Strain dependence of the Fermi surface in cadmium and rhenium from ultrasonic velocity oscillations*†

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(Received 31 May 1977)

The magnitudes of the derivatives with respect to strain ϵ of extremal cross-sectional areas A_F of the Fermi surface, $|\partial \ln A_F/\partial\epsilon|$, were determined in Cd and Re by measurement of the amplitudes of quantum oscillations in both the velocity of longitudinal sound waves and the magnetic susceptibility. For $\vec{H} \| [0001]$ in Cd, our results for the strain dependence of the α and β orbits agree with data obtained applying uniaxial stress or hydrostatic pressure. These results are also in good agreement with a rigid-parabolic-band model. For the C and D orbits, as well as for the α and β orbits, $|\partial \ln A_F/\partial\epsilon|$ scales roughly as $1/A_F$. For $\vec{H} \| [10\bar{1}0]$ in Re, our measurements of the strain dependence associated with the ellipsoids seem to be consistent with hydrostatic pressure results and in order-of-magnitude agreement with a rigid parabolic band estimate. The strain dependence of an orbit associated with 8th- or 9th-zone electrons does not scale as $1/A_F$, but seems to have an anomalously large strain dependence.

I. INTRODUCTION

The study of quantum oscillations in the magnetic susceptibility and in other physical quantities has yielded extensive information about the Fermi surfaces of metals.[1] The oscillation frequencies are related to the extremal cross-sectional areas A_F of the Fermi surface and cyclotron effective masses may be determined from the temperature dependence of the oscillation amplitudes. The magnetic field dependence of these amplitudes has also yielded information related to the electronic relaxation times. In recent years, the magnitudes of the oscillation amplitudes have been used to obtain the dependence of the Fermi surface on strain ϵ or stress σ.

The direct observation of quantum oscillations in a crystal under hydrostatic pressure or uniaxial stress requires some caution to avoid damaging the crystal. On the other hand, these dangers may be avoided and the stress dependence of the Fermi surface may be determined from the oscillatory magnetostriction alone as was done by Aron and Chandrasekhar for Bi.[2] A different approach to the problem was taken by Testardi and Condon.[3,4] These authors derived expressions relating de Haas–van Alphen (dHvA) oscillations in the ultrasonic velocity to oscillations in the magnetic susceptibility. For the case of longitudinal waves propagating along the magnetic field direction or shear waves perpendicular to the field direction, the equation relating the elastic constants C_{ij} to the differential magnetic susceptibility $\chi = \partial M/\partial H$ takes a simple form[3]

$$C_{ij}(H) = C_{ij}(H=0) - H^2 D_i D_j \chi , \tag{1}$$

where the deformation parameter $D_i \equiv \partial \ln A_F/\partial \epsilon_i$ measures the dependence of the extremal cross-sectional area A_F on the strain component ϵ_i. This expression is valid where the phase of the oscillatory free energy is rapidly varying and where $ql \ll 1$, q being the ultrasonic wave vector and l the electronic mean free path. Thus a comparison of oscillation amplitudes in two physical quantities allows the direct determination of the strain dependence of the Fermi surface. Similar comparison methods, involving the relationship between dHvA amplitudes in two physical quantities, have been used by other authors.[5-7]

In this paper, we present experimental results for the deformation parameter D_i for certain orbits in Cd and Re, determined in a manner similar to that used by Testardi and Condon. A brief account of these results has appeared elsewhere.[8] For $\vec{H} \| [0001]$ in Cd, our results for the strain dependence of α and β orbits (notation of Tsui and Stark[9]) are in agreement with data obtained applying uniaxial stress[10] or hydrostatic pressure.[11-13] The strain dependence for these orbits is also in agreement with estimates made using a rigid parabolic band (RPB) model.[5] For the C and D orbits (notation of Fletcher *et al.*[14]), as well as for the α and β orbits, $|\partial \ln A_F/\partial\epsilon|$ scales roughly as $1/A_F$, as suggested in Ref. 6. Furthermore, our data are consistent with the assignment of frequency C to a magnetic breakdown orbit involving the first- and second-zone hole surfaces. For $\vec{H} \| [10\bar{1}0]$ in Re, our measurements of the strain dependence associated with the ellipsoids seem to be consistent with hydrostatic pressure results[15] when certain predictions of a simple RPB model are ignored. The strain dependence of an orbit (P_7) associated with eighth-[16] or ninth-[17] zone electrons does not scale as $1/A_F$, but seems to have an anomalously large strain dependence. This is consistent with a model by Chu *et al.*[18] for the anomalous pressure

dependence of the superconducting transition temperature in Re.

II. EXPERIMENT

The single-crystal Cd sample used in these experiments had the form of a right circular cylinder (length = 8.9 mm), with cylinder axis parallel to the [0001] crystallographic axis to within 2°. Previously reported observations of quantum oscillations in the sound velocity in[19] Cd were made on the sample used here. Through the generosity of Dr. L. R. Testardi, a single crystal of Re was obtained. The sample had been cut from material grown in an electron beam furnace and which possessed a resistance ratio $R(300\ K)/R(4.2\ K) \approx 20\,000$. The sample had the form of a right cylinder (length = 6.8 mm) with the cylinder axis parallel to the $[10\bar{1}0]$ crystallographic axis to within 2°.

Continuous and automatic measurements of quantum oscillations in the sound velocity were made using the McSkimin pulse superposition method,[20] with modifications similar to those of Refs. 3 and 21. That is, frequency modulation and feedback techniques were used to maintain the superposed echoes in resonance (superposed so that the amplitude is maximum) and to permit continuous and automatic recording of variations in the pulse repetition rate. The system described in Ref. 21, and much more extensively in Ref. 22, has been used to study quantum oscillations in the sound velocity at repetition rates up to ~220 kHz. Shock excitation of the transducer was used at high repetition rates, whereas, at low rates, the output of a rf pulsed oscillator was applied to the transducer. In Fig. 1, we show an experimental curve of the fractional change in the velocity of longitudinal sound waves $\Delta V/V \equiv [V(H) - V(0)]/V(0)$, obtained for Cd with $\vec{H} \| [0001] \| \vec{q}_L$. This curve was obtained in the $p = 2$ mode,[20] with a pulse repetition rate of ~71 kHz and using the output of a rf pulsed oscillator. In Fig. 2, we show an experimental curve of $\Delta V/V$ for longitudinal waves in Re with $\vec{H} \| [10\bar{1}0] \| \vec{q}_L$. This curve was obtained in the $p = 2$ mode, but using shock excitation of the transducer since the pulse repetition rate was ~208 kHz.

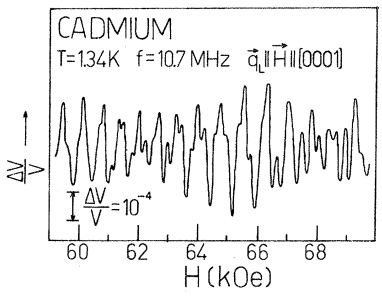

FIG. 1. Fractional change in the longitudinal sound velocity $\Delta V/V \equiv [V(H) - V(0)]/V(0)$ vs external magnetic field H for Cd with $\vec{H} \| [0001]$.

Measurements were usually made with ultrasonic frequencies ~10 MHz, although some measurements were made with frequencies near 30 MHz to verify the lack of frequency dependence for the oscillation amplitudes. We estimate that the control and recording system used to measure the velocity oscillations is accurate to ~2% for the large oscillations studied here. Effective masses and Dingle temperatures were determined from the dependence of the velocity oscillations on temperature[23] and magnetic field.[19] Temperatures between 1.3 and 4.2 K were determined by measuring the vapor pressure over a helium bath. Magnetic fields up to 70 kOe were provided by a superconducting solenoid with a field homogeneity of better than 0.01% over a sphere of 1-in. diameter.

The measure the magnetic susceptibility oscillations, the sample was placed in a pickup coil and the induced voltage V_i was measured as the magnetic field was linearly swept.[3,4] The magnetization M of the sample will be nearly uniform and may be represented by surface currents. These surface currents, in turn, may be simulated by a coil tightly wound around the sample surface and containing n turns per unit length. The current flowing in the sample coil is given by $i = J_s/n$, where J_s is a linear current density whose value is M. Then the induced voltage V_i is given by

$$V_i = L\,di/dt = (L/n)(dH/dt)(dM/dH),$$

where L is the mutual inductance between the sample and pickup coils. The mutual inductance L was measured using a Hartshorn bridge available in our laboratory. For Cd we obtained $L = 1.26 \pm 0.05$ mH and for Re, $L = 0.74 \pm 0.04$ mH. During the experiment, a buck-out coil is placed in the superconducting solenoid and is used to cancel the constant voltage induced in the pickup coil by the linear variation of the magnetic field.

It is worth mentioning that the pickup coils and

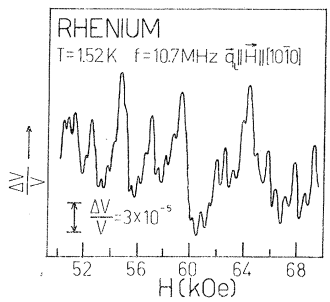

FIG. 2. $\Delta V/V$ vs H for longitudinal waves in Re with $\vec{H} \| [10\bar{1}0]$.

sample must be rigidly fixed in place to avoid noise due to vibration. The rate at which the magnetic field is swept may be increased to increase the signal-to-noise ratio. However, there are two factors which limit the rate at which the field may be swept. The response time of the amplifier (Keithley 149) and recorder (Hewlett-Packard 7004B) together is about 0.5 sec and fast field sweeps may reduce the induced voltage. Furthermore, eddy currents induced in the sample by sweeping the field rapidly may also diminish the oscillation amplitudes. The correct sweep rate was determined empirically by reducing the sweep rate until the ratio of the induced voltage to the field-sweep rate was a constnat. We thus arrived at field-sweep rates of 138 Oe/sec for Re and 223 Oe/sec for Cd. With these limitations, a noise level of about 10% was present in the induced voltage. In Fig. 3, we show an experimental curve of $\chi = \partial M/\partial H$ for Cd with $\vec{H}\|[0001]$ and, in Fig. 4, the same quantity for Re with $\vec{H}\|[10\bar{1}0]$.

Since as many as four Fermi surface orbits may be contributing simultaneously to the experimental curves, it is necessary to Fourier-analyze the data to obtain the amplitude associated with each dHvA frequency. Initially, the data were analyzed over a large field interval (~10 kOe) to determine the dHvA frequencies f_i. Then, with these frequencies, the function G was constructed, where

$$G = \sum A_i \cos(2\pi f_i/H + \phi_i) .$$

A least-sqaures fit of the function G was made to the experimental data in a smaller field interval (~4 kOe) to determine the amplitudes A_i and the phases ϕ_i. The resulting G was then subtracted from the experimental curve and this difference was again Fourier analyzed. In this way, frequencies could not be lost nor could spurious be injected. Using this procedure to analyze theoretically generated curves, it was possible to determine their oscillation amplitudes to a precision of better than 1%. For our experimental curves, with a finite noise level, this analysis contributed to the global error in a manner that depended upon the relative magnitude of the oscillation amplitude under consideration.

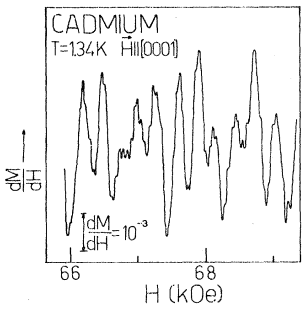

FIG. 3. Differential susceptibility $\chi = \partial M/\partial H$ vs external field H for Cd with $\vec{H}\|[0001]$.

III. RESULTS AND DISCUSSION

A. Cadmium

Experimental curves of $\Delta V/V$ and χ for Cd, such as those of Figs. 1 and 3, showed four dHvA frequencies. The data analysis procedure mentioned above allowed us to distinguish the low-frequency oscillations associated with the α and β orbits.[24] These orbit labels refer to the hole pocket at the first-zone corner H(α orbit) and the junctions at H of the diagonal arms of the second-zone hole surface (β orbit). Also present with large amplitudes were the frequencies C and D, which have been studied extensively in the ultrasonic-attenuation experiments of Fletcher *et al.*[14] Fletcher *et al.* have suggested two possible assignments for frequency C. One possibility is the third-zone electron surface referred to as the butterfly, which was predicted in nearly-free-electron (NFE) calculations.[25] Alternatively, they suggested that this frequency might be associated with a magnetic breakdown orbit involving the first-zone hole surface and the arms of the second-zone hole surface. The origin of frequency D is uncertain.

The dHvA frequencies f of Table I were obtained directly from the Fourier analysis and are in good agreement with values usually observed[14] for this orientation ($\vec{H}\|[0001]$). The sum of the frequencies corresponding to the orbits α and β is equal to the frequency C, offering the possibility that this might correspond to a magnetic breakdown orbit. The values of m^* for orbits C and D agree[24] with those of Ref. 19. The masses for orbits α and β have not been observed in cyclotron resonance experiments[26] and may be new. Our data give very different values of T_D for the various orbits and thus indicate large differences in the electron relaxation time over the Fermi sur-

TABLE I. Fermi-surface parameters for cadmium with $\vec{H}\|[0001]$.

Orbit	f (10^6 Oe)	m^*/m_0	T_D(K)	$\left\lvert \frac{\partial \ln A_F}{\partial \epsilon_3} \right\rvert$
α	6.0 ± 0.2	0.20 ± 0.01	1.5 ± 0.1	11 ±3
β	6.5 ± 0.2	0.33 ± 0.01	0.6 ± 0.1	9 ±3
C	12.6 ± 0.3	0.38 ± 0.05	0.7 ± 0.1	9 ±3
D	16.1 ± 0.3	0.40 ± 0.02	2.2 ± 0.1	5.5 ± 0.9

face. Large anisotropies in the electron relaxation time have been observed in the noble metals upon alloying at very low concentrations.[27,28] In Cu, for example, transition metal solutes have a far greater effect on the relaxation time associated with electrons in low-curvature belly orbits than on the relaxation time associated with electrons in neck orbits, and thus may produce large scattering anisotropies even at concentrations which change dHvA frequencies by less than 1%. On the other hand, anisotropies in electron relaxation time in pure Cu may be explained as a consequence of the greater sensitivity to phonon scattering for electrons on neck orbits.[29] We are unable to estimate the importance of such mechanisms in our case, but the examples indicate how large anisotropies in T_D might arise.

Before discussing our results for the strain dependence of the Fermi surface of Cd, we wish to consider the origin of frequency C. Gamble and Watts[10] assigned the corresponding frequency C of zinc to a magnetic breakdown orbit involving the first-zone hole surface and the arms of the second-zone hole surface. This assignment was made by comparing the sign and magnitude of the experimentally determined stress dependence of A_F, $\partial \ln A_F/\partial\sigma$, with theoretical values for the two possible orbits, calculated using the NFE approximation. The sign of $\partial \ln A_F/\partial\sigma$ was found to be the same as that calculated for the magnetic breakdown orbit and the magnitude was in rough agreement with the calculated value for this orbit. Furthermore, since the magnetic breakdown orbit involves, for most of its length, a path on the second-zone hole surface, it should have a sensitivity to strain similar to that of the β orbit which lies on the same surface. This is, in fact, the case; in zinc, $\partial \ln A_F/\partial\sigma$ for the orbits β and C differ by about 30%. Thus, based on these two arguments, Gamble and Watts assigned the frequency C to the magnetic breakdown orbit. Although we cannot determine the sign of the deformation parameter for C in our experiment, the additional information of Table I (m^* and T_D) make more convincing a comparison to the β orbit for the purpose of assigning the frequency C in Cd. In particular, if C is to be assigned to the magnetic breakdown orbit, involving, for the most part, the second-zone hole surface, then we would expect it to show a sensitivity to strain comparable to that of the β orbit, as well as similar values for m^* and T_D. From Table I, we see that this is the case: m^*, T_D, and $|\partial \ln A_F/\partial\epsilon_3|$ for frequency C have values close to those for the β orbit. Thus, an assignment of frequency C in Cd to the magnetic breakdown orbit seems to be consistent with our data. Considering the results of Gamble and Watts for the corresponding frequency C in zinc, this assignment becomes even more convincing.

Our results for the strain dependence of the Fermi surface of Cd can be compared directly with the stress measurements of Gamble and Watts.[10] These authors applied a compressional stress (σ_3) along the [0001] axis of a Cd single crystal and determined $\partial \ln A_F/\partial\sigma_3$ for the α and β orbits. Using the low-temperature elastic constants for Cd of Garland and Silverman,[30] we find that our experimental values of $|D_3|$ for the α and β orbits agree with values of $|D_3|$ calculated from $\partial \ln A_F/\partial\sigma_3$ of Ref. 10. However to facilitate comparison of our data with hydrostatic pressure results[11-13] as well as the uniaxial stress data,[10] we present the comparison in terms of the quantity[6,10] $\mathfrak{D} \equiv \partial \ln A_F/\partial(c/a)$, where $(c/a) = 1.886$ is the axial ratio.[1] In Table II, we present the various experimental results in terms of $\mathfrak{D}$ for the frequencies observed in our experiments.[31] For the α and β orbits, our results are in good agreement with both hydrostatic pressure and uniaxial stress results.

To obtain some feeling about our results for orbits C and D, we note that Griessen and Sor-

TABLE II. Comparison of experimental determinations of $\mathfrak{D} \equiv \partial \ln A_F/\partial(c/a)$ for Cd orbits with $\vec{H}$||[0001].

Orbit	This work[a]	Stress[b]	Pressure[c]	Pressure[d]	Pressure[e]
α	4.7 ± 1.3	4.1 ± 0.1	4.6 ± 0.5	4.5 ± 0.1	4.5 ± 0.1
β	3.8 ± 1.3	4.1 ± 0.2	4.2 ± 0.5	4.2 ± 0.2	4.2 ± 0.2
C	3.8 ± 1.3	...	...	...	...
D	2.3 ± 0.4	...	...	...	...

[a] Sign of $\mathfrak{D}$ for α and β orbits taken from Ref. 10 (See also Ref. 31). Sign for orbit C taken to agree with corresponding orbit C of zinc (Ref. 10). Sign for orbit D arbitrarily taken to be positive.

[b] Reference 10.

[c] Reference 11.

[d] Reference 12.

[e] Reference 13.

bello[6] and Griessen *et al.*[32] have demonstrated that in many cases one can expect the quantity $\partial \ln A_F / \partial P$ to scale like $1/A_F$. This result can be arrived at from a rigid-band model in which one assumes that A_F depends on the difference between the Fermi energy E_F and the energy of the band minimum.[6] Thus, we can calculate $\mathfrak{D}$ for the β orbit by scaling the value for the α orbit by the appropriate Fermi surface areas. Using, for example, $\mathfrak{D} = 4.6 \pm 0.5$ for the α orbit from the fourth column of Table II, we obtain $\mathfrak{D} = 4.2 \pm 0.5$ for the β orbit, in good agreement with the data of Table II. Calculating $\mathfrak{D}$ for the orbits C and D by scaling this same value for the α orbit, one obtains 2.2 ± 0.3 and 1.7 ± 0.2 respectively. Because of the rather large experimental errors associated with our data and those of Ref. 11, these values can be said to agree with our results of Table II. The importance point, however, is that the general trend is correct, indicating the usefulness of this type of scaling.

We have made an estimate of the deformation parameter D_3 for the α and β orbits using what might be called a RPB approximation.[5] Neglecting the dependence of the effective mass, band gaps, etc., on strain, we write $\partial \ln A_F / \partial \epsilon_3 = \partial \ln (E_F - E_H) / \partial \epsilon_3$, where E_H is the energy at the top of the valence band at the H point. Evaluating $\partial E_F / \partial \epsilon_3$ and $\partial E_H / \partial \epsilon_3$ as in Ref. 33, we obtain

$$D_3 = \frac{-2(1 + 2S_{13}/S_{33})E_F/3 + 2S_{13}E_H/S_{33}}{E_F - E_H} . \qquad (2)$$

Obtaining $E_F - E_H$ from Fig. 2 of Stark and Falicov[34] and using the bandgap of ~0.005 Ry between the bands associated with the α and β orbitals,[9] we obtain $D_3 = 8.4$ for the α orbit and $D_3 = 7.8$ for the β orbit. Keeping in mind the comments of Ref. 31 regarding sign conventions, we see that the signs of the D_3 from this simple calculation agree with the stress and pressure results. More surprisingly, however, is the fact that this simple model is in reasonable agreement with the magnitudes of the D_3. Of course, many approximations were made in obtaining Eq. (2) and the results obtained from Eq. (2) are very sensitive to the exact value $E_F - E_H$. Nevertheless, the good agreement obtained here seems to bear out the conclusion of Watts and Sundström[33] that rigid-band effects are most important in explaining uniaxial stress experiments.

B. Rhenium

A Fourier analysis of experimental curves of $\Delta V/V$ and χ (see Figs. 2 and 4) showed three dHvA frequencies, two of which are associated with the fifth-zone hole surfaces which resemble ellipsoids.[17,35,36] The third frequency which we observe in our data (64×10^5 Oe) was first observed by

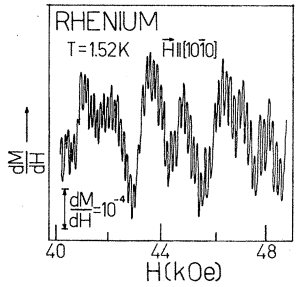

FIG. 4. χ vs H for Re with $\vec{H} \| [10\bar{1}0]$.

Thorsen *et al.*,[16] who tentatively assigned it to an orbit on the ninth-zone electron surface of Mattheis.[37] Testardi and Soden,[17] on the other hand, suggested an assignment of these oscillations to a small Γ-centered cavity in the eighth-zone electron surface on the basis of their geometric resonance data. The difficulties in making an assignment of this frequency are compounded by the extreme sensitivity of these regions of the Fermi surface to small changes in the Fermi energy.[37]

In Table III we also present values for the cyclotron effective mass m^*, the Dingle temperature T_D, and the magnitude of the deformation parameter $|D_1| = |\partial \ln A_F / \partial \epsilon_1|$. Our value of $m^*/m_0 = 0.12 \pm 0.01$ for the ellipsoids agrees with the value 0.12 estimated by Testardi and Soden from the dHvA data of Joseph and Thorsen.[35] Furthermore, our value of $m^*/m_0 = 0.17 \pm 0.01$ is roughly consistent with the value 0.19 estimated for ellipsoids tilted at 60° with respect to the field direction. In contrast to what we found for Cd, the values for T_D are equal for the several orbits under consideration, and, therefore, electron scattering is isotropic over the Fermi surface. The relatively large values of T_D are consistent with the high resistivity ratio of our crystal, since it is known[38] that the residual-resistance ratio is not necessarily a good measure of the scattering seen by an electron in a Landau level.

We now wish to consider the values for $|D_1|$ for the various orbits and compare them with our experimental results. Svechkarev and Pluzhnikov[15] measured the hydrostatic pressure dependence of

TABLE III. Fermi-surface parameters for rhenium with $\vec{H} \| [10\bar{1}0]$.

Orbit[a]	f (10^5 Oe)	m^*/m_0	T_D(K)	$\left\lvert \frac{\partial \ln A_F}{\partial \epsilon_1} \right\rvert$
P_1	7.6 ± 0.2	0.12 ± 0.01	0.8 ± 0.1	11 ± 3
P_1	13.4 ± 0.4	0.17 ± 0.01	0.8 ± 0.1	12 ± 3
P_7	64 ± 0.5	0.43 ± 0.03	0.8 ± 0.1	14 ± 3

[a] For orbit notation see Refs. 16 and 35.

A_F for the ellipsoids with major axes along the magnetic field. We immediately encounter a difficulty here, however, because we cannot calculate $\mathfrak{D}$ for the pressure results in Re as we did above for Cd. The reason for this is that, in Re, the linear compressibilities parallel and perpendicular to the c axis are equal to within experimental error[39] and, therefore, the application of hydrostatic pressure does not change the ratio c/a for Re.

To compare our results with the pressure experiments we consider the relation[3,4] $d\ln A_F = \vec{\vec{D}} \cdot \vec{\vec{S}} \cdot \vec{\sigma}$, where $\vec{\vec{D}}$ is the deformation tensor whose components were defined above, $\vec{\sigma}$ is the stress tensor, and $\vec{\vec{S}}$ is the 6×6 elastic compliance matrix for Re. For hydrostatic pressure $\sigma_1 = \sigma_2 = \sigma_3 = -P$ and $\sigma_4 = \sigma_5 = \sigma_6 = 0$. To compare our D_i with the hydrostatic pressure results, however, we use (see Ref. 31) the equation

$$\frac{\partial \ln A_F}{\partial P} = (D_1 + D_2)(S_{11} + S_{12} + S_{13}) + D_3(2S_{13} + S_{33}) . \quad (3)$$

The S_{ij} were calculated from the low-temperature elastic constants C_{ij} of Shepard and Smith,[40] as was the compressibility for converting $\partial \ln A_F/\partial \ln V$ of Svechkarev and Pluzhnikow to $\partial \ln A_F/\partial P = +2.5 \times 10^{-12}$ cm^2/dyn for use in Eq. (3). An estimate of the D_i using the RPB approach (see below) indicates that $D_3 > 0$ while $D_1 < 0$ for the ellipsoids. Taking $|D_1| = 11$ from Table III and assuming $D_1 = D_2$, we have $D_1 = D_2 = -11$. Then to obtain agreement with the work of Ref. 15 through Eq. (3), we require $D_3 = 224$. This rather large elastic anisotropy for the ellipsoids is surprising and we wish to consider the case where both D_1 and D_3 are greater than zero. In this case, with $D_1 = D_2 = 11$, we require $D_3 = 26$ to satisfy Eq. (3).

The D_i for the ellipsoids were calculated using the RPB approximation introduced above. D_3 was obtained from an expression similar to Eq. (2) with E_H replaced by E_L, the energy at the top of the valence band at the L point. The energy difference $E_F - E_L$ was taken from Fig. 3 of Ref. 37. Admittedly this value is subject to a rather large error, but this should not affect the sign of the D_i. We thus obtained $D_3 = 27$. In the same spirit we wrote $D_1 = \partial \ln A_F/\partial \epsilon_1 = (1/S_{11})\partial \ln[E_F - E_L]/\partial \sigma_1$. Calculating $\partial E_F/\partial \sigma_1$ and $\partial E_L/\partial \sigma_1$ as above (and in Ref. 33), we obtained $D_1 = -58$, where the large negative contribution from the term in E_L was partially cancelled by the term involving E_F. Thus, the RPB estimates do not show the large elastic anisotropy necessary for our data to agree with the pressure results of Ref. 15. In fact, these estimates indicate that $|D_1| > |D_3|$, a trend which would be consistent with scaling by $1/A_F$.[37] If $D_1 > 0$, contrary to the RPB predictions, then the large elastic anisotropy is reduced and the magnitude of D_3 predicted by the model is in better agreement with experiment. For these reasons, and because the model is obviously oversimplified, this possibility looks distinctly more attractive than accepting the RPB signs for the D_i. In any case, it does not seem surprising that the situation is complicated for this piece of the Fermi surface which involves such a tiny fraction of the carriers (0.0007 holes/atom). Clearly more work is needed, using a more sophisticated approach than the RPB model used here.

As for the deformation parameter $|D_1|$ for the frequency P_7, we note that it seems to be rather large. If we expect D_1 to scale as $1/A_F$,[6,32] then for P_7 with an A_F roughly one order of magnitude larger than for the ellipsoids, we expect D_1 to be roughly an order of magnitude smaller than for the ellipsoids. In Table III we see that this is not the case; $|D_1|$ is somewhat *larger* for P_7 than for the ellipsoids. Now it does not seem likely that the ellipsoids, whose Fermi surface volume is so small (0.0007 hole/atom), would have an abnormally *small* strain dependence. Furthermore, our $|D_1|$ for the ellipsoids seems to be consistent with the pressure results and, at least, in order of magnitude agreement with RPB estimates. We conclude, therefore, that $|D_1|$ for the frequency P_7 is anomalously large and that those parts of the Fermi surface associated with this orbit are unusually sensitive to strain.

This result touches on another problem. The superconducting transition temperature T_C in Re has been found to be very sensitive to inhomogeneous strain[11] and shows anomalous behavior under hydrostatic pressure.[18] To explain the anomalous behavior of T_C under pressure, Chu *et al.*[18] proposed a model involving an abrupt change in Fermi surface topology with pressure. These authors noted that certain flat regions of the Fermi surface in the eighth- and ninth-zones were very sensitive to small changes in E_F and suggested that necks might appear in the electron sheets in these zones.[37] If P_7 can be identified with electrons in the eighth- or ninth-zones, the large value of $|D_1|$ we have observed would be consistent with and tend to support the model of Chu *et al.*

ACKNOWLEDGMENTS

We wish to thank L. R. Testardi for providing the single crystal of Re and for many helpful discussions. We are grateful to H. Vargas for the Cd crystal. Discussions with N. F. Oliveira, Jr. were most helpful, especially with regard to the magnetic susceptibility measurements. P. Waki was of great assistance with the data analysis.

*Based on a thesis submitted by J. M. V. Martins in partial fulfillment of the requirements for the Sc.D. degree at Universidade de São Paulo.

†Supported by Fundação de Amparo à Pesquisa do Estado de São Paulo, Conselho Nacional de Desenvolvimento Tecnológico e Científico, and Financiadora de Estudos e Projetos.

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