PHYSICAL REVIEW B VOLUME 18, NUMBER 5 1 SEPTEMBER 1978

Temperature dependence of the spin susceptibility of nearly ferromagnetic Fermi systems

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(Received 25 May 1977)

The spin susceptibility of nearly ferromagnetic Fermi systems, e.g., liquid ^{3}He, Pd, Ni_3Ga is large ($\chi_0 \chi_{\text{Pauli}}^{-1} = \alpha_0^{-1} \gg 1$) and varies strongly with temperature on a scale much smaller than the Fermi temperature. A theory for this is presented here. Using a functional transformation, the thermodynamic potential of the fermion system is converted to that of interacting spin fluctuations (a vector-boson field). The parameters of the latter are determined by the fermion system. The susceptibility is related to the spin-fluctuation self-energy Σ. From a diagrammatic analysis of Σ, we show that there exists a convergent expansion for its temperature-dependent part, in terms of the number of correlated internal thermal spin fluctuations. The leading terms have one or two internal spin fluctuations: the three-spin-fluctuation term is of higher order in τ/α_0 (for $\tau \ll \alpha_0$) and in τ (for $\alpha_0 \lesssim \tau \ll 1$). (Here $\tau = T/T_F^0$.) Thus a calculation of $\chi^{-1}(\tau)\chi_P = \alpha(\tau)$ is possible in the entire degenerate range $\tau \ll 1$, and not merely the paramagnon range $\tau \ll \alpha_0$. $\alpha(\tau)$ is shown to go as $\alpha_0 + A(\tau^2/\alpha_0)$ for $\tau \ll \alpha_0$ and as $\alpha_0 + B\tau$ for $\alpha_0 \lesssim \tau \ll 1$, and interpolates smoothly between these two limits. (A and B are constants.) The former is the paramagnon-theory result and the latter is the result for "classical" spin fluctuations. As an illustration, we calculate, with one adjustable parameter, $\alpha(\tau)$ for liquid ^{3}He in the temperature range $0 < \tau \lesssim 0.2$. The agreement with experiment is very good.

I. INTRODUCTION

The static spin susceptibility χ_T $(q=0)$ of nearly ferromagnetic Fermi systems such as (normal) liquid ^{3}He,[1] Pd,[2] Ni_3Ga,[3] $HfZn_2$,[4] is large and is strongly temperature dependent. In terms of the free fermion or Pauli susceptibility χ_P, the quantity $\chi_0\, \chi_P^{-1} = \alpha(0)^{-1} = \alpha_0^{-1}$ (Stoner enhancement factor) is much larger than unity, for systems of interest to us. The proper temperature scale for a fermion system is $T/T_F^0 = \tau$, where T_F^0 is the free Fermi temperature [the density of states $\rho(\epsilon_F)$ at fermi energy is an equally good energy scale]. In most nearly-ferromagnetic-Fermi (NFF) systems, $\alpha(\tau)$ rises for very low temperatures ($\tau \ll \alpha_0$) as τ^2/α_0, and then as τ for $\tau \gtrsim \alpha_0$ but $\tau \ll 1$, i.e., for a degenerate system. For example, in ^{3}He at 27 atm pressure, $\alpha(\tau)$ changes in the above manner, by a factor of 6 or so, in the temperature range $0.0 < \tau < 0.2$. This behavior surprising for a degenerate Fermi system, has not been adequately explained so far. We present here a systematic microscopic theory which explains this general behavior, and leads to quite good agreement with experiment for the case of liquid ^{3}He.[5]

The dynamical model used here represents the NFF system as a system of interacting spin and density fluctuations.[5-8] This is achieved by a functional integral transformation,[9,10] where in the partition function $Z = \text{Tr} \exp[-\beta(H_0 + H_{\text{int}})]$ the two-body term H_{int} is shown to be equivalent to a one-body term with variable amplitudes $\vec{\xi}(\vec{q},u)$, $\eta(\vec{q},u)$ weighted by Gaussian factors. Here the "time" (u) and wave-vector- ($\vec{q}$) dependent fields $\vec{\xi}(\vec{q},u)$ and $\eta(\vec{q},u)$ can be identified with spin and density fluctuations, respectively. On integrating out (taking the trace) over the fast fermion degrees of freedom, one is left with an interacting spin- and density-fluctuation system. The parameters of this system, e.g., fluctuation spectrum, fluctuation coupling vertices are determined by the properties of the fermion system. Since these parameters are such that spin fluctuations are the low-lying excitations, this transformation is specially helpful for an analysis of temperature-dependent properties of NFF systems. We present here a diagrammatic many-body theory of the interacting spin-fluctuation field, a vector-boson field. The spin susceptibility is related to the boson propagator, and we thus analyze the temperature dependence of the boson self-energy Σ. For this quantity, there exists a convergent expansion in terms of the number of correlated internal thermal spin fluctuations. The leading contributions arise from diagrams involving one and two such fluctuations; that involving three fluctuations is at least one order higher in τ and thus a quantitative theory is possible for a degenerate NFF system. The fluctuation coupling vertices and the energy spectrum are strongly renormalized by the (zero-point part of) fluctuation interaction. This modification cannot be accurately calculated because of the intermediate-coupling nature of the problem. However, because the vertices depend only weakly on momentum (on a scale $2k_F$), energy (ϵ_F^0) and temperature (T_F^0), they can be simply parametrized. We parametrize the spectrum in terms of the susceptibility at $T=0$ and some general forms.

The functional-integral transformation and the interacting-spin-fluctuation formalism are described in Sec. II. Analysis of the thermal-spin-fluctuation effects and of the vertex and other renormalization effects is carried out here. In Sec. III, the one, two, and three thermal-correlated internal spin-fluctuation diagrams for Σ are discussed. It is shown that due to the first two, $\Delta\alpha(\tau) = \alpha(\tau) - \alpha_0$ goes as τ^2/α_0 (for $\tau \ll \alpha_0$). Such a result was first obtained by Béal-Monod *et al.*[11] in a paramagnon theory model. At higher temperatures ($\tau \ll 1$ but not $< \alpha_0$) the paramagnon model is inadequate. We find for $\alpha_0 \lesssim \tau \ll 1$ that $\Delta\alpha(\tau) \sim \tau$, a classical spin-fluctuation result.[6,12,13] Thus both the τ^2 and τ behavior in different temperature ranges arise from the same physical process. The higher-order three-correlated internal-spin-fluctuation contribution is shown in both the paramagnon ($\tau \ll \alpha_0$) and classical ($\alpha_0 \leq \tau \ll 1$) spin-fluctuation regimes to be of a higher order in temperature. We then present in Sec. IV a numerical calculation of the magnetic susceptibility of liquid ^{3}He for $0.0 < \tau < 0.2$ (depending on the pressure, α_0 varies from 0.05 to 0.10). The size of the quartic spin fluctuation vertex is the single free parameter and is chosen to reproduce correctly the very-low-temperature ($\tau < 0.03$) susceptibility. The calculated $\alpha(\tau)$ then agrees very well with experiment over the range $0.0 < \tau < 0.2$, the maximum deviation, at $\tau \sim 0.2$, being ~15%. Various aspects of the calculation, e.g., cutoff dependence, relative effect of one- and two-fluctuation terms, the crucial role of self-consistency are discussed, and application of the results to the large class of nearly ferromagnetic metals (e.g., Pd), intermetallic compounds (e.g., $HfZn_2$, Ni_3Ga), and alloys (e.g., Ni_xPt_{1-x}, Ni_xRh_{1-x}) is pointed out.[14] In Sec. V we discuss earlier work, and show that a $\tau^2 \ln\tau$ term in χ_T claimed by several authors[15,16] is absent from a correct calculation maintaining rotational invariance.

II. FORMALISM

The Hamiltonian of an interacting (spin-$\frac{1}{2}$) fermion system with zero-range repulsion U' between fermions of opposite spin is given by

$$H = \sum_{\vec{k},\lambda} \epsilon_k a^\dagger_{\vec{k}\lambda} a_{\vec{k}\lambda} + U' \sum_{\vec{q}} \rho_{\vec{q}\lambda} \rho_{-\vec{q}-\lambda}\,, \qquad (2.1)$$

where ϵ_k is the kinetic energy of the fermion in the momentum state $\vec{k}$ with spin component λ; $a^\dagger_{\vec{k}\lambda}$, $a_{\vec{k}\lambda}$ being operators creating and annihilating fermions in the state $\vec{k}$. $\rho_{\vec{q}\lambda}$ is the density-fluctuation operator

$$\rho_{\vec{q}\lambda} = \sum_{\vec{k}} a^\dagger_{\vec{k}\lambda} a_{\vec{k}+\vec{q}\lambda}\,.$$

The two-body term in (2.1) can be written in two ways:

$$U' \sum_{\vec{q}} \rho_{\vec{q}} \rho_{-\vec{q}} = \tfrac{1}{2} U' \sum_{\vec{q}\lambda} \rho_{\vec{q}\lambda} - \tfrac{2}{3} U' \sum_{\vec{q}} \vec{S}_{\vec{q}} \cdot \vec{S}_{-\vec{q}} \qquad (2.2a)$$

$$= \tfrac{1}{4} U' \sum_{\vec{q}} \rho_{\vec{q}} \rho_{-\vec{q}} - \tfrac{1}{3} U' \sum_{\vec{q}} \vec{S}_{\vec{q}} \cdot \vec{S}_{-\vec{q}}\,, \qquad (2.2b)$$

where $S^\alpha_{\vec{q}}$ is the spin-density-fluctuation operator

$$S^\alpha_{\vec{q}} = \sum_{\vec{k}\lambda\lambda'} a^\dagger_{\vec{k}\lambda} (\sigma^\alpha)_{\lambda\lambda'} a_{\vec{k}+\vec{q}\lambda'}$$

and $\rho_{\vec{q}}$ is the density-fluctuation operator $\rho_{\vec{q}} = \rho_{\vec{q}\lambda} + \rho_{\vec{q}-\lambda}$. Both these forms are rotationally invariant. Equation (2.2a) has a one-body term (absorbed by redefining the chemical potential μ), and a spin-spin coupling. Equation (2.2b) is a general form in which the two basic coupling terms, i.e., that between density fluctuations and between spin density fluctuations, both appear (with related coefficients in this case). Clearly Eqs. (2.2a) and (2.2b) should give the same results if proper resummation of terms is made. We use for simplicity the form (2.2a). Using the standard identity

$$\exp(|a|^2) = \int_{-\infty}^{\infty} \frac{dx_1\, dx_2}{\pi} \exp(-|x|^2 + ax^* + a^* x)\,,$$

where $x = x_1 + ix_2$, we can express the partition function $Z = \mathrm{Tr}\exp[-\beta(H - \mu N)]$ as a functional integral over boson variables $\vec{\xi}(\vec{q})$ [one for each $\vec{q}$ in Eq. (2.2a)]. The noncommutativity of operators is taken care of by the usual Feynman time ordering, i.e., we write

$$Z = \mathrm{Tr}\left[T_s \exp\left(-\frac{\beta}{N} \sum_s (H_s - \mu N_s)\right)\right],$$

where the "time" 0 to β is split into N intervals labeled s (= 1 to N), with $N \to \infty$. There is one variable $\vec{\xi}(\vec{q}s)$ for each "time." Using the periodicity condition $\vec{\xi}(\vec{q}0) = \vec{\xi}(\vec{q}\beta)$ and taking the limit $N \to \infty$, the partition function becomes

$$Z = \int \prod_{q} \frac{d\vec{\xi}(\underline{q})}{\pi} \exp\left[-\sum_{\underline{q}} |\xi^z(\underline{q})|^2 + |\xi^+(\underline{q})|^2 + |\xi^-(\underline{q})|^2 L(\vec{\xi}(\underline{q}))\right], \qquad (2.3)$$

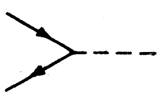
where $\underline{q} = \vec{q}, 2\pi i m/\beta = \vec{q}, z_m$ and

$$L(\vec{\xi}(\underline{q}))=\mathrm{Tr}\,T_u\Big[\exp\Big(-\int_0^\beta du\sum_{\vec{k}\lambda}\epsilon_k a^\dagger_{\vec{k}\lambda}(u)a_{\vec{k}\lambda}(u)+C^z\frac{1}{\beta}\int_0^\beta du\sum_{q}[\xi^z(\underline{q})S^z_{qu}e^{z_m u}+\mathrm{H.c.}]$$
$$+C^+\frac{1}{\beta}\int_0^\beta du\sum_{\underline{q}}[\xi^+(\underline{q})^*S^+_{qu}e^{z_m u}+\mathrm{H.c.}]+C^-\frac{1}{\beta}\int_0^\beta du\sum_{\underline{q}}[\xi^-(\underline{q})^*S^-_{qu}e^{z_m u}+\mathrm{H.c.}]\Big)\Big]. \tag{2.4}$$

In Eq. (2.4) the boson-fermion spin-fluctuation coupling amplitude is $C^z=C^+=C^-=(2U'/3\beta)^{1/2}\equiv(U/\beta)^{1/2}$. The fields transform as

$$\vec{\xi}(\vec{q}u)=\frac{1}{\beta}\sum_{m=-\infty}^{\infty}\vec{\xi}(\underline{q})e^{z_m u},$$

and since they are real, one has the relations $\xi^{z*}(\underline{q})=\xi^z(-\underline{q})$, $[\xi^+(\underline{q})]^*=\xi^-(-\underline{q})$. A diagrammatic perturbation theory for the exponent of $L(\{\vec{\xi}(\underline{q})\})$ as a power series in the coupling C^z, $C^\pm$ is easily obtained. The basic interaction vertex is diagrammatically



where the broken line represents a completely bare spin fluctuation and the straight line a fermion. The basic interaction vertex (C^z, C^+, C^-) is $(U/\beta)^{1/2}$. The diagrammatic rules are those given by Rice.[17]

On taking the trace over fermion degrees of freedom, one obtains Z as a functional integral over the boson variables $\vec{\xi}(\underline{q})$, i.e., one has

$$Z=\int\prod_{\underline{q}}\frac{d\vec{\xi}(\underline{q})}{\pi}\exp[-Y(\vec{\xi}(\underline{q}))]. \tag{2.5}$$

We now discuss the spin susceptibility. The static susceptibility is given by

$$\chi(0,0)=-\frac{1}{\beta}\frac{\partial^2(\ln Z)}{\partial h^2}\bigg|_{h\to 0}, \tag{2.6}$$

where h is the external static magnetic field. Since this adds a term $g\mu_B hS^z$ to H, a simple change of variables from $\xi^z_{0,0}$[i.e., $\xi^z(q=0,m=0)$] to $\xi^z_{0,0}-g\mu_B h/C^z$ leads to the expression

$$\chi(0,0)=g^2\mu_B^2U^{-1}[2\langle\xi^z(0,0)^2\rangle-1],$$

where $\langle\ \rangle$ denotes the functional average with the weight factor $\exp[-Y(\vec{\xi}(\underline{q}))]$. The result can be generalized to that for the dynamic susceptibility

$$\chi^{\alpha\beta}(\underline{q})=g^2\mu_B^2U^{-1}[D^{\alpha\beta}(\underline{q})-1], \tag{2.7a}$$

where

$$D^{\alpha\beta}(\underline{q})=\beta^{-1}\int_0^\beta\langle T[\xi^\alpha(\vec{q},u)\xi^\beta(-\vec{q},0)]\rangle\times e^{-z_m u}du. \tag{2.7b}$$

Because of Eq. (2.7), we identify the vector boson field $\vec{\xi}(\underline{q})$ as the spin-fluctuation field, and $\beta^{-1}Y(\vec{\xi}(\underline{q}))$ as the spin-fluctuation Hamiltonian or energy functional. The temperature dependence of χ_T can thus be found from that of $D(\underline{q}=0)$ or $D(0)$. We find $D(0)$ using diagrammatic perturbation theory for this vector boson field. The "bare" spin-fluctuation propagator is, as usual, the inverse of the quadratic term in $Y(\{\vec{\xi}\})$, i.e.,

$$[D^{\alpha\beta}(\underline{q})]^0=D^0(\underline{q})\delta_{\alpha\beta}=\delta_{\alpha\beta}[1-U\chi^0(\underline{q})]^{-1}, \tag{2.8}$$

where $\chi^0(q)$ is the Lindhard or free-fermion gas polarizability function. Diagrammatically, we denote D^0 by a wavy line and Eq. (2.8) by Fig. 1(a). This approximation, where no fluctuation interaction effects are included, is akin to the random-phase approximation (RPA) [but not identical with it as applied directly to the Hamiltonian of Eq. (2.1), see later]. The static susceptibility $\chi(0)=\chi_P[1-U\chi^0_T(0)]^{-1}$ is high if $U\chi^0(0)\lesssim 1$. Its temperature dependence arises entirely from that of $\chi^0_T(0)$. The latter is $\rho(\epsilon_F)(1-aT^2)$ where a is a positive constant of order unity. This Stoner-Wohlfarth term[18] is weakly T dependent and cannot explain the observed change in the form of T dependence.

We thus consider effects of fluctuation interaction on the propagator D, calculating as usual the self-energy Σ related to D by the Dyson equation

$$[D^{\alpha\beta}(\underline{q})]^{-1}=\delta_{\alpha\beta}D^{-1}(\underline{q})=[D^0(\underline{q})]^{-1}-\Sigma(q). \tag{2.9}$$

A few low-order terms for Σ are shown in Fig. 2, where Figs. 2(a)–2(c) show terms for Σ from the quartic spin-fluctuation coupling in Y, Figs. 2(d) and 2(e) arise from sixth-order term in Y and Fig. 2(f) is due to the eighth-order term.

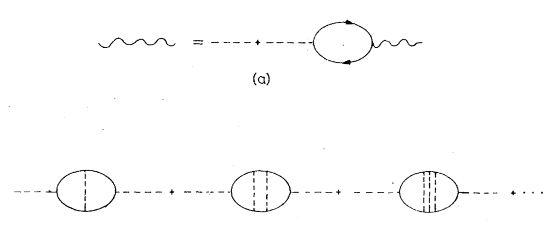


FIG. 1(a) RPA-like diagram for spin-fluctuation propagator. The dashed line represents the completely bare propagator, the wavy line the propagator in RPA. The full lines are fermion propagators. (b) Diagrams for spin-fluctuation self-energy whose summation gives conventional RPA.

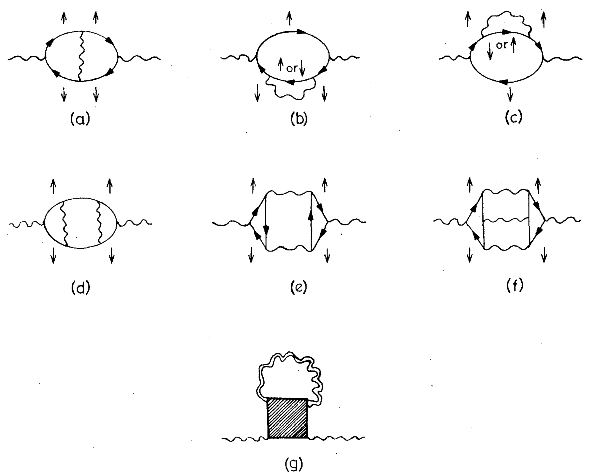


FIG. 2. Self-energy diagrams for the transverse-spin-fluctuation propagator $D(\underline{q})$. (g) Double line represents the dressed propagator $D(\underline{q})$, and the hatched square is the full vertex.

At $T=0$, each of these terms contributes significantly to Σ, of order $\tilde{U}$ [for $U\rho(\epsilon_F)=\tilde{U}\sim O(1)$]. For example, the vertex correction Fig. 2(a) is of order unity, there being no Migdal-like theorem for the spin-fluctuation self-energy.[19] The calculation of the $T=0$ or the ground-state self-energy is a genuine intermediate coupling $[U\rho(\epsilon_F)\sim 1)]$ problem with no quantitatively reliable solutions in terms of the basic parameters of the system. For example, the conventional RPA as applied directly to the Hamiltonian (2.1) gives[20] $\chi(\underline{q})=\chi^0(\underline{q})(1-U'\chi^0(\underline{q}))^{-1}$ where $U'=\frac{3}{2}U$ [compare Eq. (2.8)]. This result can be obtained in our vector-fluctuation-field scheme by resuming the series of self-energy diagrams shown in Fig. 1(b).[21] In view of this, $D_{T=0}(\underline{q})=D_g(q)$ (needed in all computations later) is best obtained directly from the experimental $\chi_g(\underline{q})$ [Eq. (2.7a)]. Since only $\chi_g(0)$ is known experimentally, we write

$$\begin{aligned} D_g^{-1}(0) &= 1-U\chi_g^0(0)-\Sigma_g(0) \\ &= \alpha_0(\tilde{U}+\alpha_0)^{-1}\simeq \alpha_0\tilde{U}^{-1}\,. \end{aligned} \tag{2.10}$$

The last equality in Eq. (2.10) is true near a ferromagnetic instability, i.e., for $\alpha_0\ll\tilde{U}\approx 1$. We also need the $\underline{q}$-dependent part of $D_g^{-1}(\underline{q})$, i.e.,

$$\begin{aligned} D_g^{-1}(\underline{q})-D_g^{-1}(0) &= -U[\chi_g^0(\underline{q})-\chi^0(0)] \\ &\quad -[\Sigma_g(\underline{q})-\Sigma_g(0)]\,. \end{aligned}$$

In order to avoid introducing further unknown constants we approximate it by $-U[\chi_g^0(\underline{q})-\chi_g^0(0)]$ and thus

$$D_g^{-1}(\underline{q})=D_g^{-1}(0)-U[\chi_g^0(q)-\chi_g^0(0)]\,. \tag{2.11}$$

We now consider the main concern of our work, namely, the thermal part Σ_T of Σ which vanishes as $T\to 0$. The susceptibility $\chi(T)$ is related to Σ_T through the equation

$$\alpha(1+\alpha)^{-1}=\tilde{U}[D_g^{-1}(0)-\Sigma_T(0)] \tag{2.12a}$$

$$=\alpha_0(1+\alpha_0)^{-1}-\tilde{U}\Sigma_T(0)\,, \tag{2.12b}$$

where as mentioned earlier, $\alpha=\alpha_T=\chi_T^{-1}\chi_P$. We use skeleton diagrams for Σ, involving true propagators $D(\underline{q})$. Now consider a simple diagram, e.g., Fig. 2(a) for Σ. The quartic coupling vertex has the value

$$\lambda^{2a}(\underline{q})=\frac{U^2}{\beta}\sum_{\underline{k}}(G_{\underline{k}}G_{\underline{k+q}})^2\,.$$

This obviously depends very weakly on temperature, i.e., as $(T/T_F^0)^2$. We neglect such a dependence. The temperature dependence of

$$\Sigma^{2a}(0)=\Sigma_g^{2a}(0)+\Sigma_T^{2a}(0)=\frac{1}{\beta}\sum_{\underline{q}}\lambda^{2a}(\underline{q})D(\underline{q})$$

thus comes from (i) explicit temperature dependence of $D(\underline{q})$ and (ii) temperature dependence from summation over Matsubara frequencies. This latter can be converted into an integral over frequency with a Bose factor, i.e., into an expression

$$\int_{-\infty}^{\infty}\phi_T(\omega)(e^{\beta\omega}-1)^{-1}d\omega\,,$$

where $\phi_T(\omega)$ involves the spectral function of D, etc. We split the integral into a zero-point and a thermal part, i.e., into

$$\int_{-\infty}^{0}\phi_T(\omega)\,d\omega$$

and

$$\int_0^{\infty}[\phi_T(\omega)-\phi_T(-\omega)](e^{\beta\omega}-1)^{-1}d\omega\,.$$

The temperature-dependent part of the former is, to leading order, seen to be of the form $A\Sigma_T^{2a}(0)$ and thus only leads to a renormalization of the size of $\Sigma_T^{2a}(0)$. This can be included by an appropriate coupling constant renormalization, and we do not consider it separately further. The essential term is the thermal part, which is large and strongly temperature dependent if $\phi_T(\omega)$ is large for low frequencies at temperatures of interest. Near the ferromagnetic instability $\mathrm{Im}[D(q,\omega^-)/\pi]$ is by definition large for $|\vec{q}|\ll 2k_F$ and $\omega\ll\epsilon_F$. We now discuss whether the thermal contribution to Σ can be systematically analyzed. The contributions $\Sigma_T^{2b}(0)$ and $\Sigma_T^{2c}(0)$ are of the same form as $\Sigma_T^{2a}(0)$ and are easily written. To obtain $\Sigma_T^{2d}(0)$, we again decompose the two frequency integrals (arising from the two boson propagators) into zero-point and thermal parts. The leading temperature dependence is due to the thermal part of one multiplying the zero-point part of the other. The term involving the product of two thermal parts is seen to be of relative order $\Sigma_T^I(\ll 1)$. This is because the momentum and energy of the two fluctuations are not correlated. Assuming these to be $\underline{q}$, $\underline{q}'$, the sixth-order coupling in $Y(\{\vec{\xi}(\underline{q})\})$ is seen to be a very flat function of $\underline{q}$ and $\underline{q}'$, with the usual range in momentum and energy $(2k_F, \epsilon_F)$. Since frequencies ω, ω' are $\lesssim k_BT$, it turns out that this flat function multiplies $D(\vec{q},0)\, D(\vec{q}',0)$. $D(q,0)$ is a sharply peaked function in q, the width in q being $\sqrt{\alpha_0}k_F$ (for $\tau \ll \alpha_0$) or $\tau^{1/3}k_F(\alpha_0 < \tau \ll 1)$ (see Sec. III A). Integrations over $\underline{q}$ and $\underline{q}'$ are thus essentially uncorrelated, and one thus has a relative factor Σ_T. This argument cannot be used for the doubly and triply thermal parts of diagrams such as Figs. 2(e) and 2(f), where the momentum and energy of fluctuations is strongly correlated, e.g., in diagrams for $\Sigma_T(0)$, $\sum_i \underline{q}_i = 0$. Here the fluctuations are all constrained to have small $\underline{q}$ where their amplitude is large. We discuss these terms separately below (Secs. III B and III C) and show that while the two correlated internal spin fluctuation term $\Sigma_T^{2e}(0)$ has the same qualitative temperature dependence as the one-spin fluctuation terms $\Sigma_T^{2a-2c}(0)$, the three-spin fluctuation term $\Sigma_T^{2f}(0)$ is of higher order in temperature. By a power counting argument this result can be extended to higher-order correlated diagrams.

Returning to the one-thermal-spin-fluctuation term $\Sigma_T^{\rm I}(0)$ in $\Sigma_T(0)$, we see that the presence of diagrams like 2(d)–2(f) renormalizes the quartic-spin-fluctuation coupling vertex, so that $\Sigma_T^{\rm I}(0)$ can be written [see Fig. 2(g)]

$$\Sigma_T^{\rm I}(0) = -\frac{1}{\beta}\sum_{\underline{q}}\lambda(\underline{q})D(\underline{q}) , \qquad (2.13)$$

where it is understood that we use the thermal part in the energy integration in Eq. (2.13). The calculation of $\lambda(\underline{q})$ is an intermediate coupling problem, and cannot be done reliably. The lowest-order (in U) estimate for $\lambda(\underline{q})$ can be obtained from Figs. 2(a)–2(c). $\lambda(\underline{q})$ is again a function of range $(2k_F,\epsilon_F)$ in $(\vec{q},\omega)$. For small q,ω, if we expand $\lambda(\underline{q})$ as a power series in (q,ω), we find that the $q=0$, $\omega=0$ term gives the leading temperature contribution, while q^2 and ω terms contribute a term of relative order τ^2/α_0 (for $\tau \ll \alpha_0$) or $\tau^{2/3}$ (for $\alpha_0 \lesssim \tau \ll 1$). Thus for small q and ω, the (q,ω) dependence of $\lambda(\underline{q})$ can be ignored. However, $\lambda(\underline{q})$ decreases sharply for $q > 2k_F$ and $\omega > \epsilon_F$. In our calculations we approximate it by a simple flat cutoff function

$$\lambda(\underline{q}) = \lambda_1\theta(q_c - |\vec{q}|) , \qquad (2.14)$$

where λ_1 and $q_c(\sim k_F)$ are parameters determined by a very-low-temperature fit. (See Sec. IV for details.)

We now calculate the thermal contribution Σ_T to self-energy arising from one, two, and three spin fluctuations.

III. THERMAL-SPIN-FLUCTUATION SELF-ENERGY

A. One-fluctuation term $\Sigma_T^{\rm I}$

The simplest thermal contribution to the spin-fluctuation self-energy arises from the interaction with one internal spin fluctuation and is diagrammatically shown in Fig. 2(g). Using the simple form (2.14) for the vertex $\lambda(\underline{q})$, one has

$$\tilde{U}\Sigma_T^{\rm I}(0) = -2\lambda_1\tilde{U}^2 \sum_{|\vec{q}|\le q_c} \int_0^\infty \omega(e^{\beta\omega}-1)^{-1} \times {\rm Im}\left(\frac{D(q,\omega^-)}{\pi\tilde{U}}\right) , \qquad (3.1)$$

where using Eqs. (2.11) and (2.12), we find

$$\tilde{U}D^{-1}(q,\omega^{\mp}) = \alpha_0(\alpha_0+1)^{-1} - \tilde{U}\Sigma_T(0) + \tilde{U}[\chi_g^0(q,\omega^{\mp}) - \chi_g^0(0)] . \qquad (3.2)$$

Equations (3.1) and (3.2) constitute a set of coupled equations for $\tilde{U}\Sigma_T^{\rm I}(0)$, knowing which $\chi(T)$ is immediately found through Eq. (2.12). In the right-hand side of Eq. (3.2), the first term is known from $\chi(0)$, and the last (except for a factor $\tilde{U}\sim 1$) is the dispersive part of $\chi_g^0(\vec{q},\omega^\pm)$. Since the frequencies ω of interest are $\lesssim k_BT$, one works in the region $\omega \lesssim qv_F$, where v_F is the Fermi velocity and so one has

$$\chi_g^0(q,\omega^\pm) \simeq \chi_g^0(q,0) \pm \tfrac{1}{2}i\pi\gamma\,\omega/q , \qquad (3.3)$$

where $\chi_g^0(\vec{q},0)$ is the static Lindhard function[22] which can be approximated for small q by

$$\chi_g^0(q,0) \simeq \chi_g^0(0,0)(1-\delta q^2) . \qquad (3.4)$$

The constants γ and δ appearing in Eqs. (3.3) and (3.4) are $\gamma=\frac{1}{2}$, $\delta=\frac{1}{12}$. In Eqs. (3.3) and (3.4), ω and q are dimensionless (in units of ϵ_F and k_F, respectively). We see that $\chi_g^0(\vec{q},0)$ decreases slowly with q.

The frequency integral in Eq. (3.1) is easily done using (3.2) and the form Eq. (3.3) for $\chi_g^0(\vec{q},\omega^\mp)$. One has

$$\tilde{U}\Sigma_T^{\rm I}(0) = -\lambda_1\tilde{U}\,\frac{1}{\pi}\sum_q C_q^{-1}[\ln y_q - (2y_q)^{-1} - \psi(y_q)] \qquad (3.5)$$

where

$$C_q = \pi\gamma/2q ,$$

$$y_q = \{\alpha_0(1+\alpha_0)^{-1} - \tilde{U}\Sigma_T^{\rm I}(0) + \tilde{U}[\chi_g^0(\vec{q},0) - \chi_g^0(0,0)]\}(\pi^2\gamma\tau/q)^{-1} \qquad (3.6)$$

and $\psi(y_q)$ is the digamma function. The limiting forms of $\Sigma_T^{\rm I}(0)$ are the following. At very low temperatures such that $y_{q=0} \gg 1$, i.e., $\tau/\alpha_0 \ll 1$, we use the asymptotic form $\ln y - (2y)^{-1} - \psi(y) = (12y^2)^{-1}$. Since only small q region contributes to the integral (3.5), we use the form Eq. (3.4), relax q_c to ∞ and find that

$$\tilde{U}\Sigma_T^{\rm I}(0) = \tilde{U}\lambda_1 \frac{\rho^2(\epsilon_F)}{\rho''(\epsilon_F)} \frac{\pi^2}{24} \frac{\tau^2}{\alpha_0}, \tag{3.7}$$

where $\rho''(\epsilon_F)$ is the second derivative of the density of states, evaluated at the Fermi surface. The inverse susceptibility [see Eq. (2.12)] is then of the form

$$\chi_T^{-1}\chi_P = \alpha(\tau) = \alpha_0 + A(\tau^2/\alpha_0). \tag{3.8}$$

This is the well-known paramagnon-regime result obtained by Béal-Monod *et al.*[11] Their method is discussed in relation to ours in Sec. V. Note that since $\Sigma_T^{\rm I}(0) \ll \alpha_0$, i.e., $\tau^2/\alpha_0 \ll \alpha_0$, it is not necessary to include in y_q the second term on the right-hand side of Eq. (3.6), i.e., $\Sigma_T^{\rm I}(0)$ does not need to be determined self-consistently. [The leading self-consistency correction is of relative order $(\tau^2/\alpha_0^3) \ln (\tau^2/\alpha_0^3)$.] For higher temperatures where typical $y_q \lesssim 1$, the limiting form is obtained by using the asymptotic expression $\ln y - (2y)^{-1} - \psi(y) = (2y)^{-1}$ (valid for $y \ll 1$). We then find that

$$\tilde{U}\Sigma_T^{\rm I}(0) = \frac{\tilde{U}\lambda_1\rho^2(\epsilon_F)}{4\rho''(\epsilon_F)} \frac{\tau}{\delta} \times \left[q_T - \left(\frac{\alpha}{\delta}\right)^{1/2} \tan^{-1} q_T \left(\frac{\delta}{\alpha}\right)^{1/2} \right], \tag{3.9}$$

where q_T is a thermal cutoff such that $y_{q_T} \sim 1$. We have used for simplicity the form Eq. (3.4). This leads to an estimate of the cutoff as $q_T^3 \sim (\tau\gamma/\delta)$, i.e., $q_T \sim \tau^{1/3}$. Thus considering only the first term in Eq. (3.9), we see that $\tilde{U}\Sigma_T^{\rm I}(0)$ and hence α, depends on τ as $\tau^{4/3}$. However, since δ is small, the thermal cutoff q_T is high, $\sim q_c$. (The spin-fluctuation energy rises only slowly with q.) Thus α rises nearly linearly with τ. This is the classical spin-fluctuation behavior, first pointed out for an itinerant electron ferromagnet by Murata and Doniach.[6] Note that we have assumed $\tau \ll 1$, i.e., the system is degenerate. Even so, since the characteristic fluctuation energy $[\alpha_0 + \tilde{U}\Sigma_T^{\rm I}(0)]$ is small, the system behaves as if it were classical with regard to spin fluctuations. We now consider the second term in Eq. (3.9). Because of its presence, the Eq. (3.9) is a transcendental equation for $\tilde{U}\Sigma_T^{\rm I}(0)$, which has to be calculated self-consistently. An estimate of its size is obtained by putting $\alpha \sim \tilde{U}\Sigma_T^{\rm I}(0) \sim \tau^{4/3}$. We then find it to be or order $\tau^{1/3}$ relative to the first term [which is the non-self-consistent estimate of $\tilde{U}\Sigma_T^{\rm I}(0)$]. Since $\tau^{1/3}$ is not very small, it is essential to do a self-consistent calculation of $\tilde{U}\Sigma_T^{\rm I}(0)$, using, say Eq. (3.9).

B. Two-fluctuation term $\Sigma_T^{\rm II}(0)$

A simple self-energy diagram for $\Sigma_T(0)$ with two internal correlated spin fluctuations is shown in Fig. 2(e). The two-internal spin fluctuations have equal and opposite momenta and energies. Thus there is only one energy integration. We are interested in the thermal part of this integral. We first note that the coupling between three static spin fluctuations vanishes due to time-reversal invariance. Examination of low-order diagrams for the three-spin-fluctuation vertex $\phi(\underline{q})$ shows it to go as ω/q^2.[11] Adding together the contribution of all the lowest-order (in U) diagrams, we find

$$\Sigma^{\rm II}(0) = \frac{1}{3U\beta} \sum_{\underline{q}} D(\underline{q}) D(-\underline{q}) \phi^2(\underline{q}), \tag{3.10}$$

where

$$\phi(\underline{q}) = \frac{U^2}{\beta} \sum_{\underline{k}} G_{\underline{k}}^2 (G_{\underline{k}-\underline{q}} - G_{\underline{k}+\underline{q}}) \tag{3.11a}$$

$$\simeq \frac{U^2 \rho(\epsilon_F)}{\epsilon_F} \frac{\omega}{q^2}, \tag{3.11b}$$

where (3.11b) is the small-ω q limit of (3.11a). After substituting Eq. (2.11) for D and doing the energy integration, we find that

$$\tilde{U}\Sigma_T^{\rm II}(0) = \bar{\lambda}_2 \frac{\tau}{2} \int_0^{q_c} dq \frac{\partial}{\partial y} \times \{ y_q^2 [\ln y_q - (2y_q)^{-1} - \psi(y_q)] \}, \tag{3.12a}$$

where

$$\bar{\lambda}_2 = 32\tilde{U}^3/3\pi^2\gamma^2 \tag{3.12b}$$

and y has been defined earlier [Eq. (3.6)]. This integral can be estimated in the regime $\tau/\alpha_0 \ll 1$, and contributes as τ^2/α_0. In the classical regime $\alpha_0 \lesssim \tau \ll 1$, one obtains for it a value $\tau^{4/3}$. Thus the two-spin-fluctuation diagram contributes terms with the same temperature dependence as the one spin-fluctuation diagram. Again, as in that case, the vertices are strongly renormalized, i.e., the three-spin-fluctuation coupling vertex is a ground-state ($T = 0$) quantity which, like the quartic vertex, is dressed by zero-point fluctuations.

C. Three-spin-fluctuation term $\Sigma_T^{\rm III}(0)$

We now consider the effect of three-correlated spin fluctuations on the self-energy. A diagram for $\Sigma^{\rm III}(0)$ is shown in Fig. 2(f) In this diagram, one of the internal spin fluctuations is transverse while the other two are longitudinal. One can also have terms with three internal transverse spin fluctuations. There are various ways of joining fluctuation lines. Taking all these into account, we find, considering only diagrams with bare quartic coupling vertices

$$\Sigma^{III}(0)=\frac{\lambda_3}{\beta^2}\sum_{\underline{q}_1,\underline{q}_2} D(\underline{q}_1)D(\underline{q}_2)D(-\underline{q}_1,-\underline{q}_2)\,, \qquad (3.13)$$

where the coupling λ_3 is evaluated for $\underline{q}_1,\underline{q}_2\equiv 0$, and is

$$\lambda_3=(8/15U^2)[\tfrac{1}{6}U^3\rho''(\epsilon_F)]^2\,. \qquad (3.14)$$

Since the quartic spin-fluctuation vertices are renormalized by the spin-fluctuation coupling, the above represents only an order of magnitude estimate. The frequency summations over z_1 and z_2 in Eq. (3.13) can be performed by writing D in terms of its spectral function $\rho(\omega)$. There are terms in Σ^{III} which contribute to Σ_g, Σ_T^{I}, and Σ_T^{II}. Omitting these, the pure thermal part is seen to be given by

$$\Sigma_T^{III}(0)=-2\lambda_3\epsilon_F^2\sum_{\vec{q}_1,\vec{q}_2}\int_0^\infty\left(\prod_i d\omega_i\,(e^{\beta\omega_i}-1)^{-1}\right)\times\rho_{q_1}(\omega_1)\rho_{q_2}(\omega_2)\rho_{-q_1-q_2}(\omega_3)\times(\omega_1+\omega_2+\omega_3)^{-1}\,. \qquad (3.15)$$

We estimate the integral by writing

$$\int_0^\infty f(\omega)(e^{\beta\omega}-1)^{-1}d\omega\simeq k_BT\int_0^{\eta k_BT}\omega^{-1}f(\omega)\,d\omega\,,$$

where η is a cutoff factor of order unity. Equation (3.15) then becomes

$$\Sigma_T^{III}(0)=-2\lambda_3\epsilon_F^2\tau^2\pi^{-3}\sum_{\vec{q}_1\vec{q}_2}(\alpha+\delta q_1^2)^{-1}(\alpha+\delta q_2^2)^{-1}\times[\alpha+\delta(\vec{q}_1+\vec{q}_2)^2]^{-1}\prod_i\tan^{-1}(\theta_i)\,, \qquad (3.16)$$

where $\theta_i=\eta\pi\gamma\tau/2(\alpha+\delta q_i^2)$. The temperature dependence of $\Sigma_T^{III}(0)$ arises from a thermal factor τ^2 outside and a thermal fluctuation phase space factor $\tan^{-1}\theta_i$ inside the q integral. In the paramagnon regime $\Sigma_T^{III}(0)\sim\tau^4/\alpha_0^3$. Since $\Sigma_T^{I}(0)$ and $\Sigma_T^{II}(0)$ go as τ^2/α_0 in this temperature regime, the three-thermal-spin-fluctuation term is of relative order $(\tau/\alpha_0)^2$, i.e., much smaller. In the classical regime the $\tan^{-1}\theta_i$ factors [in Eq. (3.16)] are of order unity. Taking them to be unity, we find

$$\Sigma_T^{III}(0)=-(\tfrac{8}{5})(\tfrac{2}{3})^4\tilde{U}^4\pi^2\tau^2\ln(1/3\alpha)\,. \qquad (3.17)$$

This is to be compared with Eq. (3.9) for $\Sigma_T^{I}(0)$ and a similar expression for $\Sigma_T^{II}(0)$. We see that $\Sigma_T^{III}(0)$ is of order $\tau\ln\tau$ relative to the simplest nonvanishing contribution. Since $\tau\ll 1$, $\Sigma_T^{III}(0)$ is of higher order. The $\ln\tau$ factor depends weakly on τ, and for α values of interest, $\ln(1/3\alpha)\sim O(1)$.

We conclude from the above analysis that the leading temperature dependence of Σ_T and therefore of the spin susceptibility is described by the terms $\Sigma_T^{I}(0)$ and $\Sigma_T^{II}(0)$. The quantities $\Sigma_T^{I}(0)$ and $\Sigma_T^{II}(0)$ are to be calculated self-consistently. $\Sigma_T(0)$ and the inverse susceptibility interpolate smoothly between the two limiting forms τ^2/α_0 (for $\tau\ll\alpha_0$) and τ (for $\alpha_0\lesssim\tau\ll 1$). We now present a self-consistent calculation of $\tilde{U}\Sigma_T$ and thus of χ_T^{-1}, valid in the range $\tau\ll 1$.

IV. COMPARISON WITH EXPERIMENT

In this section a comparison is made with the results of the theory described above, and the experimental numbers for liquid ^{3}He.[1] ^{3}He is chosen because it is a clean system for which the spin susceptibility is accurately measured. In contrast to other nearly ferromagnetic systems such as Pd and Ni_3Ga, there are no band structure or "clustering" effects. Further, the Stoner enhancement factor α_0^{-1} of ^{3}He varies from 10 to 20 as pressure varies from ~0 to ~26 atm. The density and hence T_F° do not change much. We compare here theoretical results with experiment for the two extreme pressures.

The inverse susceptibility α (i.e., $\chi_T^{-1}\chi_P$) can be found if α_0 [i.e., $\chi^{-1}(0)\chi_P$] and $\tilde{U}\Sigma_T(0)$ are known [Eq. (2.12)]. The former is taken from experiment. The important terms for the latter are the one- and two-spin-fluctuation diagrams. The first is given by Eqs. (3.5) and (3.6). Here the size λ_1 of the quartic vertex, and the wave-vector cutoff q_c [see Eq. (2.14)] are undetermined parameters. The two-spin-fluctuation term $\tilde{U}\Sigma_T^{III}(0)$ is given by Eq. (3.12). Since calculations show that the two-spin-fluctuation term, though having the same temperature dependence, is only about 15% of the one-spin-fluctuation term, the theoretical results are rather insensitive to the precise value of $\bar{\lambda}_2$ or the cutoff q_c [Eq. (3.12)], provided they are comparable to the lowest order estimate Eq. (3.12b), and $2k_F$ respectively. For simplicity, therefore, we use the equation (3.12) for $\bar{\lambda}_2$, the cutoff q_c being the same as in Eq. (3.6). Thus the free parameters are still only λ_1 and q_c. For a given λ_1 and q_c, $\tilde{U}\Sigma_T(0)$ is the sum of Eqs. (3.5) and (3.12a). The right-hand side of this sum is a function of $\tilde{U}\Sigma_T(0)$ which therefore has to be obtained self-consistently. This is achieved iteratively on a computer, for each temperature. Both the Lindhard expression and the small-q expression Eq. (3.4) were used for $\chi_g^0(\vec{q},0)$ and finally led to fits of very similar quality. The function $[\ln y-(2y)^{-1}-\psi(y)]$ is well interpolated by $(2y+12y^2)^{-1}$ which is cor-

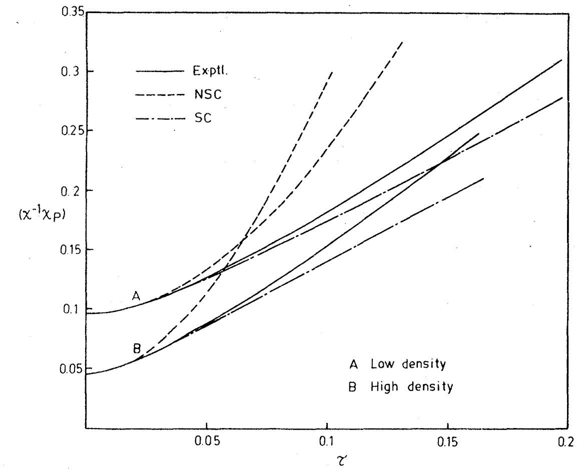


FIG. 3. Self-consistent (SC) and non-self-consistent (NSC) results for $\chi_T^{-1}\chi_P$ of liquid ^{3}He as a function of $\tau (= T/T_F^0)$ compared with experiment (the cutoff $q_c = 2K_F$).

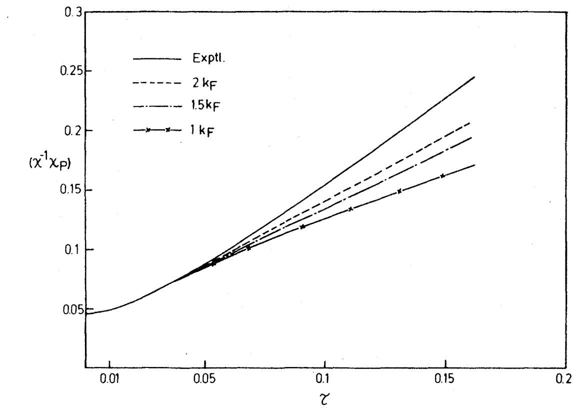


FIG. 5. Cutoff dependence of the calculated $\chi_T^{-1}\chi_P$ for high-density liquid ^{3}He. The values of cutoff q_c are shown.

rect for both $y \gg 1$ and $y \ll 1$. Choosing a particular q_c, we determine λ_1 by a least-squares fit to very low τ ($\tau < 0.03$) experimental values, $\tilde{U}\Sigma_T(0)$ being calculated self-consistently for each temperature in this range. The value of q_c is changed, and the corresponding λ_1 determined. With a given (λ_1, q_c) pair which fits very low temperature $\alpha(\tau)$, the self-consistent calculation is repeated for higher temperatures, up to $\tau \simeq 0.2$. The results are shown in Figs. 3 to 5 and in Table I.

In Fig. 3 we compare the experimental and theoretical results for $\alpha(\tau)$ as a function of τ. The cutoff q_c is taken to be $2k_F$. The experimental values of $\alpha(\tau)$ are slightly higher than the theoretical. The difference is about 15% at $\tau \simeq 0.15$, and can be accounted for by the third-order term Σ_T^{III}. A simple calculation of this term shows that it gives a positive contribution to $\alpha(\tau)$. At $\tau = 0.10$, $\tilde{U}\Sigma_T^{III}(0) \simeq -0.01$ and at $\tau = 0.15$, $\tilde{U}\Sigma_T^{III}(0) = -0.025$ (low density, case A in Fig. 3). These values are almost exactly enough to account for the difference between experiment and theory, as well as for its increase with τ. For comparison, we have also shown the non-self-consistent result for $\alpha(\tau)$, i.e., the results for $\alpha(\tau)$ obtained when on the right-hand side of Eqs. (3.6) we omit the temperature-dependent part $\Sigma_T^{I}(0)$ at all temperatures. The importance of self-consistency is clear.

In Fig. 4, we compare the results for $\alpha(\tau)$ calculated as above omitting entirely the two-spin-fluctuation term and obtaining a λ_1 which fits the low-temperature experimental values (with just

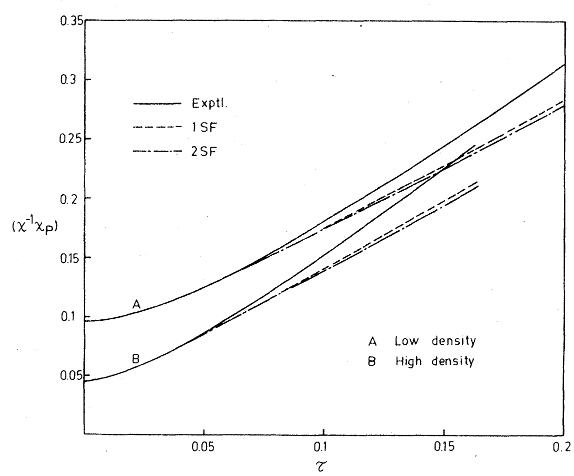


FIG. 4. $\chi_T^{-1}\chi_P$ for liquid ^{3}He as a function of τ compared with theory, assuming only the one-spin-fluctuation contribution to be nonzero (dashed line) and assuming both one- and two-spin-fluctuation contributions to be nonzero (dashed-dotted line) ($q_c = 2K_F$).

TABLE I. Spin-fluctuation coupling parameters $\lambda_1(\underline{q}) = \lambda_1\Theta(q_c - |\vec{q}|)$ for two molar volumes V_m of liquid ^{3}He. The parameter λ_1 is obtained, for each q_c, by fitting the theoretical expression (see text) to the experimental value of $\chi_T^{-1}\chi_{\text{Pauli}} = \alpha(T)$ between $0 < \tau = T/T_F^0 < 0.03$. λ_1 is shown for the case where only the one-internal-spin-fluctuation contribution to $\alpha(T)$ is included (part marked 1 SF), and where both one- and two-internal-spin-fluctuation contributions are included [marked (1+2) SF]. The zero-temperature susceptibility $\chi_0 = \chi_P\alpha_0^{-1}$.

Density	q_c	λ_1: $1\ k_F$	$1.5\ k_F$	$2\ k_F$
$V_m = 35.5$ cm^3/mole, $T_F^0 = 5.08$ °K, $\alpha_0 = 0.095$	1 SF	0.336	0.202	0.159
	(1+2) SF	0.375	0.227	0.179
$V_m = 26.5$ cm^{-3}/mole, $T_F^0 = 6.18$ °K, $\alpha_0 = 0.044$	1 SF	0.326	0.212	0.176
	(1+2) SF	0.362	0.236	0.196

the one-spin-fluctuation term). The λ_1 values necessary are shown in Table I. We see that (i) the λ_1 values for one-spin-fluctuation and two-spin-fluctuation cases are nearly the same, and (ii) the fit to experiment is nearly as good in both cases; if anything, the one-spin-fluctuation approximation fits experiment slightly better. The reasons for the above are the smallness of the two-spin-fluctuation contribution relative to the one-spin-fluctuation term $[\Sigma_T^{\mathrm{II}}(0) \sim 0.15\Sigma_T^{\mathrm{I}}(0)]$ and their similar temperature dependences. Thus for most calculations it is enough to calculate $\tilde{U}\Sigma_T^{\mathrm{I}}(0)$ self-consistently and to use it to find $\alpha(\tau)$.

The effect of the cutoff is shown in Fig. 5. We choose, for both densities, three values of the cutoff ($q_c = 1$, 1.5, and 2). For each of these we determine the λ_1 which best fits the low temperature $\alpha(\tau)$, and then calculate $\alpha(\tau)$ at higher temperatures It is seen that the fit deteriorates somewhat as q_c is reduced. Further, $\lambda_1 q_c$ is very nearly constant. Both these point to the fact that fluctuations of intermediate wave vectors ($k_F \lesssim q \lesssim 2k_F$) play a significant role. This is because the spin fluctuation energy rises slowly with q. Thus if the same effect is sought with q_c halved, one has to nearly double the (quartic) spin-fluctuation coupling vertex.

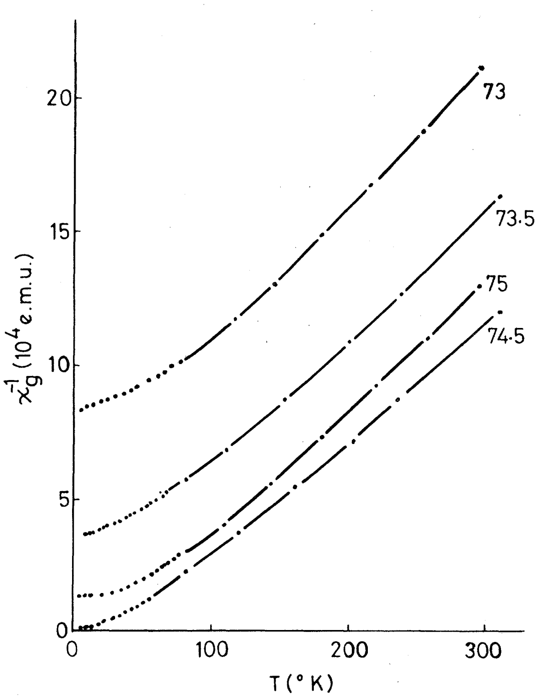


FIG. 6. Inverse susceptibility of (nominal) Ni_3Al and Ni_3Ga (in emu) as a function of temperature. The curves marked 73 and 75 refer to the Ni concentrations in Ni_3Ga. The ones marked 73.5 and 74.5 denote Ni concentration in Ni_3Al (see Ref. 23 for details).

A noteworthy feature of our calculations is that very nearly the same λ_1 is found at both densities. This is expected since λ_1 depends on free fermion properties and on U. Neither of these change much $[T_F^0(A) = 0.83\ T_F^0(B)$, see Table I].

There are many metals and alloys which also show a T^2 and T behavior for the inverse susceptibility. For example, experimental-susceptibility curves for Ni_3Al and Ni_3Ga are shown in Fig. 6. χ^{-1} increases linearly with temperature above 100 °K. At low temperatures a T^2 behavior is obvious. The high temperature slopes are nearly parallel, while the flattening begins at different temperatures.

V. COMPARISON WITH EARLIER WORK AND CONCLUSION

Earlier work on the spin susceptibility χ_T of nearly ferromagnetic Fermi systems is confined to the very low temperature or paramagnon region ($\tau \ll \alpha_0$). Here, Béal-Monod *et al.* showed that $\chi_P^{-1}(\Delta\chi) \sim -\tau^2\alpha_0^2$. There is a one-to-one diagrammatic correspondence between our expression for $D(0) \simeq D_g(0) + D_g^2\Sigma_T(0)$ with $\Sigma_T(0)$ evaluated non-self-consistently, and $(\partial^2 F/\partial B^2)_{B=0}$ evaluated in the ladder bubble approximation for F. The latter approximation corresponds to evaluating $D_g(0)$ in the RPA, and the quartic-spin-fluctuation coupling λ_1 in the lowest nonvanishing order [Figs. 2(a)–2(c)]. Our analysis shows that the validity of the paramagnon form is not tied to the approximation scheme mentioned above. Further, we show that the vertex λ_1 cannot be determined accurately from theory. Kawabata[24] has pointed out that the Béal-Monod estimate of λ_1 is incorrect because it omits the effect due to chemical potential changing with temperature. Because of what we have said above the objection, while valid, is not serious.

There have been a number of papers in which the existence of a $\tau^2 \ln\tau$ term for $\chi_P^{-1}\chi_T$ has been claimed. The coefficient is, according to Misawa,[15] proportional to α_0^{-1}, while according to Barnea[16] it is proportional to α_0^{-3}. The former is obtained from phenomenological Landau Fermi-liquid theory, and the latter from microscopic many-body theory. We have analyzed the origin of the $\tau^2 \ln\tau$ term and find it to be in an improper spin-nonconserving approximation. The argument is as follows. Ignoring μ_B, etc., the susceptibility χ is

$$\chi = \frac{\partial}{\partial B}\left(\langle n_+\rangle - \langle n_-\rangle\right)\Big|_{B=0} = \sum_{p\sigma} \sigma\left(\frac{\partial G_{p\sigma}}{\partial B}\right)\Big|_{B=0} = \left[\sum_{p} G_{p\sigma}^2\left(-1 + \sigma\frac{\partial \Sigma_\sigma(p,B)}{\partial B}\right)\right]_{B=0}.$$

Assuming the temperature dependence to arise from $\Sigma_\sigma(\underline{p},B)$, a correct approximation is

$$\chi \simeq \sum_{p\sigma} G_{\underline{p}}^{0^2}\left(G_{\underline{p}}^0 \Sigma_{\underline{p}} + \sigma \frac{\partial \Sigma_\sigma(\underline{p},B)}{\partial B}\bigg|_{B\to 0}\right),$$

where $G_{\underline{p}}^0$ and $\Sigma_{\underline{p}}$ are at $B=0$. The $\tau^2 \ln\tau$ term arises if only the first term on the right is retained (Barnea). When both the self-energy and the vertex-correction terms are retained, as they will be in a conserving or vector-field approximation, there is no $\tau^2\ln\tau$ term. This is most easily seen here by using the paramagnon approximation for $\Sigma_\sigma(\underline{p},B)$ (with zero range U). The $\tau^2 \ln\tau$ parts cancel.

The form of $\Delta\chi_T$ for $\tau \ll \alpha_0$ has led to the belief that the expansion parameter is τ/α_0. For example, deviations from τ^2 behavior for χ_T appear at temperatures $\tau \sim 0.3\alpha_0$, i.e., $\tau \sim 0.015$ or $\tau \sim 0.03$ for liquid ^{3}He (high and low density, respectively). Thus the impression is that the situation is very complicated for $\alpha_0 \lesssim \tau \ll 1$. One of our aims here has been to show that this is not so. The processes involving one- and two- (correlated-) spin-fluctuations dominate the temperature dependence of Σ_T at *all* temperatures $\tau \ll 1$. The three and higher thermal-spin-correlation corrections to Σ_T are of a higher order in temperature. This is a quantum-statistical effect. For example, consider a low-T_c itinerant-electron ferromagnet. The critical regime can be defined as one where the three-fluctuation correlation term $[\sim\tau^2 \ln(\tau-\tau_c)]$ is of the same size as the Hartree term τ for χ^1.[13] Thus the width of the critical correlated fluctuation regime is $\Delta\tau \sim e^{-1/\tau_c}$, i.e., it shrinks exponentially to zero as $\tau_c \to 0$. In our case (τ_c negative), the correlated-fluctuation term is always smaller than the Hartree term. The extra powers of T that reduce the fluctuation-correlation effect are obtained from the Bose factor, one for each thermal-fluctuation energy. This T is scaled with respect to the Fermi energy, as can be seen by considering the case $\alpha_0 = 0$. There are additional phase-space factors. These give an extra factor $(\tau/\alpha_0)^2$ for $\tau \ll \alpha_0$, and a logarithmic term $\ln(1/3\alpha)$ in the classical regime. Thus quantum statistics suppress fluctuation correlations. This has been shown for the present system by Hertz[25] using a renormalization-group method. We have explicitly calculated correlation effects above using diagrammatic perturbation theory, showing it to converge.

The effects of spin fluctuations on the specific heat of a nearly-ferromagnetic-Fermi system can be analyzed in a way similar to that of this paper.[26] The results are again valid for the temperature range $\tau \ll 1$, and not just the range $\tau \ll \alpha_0$. The observed Schottky-like peak in $C_v(T)$ of ^{3}He, occurring at about $\tau \sim 0.3\alpha_0$, its tailing off and levelling to half the free Fermi-gas value $\frac{1}{4}k_B\pi^2\tau$, can all be explained by considering the behavior of spin and density fluctuations. The spontaneous magnetization squared of weak ferromagnets also shows a τ^2 behavior for $\tau \ll \tau_c$ and a classical $(\tau_c - \tau)$ behavior for $\tau \sim \tau_c$. This will be discussed in a forthcoming paper.[27]

ACKNOWLEDGMENT

One of the authors (S.G.M.) gratefully acknowledges financial assistance from the Council of Scientific and Industrial Research (India).

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