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Some properties of a charged two-dimensional isotropic Fermi liquid

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The results obtained by using the Landau-Silin theory of a charged Fermi liquid to calculate some properties of an interacting two-dimensional electron gas are presented. The static properties considered are the specific heat, spin susceptibility, and compressibility. The dynamical properties discussed are the frequency-dependent magnetoconductivity tensor and the wave-vector-dependent transverse dynamic spin susceptibility in the long-wavelength limit. We compare these results with the analogous well-known formulas valid for a three-dimensional Fermi liquid and conclude that there are no new effects associated with the reduced dimensionality.

I. INTRODUCTION

The purpose of this note is to present the results of calculations of some properties of a two-dimensional Fermi liquid and to compare these results with analogous standard formulas valid for a three-dimensional system. I find that, aside from some unimportant changes in overall multiplicative factors, there are no new or unexpected effects associated with the reduced dimensionality. The details of the calculations are omitted because the Landau-Silin[1,2] theory of a charged Fermi liquid has been fully discussed in the literature[3] and the extension to two-dimensional systems is straightforward.[4]

II. NOTATION

The energy $\tilde{\mathcal{E}}_{\alpha\beta}(\vec{k})$ of a quasiparticle of momentum $\vec{k}$ can be written in the usual form

$$\tilde{\mathcal{E}}_{\alpha\beta}(\vec{k}) = \mathcal{E}_{\alpha\beta}(\vec{k}) + 2\delta_{\alpha,\beta}\sum_{k'} a(\hat{k}\cdot\hat{k}')\,\delta f(\vec{k}') + 2\,\vec{\sigma}_{\alpha\beta}\cdot\sum_{k'} b(\hat{k}\cdot\hat{k}')\,\delta\vec{\psi}(\vec{k}')\,, \tag{2.1}$$

where $\mathcal{E}_{\alpha\beta}(\vec{k})$ is the energy of a single excited quasiparticle which in the presence of an external magnetic field $\vec{H}_0$ is given by

$$\mathcal{E}_{\alpha\beta}(\vec{k}) = (k^2/2m^*)\,\delta_{\alpha,\beta} - g\mu_B\,\vec{H}_0\cdot\vec{\sigma}_{\alpha\beta}\,, \tag{2.1a}$$

where m^* is the quasiparticle mass and the other quantities have their usual meanings. The functions $a(\hat{k}\cdot\hat{k}')$ and $b(\hat{k}\cdot\hat{k}')$ describe, respectively, the spin-independent and exchange interactions among the quasiparticles. The quasiparticle number and spin-density functions are denoted, respectively, by $\delta f(\vec{k})$ and $\delta\vec{\psi}(\vec{k})$. In an isotropic two-dimensional Fermi liquid $\vec{k} = |\vec{k}|\,(\cos\phi, \sin\phi)$ so that the Fermi perimeter (the analog of the Fermi surface in three dimensions) is a circle of radius k_F. As in the case of a three-dimensional Fermi liquid, it is useful to define two sets of dimensionless parameters $\{A_l\}$ and $\{B_l\}$ to characterize the interactions. These parameters are defined by the expansions

$$\nu\,a(\hat{k}\cdot\hat{k}') = \sum_{l=0}^{\infty} A_l\, g_l \cos l(\phi-\phi')\,, \tag{2.2a}$$

and similarly for $b(\hat{k}\cdot\hat{k}')$. We have introduced, $\nu = m^*/\pi$, the quasiparticle density of states per unit area for both spin directions and have defined the weight function

$$g_l = \delta_{l,0} + 2(1-\delta_{l,0})\,, \tag{2.2b}$$

for $l = (0,1,2,\ldots,\infty)$ and $g_l = 0$ otherwise. On inverting the above expansions, we find that

$$A_l = \nu\int_0^{2\pi}\frac{d\phi}{2\pi}\,a(\hat{k}\cdot\hat{k}')\cos l(\phi-\phi') \tag{2.2c}$$

and similarly for B_l.

III. STATIC PROPERTIES

A. Specific heat

The specific heat at constant volume C_v is given by

$$C_v = \left(\frac{\partial F}{\partial T}\right)_\mu\,, \tag{3.1}$$

where $F = E - \mu N$ is the free energy of the system. A simple calculation shows that the low-temperature specific heat is given by

$$C_v = \tfrac{1}{3}\,\nu\,\pi^2 k_B^2\, T\,, \tag{3.2}$$

which is identical in form to the standard result valid for a three-dimensional Fermi liquid.

B. Spin susceptibility

The susceptibility χ is easily obtained by calculating the magnetization $\vec{M} = \chi\,\vec{H}_0$ produced by applying an external magnetic field. We find that

$$\chi = (g\mu_B)^2 \nu / 4(1 + B_0)\,, \tag{3.3}$$

which is identical in form to the spin susceptibility of an isotropic three-dimensional Fermi liquid.

It is useful to compare χ with $\chi^{(0)}$, the spin susceptibility of noninteracting electrons. The ratio $\chi/\chi^{(0)}$ is given by

$$\chi/\chi^{(0)} = (m^*/m)/(1 + B_0)\,, \tag{3.4}$$

which is the same as for a three-dimensional Fermi liquid.

C. Compressibility

The compressibility, κ, of a two-dimensional quasiparticle gas can be defined by

$$\frac{1}{\kappa} = -A\,\frac{\partial P}{\partial A}\,, \tag{3.5}$$

where P is the pressure and A is the area of the sample. Performing a calculation which parallels one given by Pines and Nozières[3] in discussing the compressibility of a three-dimensional charged Fermi liquid, we find that

$$\kappa = \nu / n_s^2 (1 + A_0)\,, \tag{3.6}$$

where n_s is the number of quasiparticles per unit area. The ratio of κ to the compressibility, $\kappa^{(0)}$, of noninteracting electrons is

$$\kappa/\kappa^{(0)} = (m^*/m)/(1 + A_0)\,, \tag{3.7}$$

which is the same as for a three-dimensional Fermi liquid. From the compressibility we calculate the speed of sound c_s. We find that

$$c_s^2 = k_F^2 (1 + A_0)/2mm^* = \tfrac{1}{2}(m^*/m)\, v_F^2 (1 + A_0)\,. \tag{3.8}$$

The above equation is identical to that which one finds for a three-dimensional Fermi liquid if the factor of $\frac{1}{2}$ is replaced by $\frac{1}{3}$.

IV. TIME-DEPENDENT PROPERTIES

A. Magnetoconductivity tensor

The frequency-dependent conductivity tensor has been calculated in the presence of an applied magnetic field $\vec{H}_0 = H_0 \hat{z}$. We find that

$$\sigma_{xx}(\omega) = \sigma_{yy}(\omega) = i n_s e^2\, \frac{1 + A_1}{m^*}\, \frac{\omega + i/\tau^*}{(\omega + i/\tau^*)^2 - \omega_c^{*2}}\,, \tag{4.1}$$

and

$$\sigma_{xy}(\omega) = -\sigma_{yx} = n_s e^2\, \frac{1 + A_1}{m^*}\, \frac{\omega_c^*}{(\omega + i/\tau^*)^2 - \omega_c^{*2}}\,, \tag{4.2}$$

with $\omega_c^* = \omega_c(1 + A_1)$ and $\tau^* = \tau/(1 + A_1)$ where $\omega_c = |e|H_0/m^*c$ is the quasiparticle cyclotron frequency and τ is a phenomenological momentum scattering time. For a uniform Fermi liquid it is easy to prove that the well-known result,

$$m^*/m = 1 + A_1\,, \tag{4.3}$$

is still valid in two dimensions.

B. Wave-vector-dependent transverse dynamic-spin susceptibility

The transverse wave vector and frequency-dependent spin susceptibility, $\chi_+(q, \omega)$, has been calculated to $O(q^2)$ in the presence of an external magnetic field $\vec{H}_0 = H_0 \hat{z}$. This response function is obtained by calculating the transverse magnetization, $M_+(q, \omega) = \chi_+(q, \omega) h_+(q, \omega)$, produced by applying a weak frequency and wave-number dependent magnetic field, $h_+(q, \omega) = h_x(q, \omega) + i h_y(q, \omega)$, transverse to the static field $\vec{H}_0$. We find that

$$\chi_+(q, \omega) = \frac{\chi(-\omega_L + 1/\tau_s^*)}{\omega - \omega_L + i/\tau_s^* + i D_s q^2}\,, \tag{4.4}$$

where χ is the static susceptibility calculated previously [e.g., see Eq. (3.3)], $\omega_L = g\mu_B H_0$ is the Larmor spin-precession frequency, $1/\tau_s^* = 1/\tau_s(1 + B_0)$ where τ_s is a phenomenological spin-flip scattering time, and D_s is a spin-diffusion constant. To see the effects of the quasiparticle interactions, we consider the case B_0 finite and $B_1 = 0$. In this case D_s is complex and in the absence of collisions (i.e., $\tau \to \infty$) is pure imaginary. The susceptibility then has a pole at

$$\omega = \omega_L + q^2 B_0 v_F^2 / 2[(\omega_L \tilde{B}_0)^2 - \omega_c^2]\,, \tag{4.5}$$

which gives the dispersion relation for paramagnetic spin waves. The dispersion relation (4.5) is essentially identical, in the corresponding limit, to that of a charged three-dimensional Fermi liquid (the factor of 2 in the second term is replaced by a factor of 3).

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[4] The details of these calculations are available from the author upon request.