10^5 the deviation from a straight line through the high-temperature data, i.e., the part of the total resistance which would account for an exponent, is observable only below about 312.1 K. Still at the minimum, at about 0.5 K above T_N , this deviation is below 10^{-3} of the measured resistance which itself is at most of the order of a

few m Ω for reasons discussed previously. Thus it is a quite formidable experimental task to obtain a power law for the resistivity of chromium above T_N . A very large density of data points combined with even higher resolution seem to be required for this limited-temperature interval. Therefore we conclude that *if* there is a power law in the resistivity of chromium above T_N , this power law is applicable in a restricted-temperature range only, say, within 2 K above T_N . This conclusion is consistent with other attempts^{9,11-13} to fit the power law to the resistivity above T_N .

IV. RANGE OF THE POWER-LAW FITS

The development of the Suezaki-Mori theory¹ appears to be at least partially founded on the mean-field theory. Fluctuations in the order parameter are considered but they are taken to be small, certainly smaller than the order parameter itself. Therefore, approaching the critical point, a temperature region is reached where this condition is no longer satisfied. We now try to estimate this temperature limit ϵ_c in order to check independently that the temperature region over which our power laws were obtained are consistent with the theories on which they rely.

Kadanoff *et al.* have given²⁰ as an order of magnitude estimate of ϵ_c , the Ginzburg temperature,

$$\epsilon_c = \frac{1}{32\pi^2} \left(\frac{k_B}{\Delta C} \right)^2 \frac{1}{\xi_0^6}, \quad (5)$$

where ξ_0 is the zero-temperature-coherence length and ΔC the jump in specific heat per unit volume. The quantity ξ_0 can be estimated from²¹

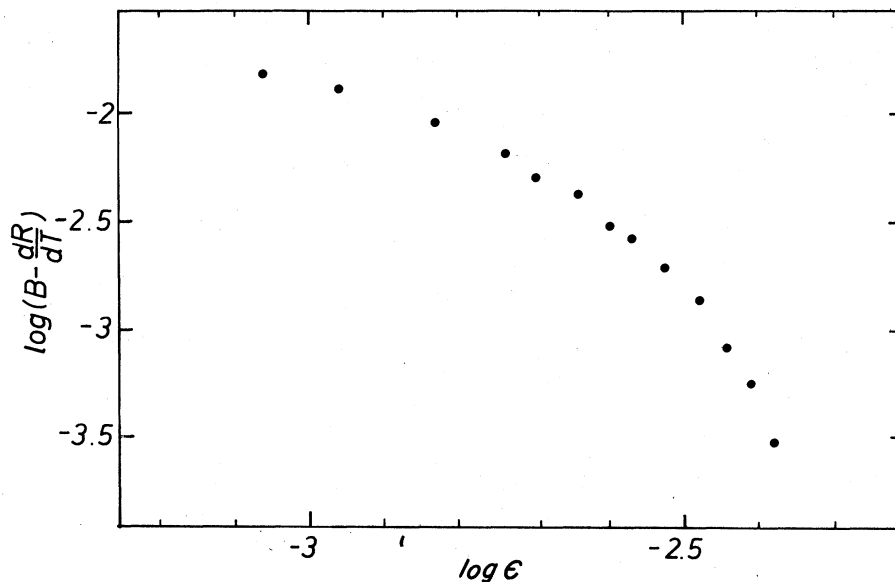


FIG. 6. $\log(B - d\rho/dT)$ vs $\log\epsilon$ for Cr (3) above T_N .

$$\xi_0 = \hbar v_F / \pi \Delta, \quad (6)$$

where Δ is the energy gap and v_F the Fermi velocity. We obtain $\xi_0 = 6 \text{ \AA}$ using the measured²² gap of 0.14 eV at low temperatures and Fermi velocity, determined from neutron-scattering experiments,²³ of $3.8 \times 10^5 \text{ m/sec}$, which is considerably lower than that obtained from more recent calculations²⁴ but in agreement with other experimental data.²⁵ This value of ξ_0 is 40% smaller than one previously used,¹¹ which implies about one order of magnitude differences in ϵ_c . Various values of ΔC have been reported. Some results in the literature, expressed in $\text{J cm}^{-3} \text{ K}^{-1}$, are 4×10^{-3} ,¹¹ 9×10^{-2} ,²⁶ 0.5,²⁷ 0.1,²⁸ and 4.6×10^{-2} .²⁹ These large differences further illustrate the extreme sensitivity of some of the properties of chromium. However, one would not expect the factors in Eq. (6) to depend strongly on strain. Thus it is the estimate of ϵ_c [Eq. (5)] that appears to be very sample dependent.

If we use $\Delta C = 0.1 \text{ J cm}^{-3} \text{ K}^{-1}$ (Ref. 28), which is approximately consistent with values measured by some of us³⁰ on polycrystalline samples of the same stock as presently used and with a heat treatment similar to that of chromium (3) sample, we obtain $\epsilon_c \approx 1.1 \times 10^{-3}$. This value is consistent with the range of the power-law fit for the chromium (2) and (3) samples. In fact, it may be an open question whether the failure of our power laws below about $\epsilon = 10^{-3}$ is due to breakdown of mean-field theory approximations or the handling of data. These qualitative arguments also apply to the heavily strained chromium (1) sample. When strains are released by annealing, ΔC is known to increase.^{27,30} Our chromium (1) sample may therefore be expected to have a low ΔC which from Eq. (5) implies a high value of ϵ_c and a restricted range of applicability of Eq. (2), in agreement with the observations illustrated in Fig. 5.

It should be remarked that our experimental results seem to imply that $\beta = \frac{1}{3}$, not $\frac{1}{2}$ as expected from pure-molecular-field approximation. It is possible that Eq. (5) is not directly applicable to our data. However, we feel that the above simple estimate of ϵ_c provides some insight into the problem of the validity of a power law. Lastly, Ref. 11 also uses Eq. (5) for the same purposes in spite of the fact that β is different from $\frac{1}{2}$.

We also ask ourselves why there are deviations from Eq. (2) at the low-temperature end of the straight line in Fig. 3 for about $\epsilon > 3 \times 10^{-2}$. In view of the agreement between our data and the Suezaki-Mori theory in the range $10^{-3} \leq \epsilon \leq 2 \times 10^{-2}$, we attempt to apply the complete expression of this theory for $d\rho/dT$ below T_N [Ref. 1,

Eq. (4.10)], i.e.,

$$\frac{1}{\rho(T_N)} \frac{d\rho}{dT} = B - A\epsilon^{-2/3} + C\epsilon^{-1/3}. \quad (7)$$

This analysis takes into account the phonon term and uses the numerical value of $\beta = \frac{1}{3}$, which is justified from our data. The constant C is determined from the observed maximum in ρ . If the slope λ of $\log\{B - [1/\rho(T_N)](d\rho/dT)\}$ vs $\log\epsilon$ is now evaluated, we find as required, that λ at $\epsilon = 10^{-3}$ differs only slightly (below 3%) from the value in Eq. (2) ($-\frac{2}{3}$ in this model). Far from T_N , however, at $\epsilon = 5 \times 10^{-2}$, the value of λ obtained from Eq. (7) is -0.8 , which gives a correction of the right sign, but smaller than required to account for the experimental slope of about -1 . A further reason for discrepancies in this temperature region may be the choice of the B value. The ordinate in Fig. 3 is very sensitive to B for low values of the argument. Moreover, the measuring temperature is only about half the Debye temperature of chromium³¹ and variations in the phonon resistivity may be expected to be observable over a 15-K temperature interval.

V. CONCLUSIONS

We have found that the electrical resistivity of chromium below T_N is of the form of $\rho \sim \epsilon^{1-\lambda}$, with λ close to $\frac{2}{3}$ for several samples with different thermal histories. Although the phase transition is somewhat smeared in a strained sample, this conclusion remains valid over a more limited temperature interval. The ranges of the fits are consistent with mean-field theory estimates and the value of λ agrees with that obtained in the Suezaki-Mori theory³ for the dominant term in the resistivity below T_N . Thus we conclude that in the temperature region of the observed power law, smearing of the order parameter Δ as considered by Takada³ is negligible. The effective number of charge carriers n_{eff} follows the relation

$$n_{\text{eff}} \sim n(1 - b\Delta),$$

with $\Delta \sim \epsilon^b$. b here is an appropriate constant. From a Heisenberg large-spin model and scaling laws one then obtains $\beta = \frac{1}{3}$, in accordance with the experimental results and Eq. (2).

Unresolved questions include the departure from a single power law for $\epsilon \geq 2 \times 10^{-2}$ below T_N (which apparently can only be partly accounted for by considering an additional term in the Suezaki-Mori theory). Furthermore, we have not resolved the behavior of the resistivity above T_N . The experimental difficulty of this problem is stressed.

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