Effect of a parallel magnetic field on interband transitions in two-dimensional systems

P. M. Platzman
Bell Laboratories, Murray Hill, New Jersey 07974

B. I. Halperin
Harvard University, Cambridge, Massachusetts 02138

(Received 30 November 1977)

In the presence of a parallel magnetic field the interband transition connected with electrons trapped at a helium-vacuum interface shows interesting structure. We show how the shape of this interband absorption line, in the absence of collisions with gas atoms or surface ripplons, may be directly related to the Fourier transform of the velocity autocorrelation function.

I. INTRODUCTION

For electrons trapped on a liquid-helium surface, the one-electron energy spectrum consists of several discrete quantum levels for motion perpendicular to a surface of which only the lowest level is occupied, and a free-electron-like continuum for motion in the plane of the surface.[1,2] In a typical high-frequency experiment, one observes interband electronic dipole transitions, involving the promotion of an electron from the ground-state quantum level to an excited level for perpendicular motion (see Fig. 1). When a magnetic field B is applied parallel to the surface, the motion of the electron in the plane is coupled, by virtue of the $\vec{v}\times\vec{B}$ force, to this interband transition. This coupling can induce a rather complicated and interesting broadening and/or structure into the shape of the absorption line.[3]

In a recent experiment[3] Zipfel *et al.* observed such effects by applying fields of the order of 1 kG parallel to the surface of liquid He. They investigated, at temperatures near 1 °K, the width of the interband transition as a function of electron density and magnetic field. From these measurements they extracted a so-called velocity autocorrelation time.

In this paper we would like to show how the shape of the interband absorption line, in the absence of collisions with gas atoms or surface ripplons, may be directly related to the Fourier transform of the velocity autocorrelation function. We will show that it is possible (by varying the magnetic-field strength) to extract detailed information about this velocity autocorrelation function. These experiments correspond in many respects to Mössbauer studies of vacancy diffusion[4] and to incoherent-neutron-scattering experiments where the momentum transfer is analogous to the strength of the dc magnetic field.[5] Since the system of interest, electrons on helium, can be strongly correlated, the behavior of this correlation function is of great interest. For example, these experiments can be used to study changes in the electron self-diffusion constant as one passes from a correlated electron liquid to a two-dimensional Wigner crystal.[6]

II. FORMULATION

The essential aspect of our problem is that the perpendicular energy levels have a large separation compared to the characteristic energies for electronic motion in the plane. In this "Born-Oppenheimer" limit the electron undergoing the transition is identifiable and the absorption probability is proportional to[7]

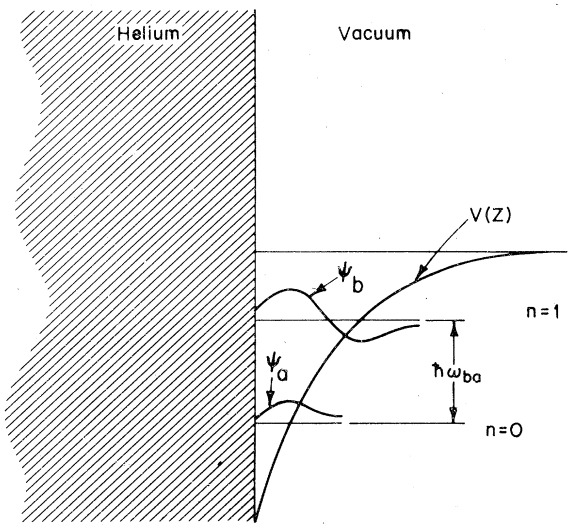

FIG. 1. Energy-level picture for electrons on the surface of liquid helium. The potential $V(z)$, the wave functions $\psi_{a,b}$, and the energy levels are shown schematically.

$$I_{ba}(\omega) = |M_{ba}|^2 \left\langle \sum_n \left| \int d\{\vec{x}_i\} X^*_{1n}(\{\vec{x}_i\}) X_{0m}(\{\vec{x}_i\}) \right|^2 \times \delta(\omega - E_{1n} + E_{0m}) \right\rangle . \quad (1)$$

Here M_{ba} is the one-electron interband dipole matrix element,

$$M_{ba} = \int \phi_b^*(z) z \phi_a(z)\, dz , \quad (2)$$

where ϕ_a and ϕ_b are the one-electron ground state and first excited states for perpendicular motion, while X_{0m} and X_{1n} are the eigenfunctions of the many-body Hamiltonians H_0 and H_1 for the in-plane motion, with zero or one excited electron, respectively, and the remaining electrons in the ground state for perpendicular motion.[8] The summation in (1) is over all the possible final states X_{1n}, while the angular brackets denote a thermal average over the initial states X_{0m}. We use the vector $\vec{x}_i$ to denote the coordinates (x_i, y_i) of the ith electron within the plane, and the integration in (1) is over the set $\{\vec{x}_i\}$ giving the positions of the N electrons. The quantities E_{1n} and E_{0m} are the energies of the wave functions X_{1n} and X_{0m}. We have set $\hbar = 1$. In deriving (1) we have assumed that the perpendicular potential is sufficiently strong so that it is valid to ignore the dependence of the matrix elements M_{ba} on the coordinates $\vec{x}_i$ (so-called Condon approximation[7]).

Equation (1) may be written

$$I_{ba}(\omega) = |M_{ba}|^2 G(\omega) , \quad (3)$$

where

$$G(\omega) = \int e^{i\omega t} G(t)\, dt \quad (4)$$

and

$$G(t) = \langle e^{iH_1 t} e^{-iH_0 t} \rangle . \quad (5)$$

The Hamiltonians H_1 and H_0 (in the absence of coupling to external scatterers) are given by

$$H_0 = \sum_i \frac{p_i^2}{2m} + \frac{1}{2} \sum_{i \neq j} \frac{e^2}{|\vec{x}_i - \vec{x}_j|} + \omega_c \langle z \rangle_a \sum_i p_{ix} + V(\{\vec{x}_i\}) + E_{0a} , \quad (6)$$

and

$$H_1 = H_0 + \omega_c(\langle z \rangle_b - \langle z \rangle_a) p_x + \omega_{ba} , \quad (7)$$

where $\omega_c \equiv eB/mc$ is the cyclotron frequency and $\langle z \rangle_b$ and $\langle z \rangle_a$ are the expectation values of z in the states ϕ_b and ϕ_a, respectively. The energy $\omega_{ba} \equiv E_{0b} - E_{0a}$ is the one-electron level splitting in the absence of the magnetic field, p_{ix} is the x component of the two-dimensional momentum $\vec{p}_i$ for the ith electron, while p_x denotes the x component of momentum for the single electron which has been promoted to the excited state.[8] We have assumed that the magnetic field $\vec{B}$ lies in the y direction, and we have chosen a gauge with vector potential $\vec{A}$ parallel to x and proportional to z. The term $V(\{\vec{x}_i\})$ is the potential of the uniform positive background from the charges on the positive plate necessary to obtain a uniform electron density. In writing down (6) and (7) we have again assumed that the localization in the z direction is small compared to the interparticle spacing, so that we may neglect the dependence of the electron-electron interaction on the z coordinates of the electrons, and we have neglected energy shifts of order ω_c^2/ω_{ba}.

We shall choose the origin of z such at $\langle z \rangle_a = 0$; thus, H_0 is identical to the electronic Hamiltonian in the absence of the magnetic field. Note however that the second term in (7), which changes suddenly when the tagged electron is promoted, couples the z transition to the in plane motion.

Using standard techniques,[9] and taking the center of the line as the origin in frequency space, the function $G(t)$ may be written in the form

$$G(t) = \left\langle T \exp\left(i\lambda \int_0^t p_x(s)\, ds \right) \right\rangle , \quad (8)$$

where

$$\lambda = \omega_c(\langle z \rangle_b - \langle z \rangle_a) , \quad (9)$$

$$p_x(s) = e^{iH_0 s} p_x e^{-iH_0 s} , \quad (10)$$

and T denotes the appropriate time-ordering operation.

Equation (8) may be compared with the form factor for *incoherent* neutron scattering at momentum and energy transfer $(\vec{k}, \omega)$, which is given by[5]

$$S_{\text{inc}}(\vec{k}, \omega) = \int e^{i\omega t} \langle e^{it\vec{k}\cdot\vec{r}(t)} e^{-it\vec{k}\cdot\vec{r}(0)} \rangle\, dt , \quad (11)$$

$$\langle e^{it\vec{k}\cdot\vec{r}(t)} e^{-it\vec{k}\cdot\vec{r}(0)} \rangle = \left\langle T \exp\left(i\vec{k}/m \cdot \int_0^t \vec{p}(s)\, ds \right) \right\rangle . \quad (12)$$

The equivalence between (8) and (12) is evident if we set $km^{-1} = \hat{u}_x \omega_c(\langle z \rangle_b - \langle z \rangle_a)$.

III. APPROXIMATE FORMULAS AND LIMITING CASES

In general we cannot evaluate Eq. (8) exactly. However, it is possible to look in more detail at the behavior of $G(t)$ in a number of interesting physical cases. Below we shall write down some approximate expressions which are probably quite accurate for a wide range of systems, and we shall then use these expressions to examine line shapes in a variety of cases. We shall limit our-

selves primarily to the classical case, where $k_B T/\hbar$ is large compared to the characteristic frequencies of motion in the plane, since this condition is applicable to current experiments on electrons on helium. The quantum case will be mentioned where the results are applicable.

In the classical case, we may neglect the time ordering operation in (8); i.e.,

$$G(t) = G(-t) = \langle e^{i\lambda m x(t)} \rangle . \tag{13}$$

Here

$$x(t) = \int_0^1 v(s)\, ds \tag{14}$$

is the displacement of the excited electron relative to its position at $t=0$, and $v(s)$ is the x component of the velocity at time s.

A. Short-time behavior of $G(t)$

Suppose that $\bar{\omega}$ is some characteristic frequency for collisions or oscillations in the plane. Then for $\bar{\omega}t \ll 1$, we have $v(t) \simeq v(0)$, and hence $x(t) = tv(0)$. Since $v(0)$ has a Gaussian distribution with

$$\langle v^2(0) \rangle = k_B T/m \equiv v_{\rm th}^2 , \tag{15}$$

$$\langle e^{i\lambda m x(t)} \rangle = e^{-(m^2\lambda^2/2)\langle x^2 \rangle} = e^{-\Omega^2 t^2/2} . \tag{16}$$

The frequency

$$\Omega \equiv \lambda (k_B T m)^{1/2} . \tag{17}$$

In this short-time limit the line-shape function $G(t)$ is independent of the interactions among the particles, i.e., we can think of the system as a set of free particles.

B. Long-time behavior of $G(t)$

For sufficiently long times, we again expect $x(t)$ to have a Gaussian distribution. This time, however,

$$\langle x^2 \rangle = D_s t , \tag{18}$$

where D_s, is the self-diffusion coefficient for the tagged electron. If we define the velocity autocorrelation function

$$Z(t) = \langle v(t)v(0) \rangle / v_{\rm th}^2 , \tag{19}$$

then

$$D_s = 2v_{\rm th}^2 \tau , \tag{20}$$

where τ is the "velocity autocorrelation time"[10]

$$\tau = \int_0^\infty Z(t)\, dt . \tag{21}$$

We note that if $\tilde{Z}(\omega)$ is the Fourier transform of $Z(t)$, then

$$\tau = \tfrac{1}{2}\tilde{Z}(0) . \tag{22}$$

The time τ is not simply related to any of the short-time properties of the system. In fact, as we shall see, τ vanishes for a perfect two-dimensional solid, while the short-time behavior is characterized by the zone-boundary longitudinal plasmon frequency. At large times, when (18) applies, we have

$$G(t) = e^{-\Gamma t} , \tag{23a}$$

where

$$\Gamma = \Omega^2 \tau = \tfrac{1}{2}\lambda^2 m^2 D_s . \tag{23b}$$

C. General-time Gaussian approximation

Since $x(t)$ has a Gaussian distribution for small and large time, it is natural to make a Gaussian approximation for all times. In that case for all systems both quantum mechanical and classical we have

$$G(t) = e^{-\Omega^2 A(t)} , \tag{24a}$$

$$A(t) = \frac{1}{2}\int_0^t ds \int_0^t ds'\, Z(s-s') \tag{24b}$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\tilde{Z}(\omega)}{\omega^2} (1 - \cos\omega t) . \tag{24c}$$

Thus, in the Gaussian approximation, the line shape $G(\omega)$ is completely determined, for all values of the field strength B, by the velocity autocorrelation function $Z(t)$ or its Fourier transform $\tilde{Z}(\omega)$. The limiting forms (16) and (23) are special cases of (24).

D. Strong magnetic fields

When Ω is much greater than $\bar{\omega}$, we expect that $G(t)$ will be negligibly small except for $t \ll \bar{\omega}^{-1}$. Thus, for magnetic fields large enough so that $\Omega/\bar{\omega} \gg 1$, we can use Eq. (16) in (4) to obtain a Gaussian line shape:

$$G(\omega) = (2\pi)^{1/2}\Omega^{-1} \exp(-\omega^2/2\Omega^2) . \tag{25}$$

Equation (25) is one of the limiting cases described by Zipfel *et al.* and is also the free-electron result.

E. Weak magnetic fields

When the magnetic field is sufficiently weak, the function $G(t)$ does not deviate greatly from unity until very long times, at which point $G(t)$ decreases exponentially according to Eq. (23), provided that D_s is not equal to zero for the system of interest. Inserting (23) into (4), we find a "motionally narrowed" Lorentzian line shape,

$$G(\omega) = 2\Gamma/(\omega^2 + \Gamma^2) , \tag{26}$$

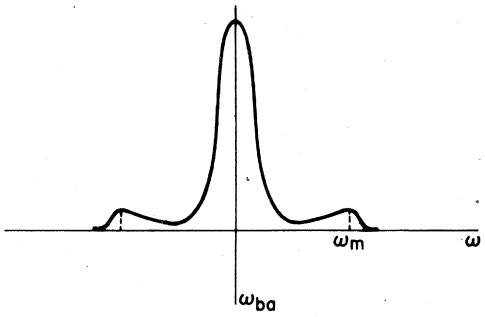

FIG. 2. Schematic of the interband absorption line for a strongly correlated liquid when $\Omega^2/\omega_m^2 \cong 1$. Here ω_m is approximately given by the zone-boundary plasmon in the solid and $\Omega = (k_B T/m)^{1/2}\omega_c\,(\langle z\rangle_b - \langle z\rangle_a)$.

where Γ, given by (23b), is proportional to the square of the magnetic field B. This form, which was also given by Zipfel *et al.*[3] is valid in the limit $B \to 0$, for frequencies ω of the order of Γ, or for any other sufficiently low frequency provided that the self-diffusion constant D_s does not vanish for the system of interest. As we shall see below, however, if one looks at a fixed frequency ω in the tail of the line, one finds

$$G(\omega) = (\Omega^2/2\omega^2)\tilde{Z}(\omega)\,, \tag{27}$$

which will reflect structure present in $\tilde{Z}(\omega)$ at the given frequency. In particular, if one has sufficient experimental sensitivity to investigate $G(\omega)$ in the appropriate region of ω and B, one can study the vibrational density of states of the system (see Fig. 2).

It should also be remarked that for a solid or very viscous liquid, where the velocity autocorrelation time τ may be very small compared to the characteristic vibrational frequency ω_m^{-1}, the conditions for a Lorentzian line shape will be very restricted. The behavior of $G(\omega)$ in the relatively complicated intermediate regime will be discussed below, with the aid of the general-time Gaussian approximation given by Eq. (24).

There has been some numerical and theoretical work on the behavior of $Z(t)$ for three dimensional systems[11] but no published work on two-dimensional plasmas.[12] Because of this we will apply some of the general considerations above to a qualitative discussion of several specific cases.

IV. APPLICATIONS

A. Dilute electron gas

Let us first consider a dilute two-dimensional electron gas, for which the thermal kinetic energy is large compared to the Coulomb energy of the electrons ($k_B T > e^2 n^{1/2}\pi^{1/2}$). In this case we may approximate $Z(t)$ by

$$Z(t) \approx e^{-\gamma t}\,, \tag{28}$$

where γ is the collision frequency for loss of momentum by the tagged electron, due to collisions with other electrons. In this case, $Z(\omega)$ is a Lorentzian with width γ, and the autocorrelation time τ is given by

$$\tau = \gamma^{-1}\,. \tag{29}$$

It follows from (24) that the line shape $G(\omega)$ is a Gaussian with width Ω, for fields such that $\Omega\tau \gg 1$, that $G(\omega)$ is a Lorentzian of width $\Omega^2\tau$, when $\Omega\tau \ll 1$, and that $G(\omega)$ has a relatively featureless intermediate form when $\Omega\tau$ is of the order of 1.

B. Two-dimensional harmonic solid

When the electron density is sufficiently large ($\Gamma_0 \equiv e^2 n^{1/2}\pi^{1/2}/k_B T \geq 100$) we expect the electrons to form a two-dimensional Wigner solid, with a triangular lattice.[5] At sufficiently high densities, the crystal is far from the melting point, and we may neglect the possibility of interchange of electrons between different lattice sites or of migration through vacancies or dislocations.[13] In this case, no diffusion is possible and $D_s = 0$. The correlation function $Z(t)$ becomes negative before approaching zero at long times, and the velocity autocorrelation time τ, defined by the integral in Eq. (21), is zero. The velocity autocorrelation function in this case may be approximated by that of an ideal two-dimensional harmonic solid.

It is straightforward to demonstrate that for a d-dimensional classical harmonic solid ($k_B T > \hbar\omega_m$) with one type of atom

$$\tilde{Z}(\omega) = (1/2d)\rho(|\omega|)\,, \tag{30}$$

where ρ is the phonon density of states, normalized so that

$$\frac{1}{2\pi}\int_0^\infty \rho(\omega)\,d\omega = d\,. \tag{31}$$

Some general characteristics of $\rho(\omega)$ are well known. The phonon density ρ will always vanish for ω larger than some maximum frequency ω_m, and $\rho(\omega)$ will generally have one or more maxima at frequencies somewhat lower than ω_m. In the limit $\omega \ll \omega_m$ contributions to ρ come from long-wavelength acoustical phonons, whose linear spectrum gives, for the d-dimensional crystal,

$$\rho(\omega) \propto \omega^{d-1}/\omega_m^d\,. \tag{32}$$

In the special case of the two-dimensional Coulomb crystal, the longitudinal phonon velocity diverges[6] at long wavelengths ($\omega \sim k^{1/2}$) and the corresponding density of states vanishes as ω^3, for small ω. Nevertheless, the transverse phonon velocity is

finite, so that Eq. (32) applies.

The behavior of $G(\omega)$ for $\omega \to 0$ is determined by the long-time behavior of the function $A(t)$, defined in Eq. (24b). For a three-dimensional solid, where $\tilde{Z} \propto \omega^2$, as $\omega \to 0$, the integral in Eq. (24b) is finite for $t \to \infty$, i.e.,

$$A(\infty) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{Z(\omega)}{\omega^2} . \tag{33}$$

Thus, the line-shape function $G(\omega)$ contains a δ function at $\omega = 0$,

$$G(\omega) = 2\pi e^{-2W} \delta(\omega) + \text{continuum}. \tag{34}$$

Here e^{-2W} is the Debye-Waller factor with

$$W = \Omega^2 A(\infty). \tag{35}$$

We may obtain the continuous part of the spectrum by expanding $G(t) - G(\infty)$ in powers of Ω^2, using (24) and substituting in (4). The one-phonon contribution to $G(\omega)$, of order Ω^2, may be written

$$G_{1p}(\omega) = e^{-2W} (\Omega^2/\omega^2) \tilde{Z}(\omega). \tag{36}$$

In the limit $\Omega^2 \to 0$, where $e^{-2W} = 1$, this yields the result given in Eq. (27).

In two dimensions, the integral (24b) diverges as $t \to \infty$, and there is no δ-function contribution to $G(\omega)$. Let us write Eq. (32) in the form

$$\tilde{Z}(\omega) = \alpha\omega/\omega_m^2, \tag{37}$$

for $\omega \ll \omega_m$, where α is a constant. For a two-dimensional Coulomb solid crystallized as a triangular lattice with lattice constant b and density $n \equiv (\frac{1}{2}\sqrt{3}b^2)^{-1}$, we have $\alpha = 32\sqrt{3}$ and,

$$\omega_m^2 = 8e^2/mb^3. \tag{38}$$

Then, for large times t, we have, from (24c),

$$A(t) = c_1 + (\alpha/\pi\omega_m^2) \ln(\omega_m t). \tag{39}$$

Using (4) and (24a), we find that for $\omega \ll \omega_m$, and Ω not too large,[13]

$$G(\omega) \sim C\omega_m^{-x}/\omega^{1-x}, \tag{40}$$

where C is a constant and

$$x = \Omega^2\alpha/\pi\omega_m^2. \tag{41}$$

For sufficiently small values of Ω, we have

$$C \sim \pi x, \tag{42}$$

which insures that $G(\omega)$ is equal to $2\pi\delta(\omega)$, in the limit $\Omega \to 0$.

Thus, for a two-dimensional solid far from its melting point, where self-diffusion can be neglected, and for weak magnetic fields, we expect a line shape $G(\omega)$ that has the form (40), for $\omega \ll \omega_m$, and the form given in (27) for ω of the order ω_m. Note that (40) joins smoothly onto the result obtained by use of (37) in (27), for ω/ω_m small but nonvanishing, when Ω/ω_m is sufficiently small.

C. Two-dimensional solid near the melting point

For the electron solid near the melting density, we may assume that there is a small but finite value of D_s, so that the velocity autocorrelation time τ defined by (21) is small compared to the value of ω_m^{-1}. For a three-dimensional solid, the effect of finite D_s on the line shape $G(\omega)$ is simple. The δ function appearing in (34) is broadened into a Lorentzian of width Γ, given by (23b).[14] For the two-dimensional case the situation is somewhat more complicated. At large times we now have

$$\frac{\langle x^2 \rangle}{2v_{\text{th}}^2} = A(t) \sim t\tau + \frac{\alpha}{\pi\omega_m^2} \ln(\omega_m t) + \text{const}, \tag{43}$$

where the first term arises from electronic interchanges or other processes that lead to self-diffusion, and the second term arises from thermal fluctuations of the long-wavelength transverse acoustical phonons. Note that these stochastic processes are independent of each other, and are both Gaussian at long times, so that Eq. (24a) is exact at large t. For the function $Z(\omega)$, we find, using (22), (37), (43), and (24b) that

$$\tilde{Z}(\omega) = 2\tau, \quad \text{for } \omega < 2\alpha^{-1}\omega_m^2\tau, \tag{44}$$

$$\tilde{Z}(\omega) = \alpha\omega/\omega_m^2, \quad \text{for } 2\alpha^{-1}\omega_m^2\tau < \omega \ll \omega_m.$$

For the line-shape function $G(\omega)$, there are several regimes. When

$$\Omega < \omega_m [\ln(\omega_m\tau)]^{-1/2}, \tag{45}$$

we find

$$G(\omega) = 2\Gamma/(\omega^2 + \Gamma^2), \quad \text{for } \omega < 2\alpha^{-1}\omega_m^2\tau, \tag{46a}$$

$$G(\omega) = (\Omega^2/\omega^2)\tilde{Z}(\omega), \quad \text{for } \omega > \Gamma, \tag{46b}$$

where $\Gamma = \Omega^2\tau$, as in (23b). Note that Γ is less than $\omega_m^2\tau$, in this regime, so that the regions of validity of (46a) and (46b) can overlap. For field strength such that

$$\omega_m [\ln(\omega_m\tau)]^{-1/2} < \Omega \ll \omega_m, \tag{47}$$

we find

$$G(\omega) = (\omega_m\tau) \frac{2\Gamma}{\omega^2 + \Gamma^2} \quad \text{for } \omega < 2\alpha^{-1}\omega_m^2\tau, \tag{48}$$

$$\approx \frac{\Omega^2\tilde{Z}(\omega)}{\omega^2} \left(\frac{\omega}{\omega_m}\right)^x, \quad \text{for } \omega > 2\alpha^{-1}\omega_m^2\tau, \tag{49}$$

where the exponent x, given by (41), is small compared to unity. Note that the line shape in both (46) and (48) may be described as a Lorentzian central peak with non-Lorentzian wings. In (46), most of the weight is in the central peak, whereas in (47) there may be substantial weight in the tails.

Note that (46) agrees with (40), for $2\alpha^{-1}\omega_m^2\tau < \omega \ll \omega_m$. When the magnetic field is sufficiently strong so that $\Omega \gg \omega_m$, then the line shape has a Gaussian form, with width Ω, as given in (25).

Many features of the line shape for the anharmonic solid may persist in the strongly correlated liquid, close to the melting point. In particular, for weak magnetic fields, we expect a Lorentzian central peak, whose width Γ may be small compared to $\Omega^2\omega_m^{-1}$, if self-diffusion is strongly hindered by correlations effects. On the other hand, coupling to long-wavelength transverse-momentum modes will tend to increase Γ in the strongly correlated electron liquid, if momentum transfer to the helium is sufficiently slow.[10] It should be emphasized once again that the velocity autocorrelation time τ is not simply related to any positive moment of the frequency distribution, in these circumstances.

It is expected that $\tilde{Z}(\omega)$ for the dense liquid will have a vibrational structure, for ω near ω_m, that will be similar to that found in the solid. This structure should be reflected in $G(\omega)$, for magnetic fields such that $\Omega \leq \omega_m$.

V. CONCLUSION AND EXPERIMENTAL OUTLOOK

From the analysis presented it is clear that the detailed shape of the interband absorption line has a great deal of important information in it concerning the strongly correlated motion (self-diffusion) of the particle which has been promoted. This motion in turn mirrors many of the dynamical properties of the electron fluid or solid. Questions concerning the existence of long-wavelength transverse modes, defect motion, and zone-boundary phonons in the solid, and the effect of melting on self-diffusion, all directly influence the line shape.

The experimental situation is in its earliest stages and much work at low temperatures and high magnetic fields remains to be done. The one experiment[3] which has been done was carried out at 1.2°K. Parallel fields from 200 G to 3 kG were applied to the electrons with carrier concentrations ranging from $1.5\times10^7 < n < 2.2\times10^8$. For this range of densities and temperatures the parameter Γ_0 (Sec. IV B), which measures the strength of the Coulomb interactions, runs from a low of 9 to a high of 36 so that the electron fluid is strongly interacting though not in the solid regime $\Gamma_0 > 95$.[6] In addition, the vapor density of helium atoms above the liquid at 1.2°K is sufficiently large that interaction of electrons with such centers is nonnegligible.[15] The absorption line in zero magnetic field is a Lorentzian whose width varies linearly with the number of helium atoms in the vapor. The linewidth has been attributed to two mechanisms, direct scattering of the electron from the helium atoms and an inhomogeneous broadening connected with the fluctuations in dielectric constant of the gas above the surface.[16] This scattering complicates an analysis of line shapes in terms of the self diffusion constant as discussed here.

Despite such difficulties the experimental results in Ref. 3 showed that the excess linewidth, in the presence of a field depended quadratically on the field. From these rough measurements of linewidth the authors extracted a velocity autocorrelation time τ which, over their limited range of density, seemed to crudely correlate with ω_m^{-1} the inverse of the zone-boundary plasmon frequency for the solid.

While we do not know precisely the details of the velocity autocorrelation function for this liquid we point out again that it is incorrect to compare τ with a zone-boundary short-time property of the correlation function. Of course in this classical problem the zone-boundary frequency

$$\omega_m = 2.5(e^2/m)^{1/2}n^{3/4},$$

in some sense sets the time scale. For example, if we assume weak scattering of electrons from one another it is a simple matter to show, taking the distance of closest approach $r_c = e^2/k_BT$ as an effective cross section (in two dimensions) for 90° scattering, that

$$\tau_\omega \cong (1/\omega_m)(3/\Gamma_0^{1/2}). \tag{51}$$

Numerically, τ_ω is not too different from ω_m^{-1} for $9 < \Gamma_0 < 36$.

In order to eliminate complications from collisions with the helium vapor and in order to reach larger values of the parameter Γ_0, it is desirable to perform the experiments at lower temperatures ($T \sim 0.3$ K). In addition, fields such that $0.1 < \Omega^2/\omega_m^2 < 1$ are desirable to order to see the one-phonon sidebands and to measure directly the function $\tilde{Z}(\omega)$ [see Eq. (27)]. For density $n \sim 2.5\times10^8$, we have $\omega_m \sim 1\times10^{11}$/sec, and $\Omega = 10^{10}$ for $B = 1$ kG at $T = 1$°K. Thus, fields of 3–10 kG are of the correct size, to look for this effect. In order to study the narrow central peak and to measure D_s in the solid phase, smaller fields should be used.[16]

ACKNOWLEDGMENTS

We would like to thank C. C. Grimes, T. R. Brown, and C. L. Zipfel for many informative discussions. We are grateful also to L. Gunther and M. E. Fisher for helpful comments.

[1]V. B. Shikin and Yu P. Monarkha, Sov. J. Low Temp. Phys. 1, 459 (1975); R. S. Crandall, Surf. Sci. 58, 266 (1976); C. C. Grimes, Surf. Sci. (to be published).
[2]C. C. Grimes and T. R. Brown, Phys. Rev. Lett. 23, 1238 (1969).
[3]C. L. Zipfel, T. R. Brown, and C. C. Grimes, Phys. Rev. Lett. 37, 1760 (1976).
[4]See, for example, C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1963), Chap. 20.
[5]W. Marshall and S. Lovesy, *Theory of Thermal Neutron Scattering* (Clarendon, Oxford, 1971).
[6]P. M. Platzman and H. Fukuyama, Phys. Rev. B 10, 3150 (1974); R. W. Hockney and T. R. Brown, J. Phys. C 8, 1818 (1975).
[7]M. Lax, J. Chem. Phys. 20, 1752 (1952).
[8]In Eq. (1) we have neglected the process where excitation energy may be transferred from one electron to another by mean of the dipole-dipole coupling. This approximation is correct in the Born-Oppenheimer limit, where the dipole moment M_{ba} is small compared to the interelectron spacing.
[9]In the book, *Polarons and Excitons*, edited by C. G. Kuper and G. D. Whitfield (Plenum, New York, 1973), see the articles by T. D. Schultz and P. M. Platzman.
[10]For a two-dimensional fluid on an ideal translationally invariant substrate, where the momentum of the fluid is conserved, it is known that $Z(t)$ decays as $t^{-1}(\ln t)^{-1/2}$ for long times. See, for example, Y. Pomeau and P. Résibois, Phys. Rep. C 19, 64 (1975); D. Forster, D. R.Nelson, and M. Stephen, Phys. Rev. A 16, 732 (1977). In this ideal situation, the self-diffusion constant D_s diverges as $(\ln t_m)^{1/2}$, where the cutoff time t_m is determined by the inverse frequency resolution of the experiments. In the present case, the maximum time t_m is also limited by the lifetime for momentum transfer from the electrons to excitations of the helium and to residual atoms in the vapor. In practice, the long-time tail in $Z(t)$ may enhance D_s and may affect the line shape $G(\omega)$ for the dense electron liquid, but should have little observable effect in the dilute regime.
[11]J. P. Hansen, I. R. McDonald, and E. L. Pollock, Phys. Rev. A 11, 1025 (1975); H. Gould and G. Mazenko, Phys. Rev. Lett. 21, 1455 (1975).
[12]H. Gould (private communication) has begun work using the formalism of Mazenko and Gould (Ref. 11) in two-dimensional systems. H. Ichimaru and N. Itoh (private communication) have analyzed the static structure factor and short-time behavior of the autocorrelation function using a Wigner disk model of the two-dimensional liquid.
[13]A result equivalent to our Eq. (40) has been obtained for the Mössbauer line shape in an ideal two-dimensional solid by Y. Imry and L. Gunther, Phys. Rev. B 3, 3939 (1971).
[14]The width of the central peak, as measured in Mössbauer experiments, has been used to study diffusion in three dimensional solids near the melting point. For recent discussions, see L. Gunther, J. Phys. (Paris) Suppl. 37, 12 C6-15 (1977), and references therein.
[15]T. R. Brown and C. C. Grimes, Phys. Rev. Lett. 29, 1233 (1972); C. C. Grimes, T. R. Brown, M. L. Burns, and C. L. Zipfel, Phys. Rev. B 13, 140 (1976).
[16]The dipole coupling mentioned in Ref. 8 may affect high-resolution measurements of the central-peak width in the solid phase, if the peak width Γ and the electron rate D_s/b^2 become smaller than the excitation-transfer rate caused by the dipolar coupling. To estimate this rate, we note that for electrons on a perfect triangular lattice, the dipolar coupling leads to a shift in the excitation frequency and an exciton bandwidth approximately given by $\Delta \approx 6M_{ba}^2/\hbar b^3 = 2\times 10^8 n^{3/2}\ \mathrm{sec}^{-1}$, where n is measured in units of $10^8\ \mathrm{cm}^{-2}$. We may note that the ratio Δ/ω_m is $\approx 10^{-2}$, for $n = 2.5\times 10^8$, and decreases proportional to $n^{3/4}$ for decreasing n.