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Bulk boundary conditions for injection and extraction in trap-free lifetime and relaxation semiconductors

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Small-carrier perturbations close to the unperturbed bulk are analyzed which represent boundary conditions for the perturbations produced by injection and extraction in thick enough semiconductor samples. On this basis concentration contours for the strong-perturbation range can be qualitatively drawn and even numerically evaluated. The results allow for a clear distinction to be made between lifetime and relaxation behavior. There is also an intermediary regime between these, and within it a new kind of steady state with oscillations in space is proved to be possible for appropriately chosen currents. Two new conditions have also been evidenced for the high current conduction in the asymptotic range: "equality recombination" in the deep-lifetime case, and "unperturbed conductivity" in the deep-relaxation case. Necessary conditions for obtaining an increase in resistance through injection have also been outlined.

I. INTRODUCTION

The paper deals with small perturbations of carrier concentrations produced by injection or extraction in trap-free semiconductors. Such behaviors are not restricted to small currents, but to those regions of the material close to the unperturbed bulk. The behaviors within these regions represent boundary conditions for the associated strong-perturbation regions.

The small-carrier-perturbation range is analyzed through the asymptotic-range solutions in a semi-infinite sample, solutions which can be obtained from linearized equations.

The asymptotic analysis allows one to obtain information concerning the injection-extraction behavior through analytic treatment, but without having recourse to approximations like neutrality, $NP \cong N_i^2$,[1] $NP = N_i^2$,[2] negligible diffusion,[1] or unity injection ratio.[3,4] It can quantitatively decide whether and when the above-mentioned approximations are valid for the low-perturbation range. Of course, the conclusions can be applied only to carrier perturbation in thick-enough samples, so that the innermost material remains practically unperturbed by the injection or extraction through the contact surface.

Our analytic treatment investigates all possible values of the ratio between dielectric relaxation time τ_D and diffusion-length lifetime τ_0, showing that not only lifetime and relaxation regimes are possible, but also an intermediary one; the τ_0/τ_0 limits of this intermediary regime have first emerged from the analysis Van Roosbroeck gave of the transient drift with recombination.[5] Within the intermediary regime, whose steady states we analyze for the first time, depending on current, either lifetime or relaxation behaviors are possible, or even a new kind of steady state with oscillations in space. Our treatment also covers the whole range of possible currents (from $-\infty$ to $+\infty$) and investigates new conditions of carrier perturbation which appear in the asymptotic range at high enough currents in extreme-lifetime and -relaxation cases.

The analysis of the asymptotic range of carrier perturbation in pronounced extrinsic materials shows that in this case recombination and space-charge effects act as independent processes which compete in controlling the penetration of carrier perturbations inside the materials.

One more advantage of our treatment is that it brings additional insight into the problem of resistance enhancement through minority-carrier injection; injections that result in aperiodic field contours enhance the resistance only if within their asymptotic range the field decreases towards its limit; quantitative conditions to obtain such asymptotic behaviors are deduced.

II. MODEL AND EQUATIONS

The model is that a semi-infinite trap-free semiconductor limited by an injection-extraction plane at the space origin and within which a steady-state conduction is established through injection or extraction of charge carriers. These result in a state quantitatively defined through departures of electron and hole concentrations from their unperturbed bulk values.

The current is positive when it enters the material through the injection-extraction surface. For a semi-infinite arbitrary (n- or p-type) semiconductor *the current is in the forward direction* when minority carriers enter (are injected), or majority carriers are taken out (are extracted), through the contact interface; *the current is in the re-*

verse direction in the opposite cases. We define these directions as they correspond to the characteristic behaviors of the material, but they are not necessarily connected to a p-n junction contact. Thus we may have very high currents in the reverse direction that are not necessarily very high reverse currents of a junction (which might be confusing). For instance, they can result from a majority-carrier injection. For an n-type material the currents in the forward direction are positive.

As usual,[1,3] a recombination term

$$R=(np-n_0p_0)/\tau_0(n_0+p_0) \tag{1}$$

is considered, in which n_0 and p_0 are the equilibrium concentrations and τ_0 is a diffusion-length lifetime.[6]

In the steady state the usual current, continuity, and Poisson equations, for the one-dimensional case, can be written

$$J_n=\nu_n\left(NE+\frac{dN}{dX}\right), \tag{2}$$

$$J_p=\nu_p\left(PE-\frac{dP}{dX}\right),\quad J=J_n+J_p\,, \tag{3}$$

$$\frac{dJ_p}{dX}=-\frac{dJ_n}{dX}=-\frac{\tau_D}{\tau_0}\,(NP-N_0P_0), \tag{4}$$

$$\frac{dE}{dX}=(P-P_0)-(N-N_0) \tag{5}$$

in a suitable normalization, in which N and P are carrier concentrations n,p normalized to the total concentration m_0:

$$m_0=n_0+p_0\quad(N_0+P_0=1); \tag{6}$$

ν_n and ν_p are electron and hole mobilities normalized to the average mobility μ_0:

$$\mu_0=\sigma_0/em_0=(n_0\mu_n+p_0\mu_p)/m_0\quad(\nu_nN_0+\nu_pP_0=1); \tag{7}$$

where X is the spatial coordinate, $x\in(0,\infty)$, normalized to the Debye length L_D:

$$L_D=(kT\epsilon/e^2m_0)^{1/2}, \tag{8}$$

E is the electric field normalized to kT/eL_D, J,J_n,J_p are current densities normalized to eL_Dm_0/τ_D, where τ_D is the dielectric-relaxation time defined through the expression

$$\tau_D=\epsilon/e\,\mu_0m_0=\epsilon/\sigma_0. \tag{9}$$

This relaxation time relates to the Debye length through the usual expression $L_D=(D_0\tau_D)^{1/2}$, where $D_0=kT\mu_0/e$.

It can easily be shown that the solutions which remain finite over the semi-infinite range should asymptotically approach the unperturbed bulk condition, $\Delta N=N-N_0=0$, $\Delta P=P-P_0=0$, $\Delta J_p=J_p-J_{p0}=J_{n0}-J_n=0$, and $\Delta E=E-E_0=0$, with

$$J=J_n+J_p=J_{n0}+J_{p0}=\text{const}, \tag{10}$$

and given by

$$J=(\nu_nN_0+\nu_pP_0)E_0=E_0, \tag{11}$$

E_0 being the normalized field which carries the current J through the unperturbed bulk.

The following analysis is concerned with such solutions in the asymptotic range in which linear approximations are valid whatever the current. The information obtained through the linear approximation in the asymptotic range characterizes also the solution outside this range. For instance, boundary conditions can be obtained for numerical evaluation in the nonlinear range in a more efficient way than previously done.[3]

In the asymptotic range[7] and in terms of ΔN, ΔP, ΔE, and ΔJ_p, Eqs. (2)–(5) become

$$\frac{d\Delta N}{dX}=-E_0\Delta N-N_0\Delta E-\frac{1}{\nu_n}\,\Delta J_p, \tag{12}$$

$$\frac{d\Delta P}{dX}=E_0\Delta P+P_0\Delta E-\frac{1}{\nu_p}\,\Delta J_p, \tag{13}$$

$$\frac{d\Delta J_p}{dX}=-\frac{\tau_D}{\tau_0}(P_0\Delta N+N_0\Delta P)\,, \tag{14}$$

$$\frac{d\Delta E}{dX}=\Delta P-\Delta N. \tag{15}$$

The sought-after solutions look like

$$\Delta N,\Delta P,\Delta J_p,\Delta E=C_1^{N,P,J,E}\exp(-\alpha_1X)+C_2^{N,P,J,E}\exp(-\alpha_2X), \tag{16}$$

where (a) α_1 and α_2 are the solutions with a positive real part of the characteristic equation

$$\alpha^4-(1+\lambda_1^2+E_0^2)\alpha^2+aE_0\alpha+\lambda_1^2=0\,, \tag{17}$$

in which

$$\lambda_1^2=\frac{1}{\nu_n\nu_p}\,\frac{\tau_D}{\tau_0}=\left(\frac{N_0}{\nu_p}+\frac{P_0}{\nu_n}\right)\frac{\tau_D}{\tau_0}\,, \tag{18}$$

$$a=N_0\left(\frac{1}{\nu_p}\,\frac{\tau_D}{\tau_0}-1\right)-P_0\left(\frac{1}{\nu_n}\,\frac{\tau_D}{\tau_0}-1\right), \tag{19}$$

and (b) $C_1^{N,P,J,E}$ and $C_2^{N,P,J,E}$ are, respectively, proportional to two arbitrary chosen constants C_1,C_2 $(C_{1,2}^{N,P,J,E}=\beta_{N,P,J,E}^{1,2}C_{1,2})$.

We shall be mainly concerned with distinct real solutions and we chose $\alpha_2>\alpha_1>0$. We shall call α_1, and its corresponding component in the general solution $C_1\exp(-\alpha_1X)$, *the main solution*, as the asymptotic behavior reduces to it towards infinity. There are only few exceptional physical conditions which impose $C_1=0$, $C_2\neq0$ and for which *the secondary solution* α_2 and $C_2\exp(-\alpha_2X)$ describe the asymptotic behavior, e.g., a lifetime p-n junction in thermal equilibrium.

Of all different proportionality coefficients we shall discuss only

$$\beta_P = \beta_P^1 = C_1^P / C_1^N = \Delta P / \Delta N , \tag{20}$$

in which ΔN and ΔP are both for the main solution, and which represents an important feature of a certain injection-extraction behavior. For $\beta_P^{1,2}$ the following expression can be derived (Appendix B):

$$\beta_P^{1,2} = \frac{\nu_n}{\nu_p} \frac{\alpha^2 - \alpha E_0 - 1/\nu_n}{\alpha^2 + \alpha E_0 - 1/\nu_p} = \frac{1 + \nu_n(\alpha E_0 - \alpha^2)}{1 - \nu_p(\alpha E_0 + \alpha^2)} , \quad \alpha = \alpha_{1,2} . \tag{21}$$

Under particular conditions in which degeneracy appears $\alpha_1 = \alpha_2$ and no expression can be written for $\beta_P^{1,2}$ as it has an arbitrary value.

III. GENERAL DISCUSSION OF THE SOLUTIONS

We shall further present some features of the solutions, deduced from the equations shown above, for various values of the ratio τ_D/τ_0, and of the total current J (or bulk field E_0). Various behaviors of the solutions will be translated in terms of small-perturbation behavior of an n-type material ($N_0 > P_0$). Extension to p-type materials can easily be done through the symmetry of Eq. (17) with respect to simultaneous changes $N_0 \rightleftarrows P_0$, and $E_0 \rightleftarrows -E_0$ (or $J \rightleftarrows -J$). None of the effects which will be further discussed is restricted to n-type materials.

It is time to emphasize that, throughout this paper, we quantitatively define the carrier-perturbation condition through ΔN or $\Delta P \neq 0$, not necessarily associated to a nonzero current J. For instance, in a p-n junction at thermal equilibrium, or for a steady-state distribution of equal mobility carriers generated by a nonuniform illumination, there is carrier perturbation, but no net current.

Throughout this paper we talk about contributions of the perturbations of electrons (ΔN) or holes (ΔP) to the net recombination rate, i.e., to the departure of the generation-recombination process from equilibrium. In the asymptotic range these contributions are proportional to the respective perturbations weighted with the equilibrium concentrations of opposite-sign carriers [the terms $P_0 \Delta N$ and $N_0 \Delta P$ in Eq. (14)]. It is as if excess electrons and holes do not see each other and independently recombine with the corresponding equilibrium carriers. Two particular cases are worth mentioning: the zero-recombination case, in which carrier-perturbation contributions cancel each other, $P_0 \Delta N = -N_0 \Delta P$, which approximates $NP = N_0 P_0 = N_i^2$ in the asymptotic range; and the "equality-recombination" case,[8] with equal carrier-perturbation contributions in the net recombination rate $P_0 \Delta N = N_0 \Delta P$. The conditions under which these cases appear will be discussed later.

From the characteristic equation (17), some general features of the solutions can be derived (Appendix A) and will be presented below:

(a) For zero current, $E_0 = 0$,

$$\alpha(0) = \begin{cases} 1 , \\ \lambda_1 = \left(\dfrac{1}{\nu_n \nu_p} \dfrac{\tau_D}{\tau_0} \right)^{1/2} = \dfrac{L_D}{L_a} , \end{cases} \tag{22}$$

the lower of which becomes the main solution α_1, the other being the secondary one α_2.

The solution $\alpha = 1$ corresponds to a characteristic length of the spatial distribution equal to the Debye length $L_D = (D_0 \tau_D)^{1/2}$.

The solution $\alpha = \lambda_1$ corresponds to a characteristic length equal to the ambipolar diffusion length $L_a = (D_a \tau_0)^{1/2}$, the ambipolar diffusion coefficient D_a having its usual meaning, $D_a = (n_0 + p_0)/(n_0/D_p + p_0/D_n)$.

We see, therefore, that the main solution for zero current relaxes in space with the Debye length or the ambipolar diffusion length, depending on whether τ_D/τ_0 is greater or smaller than $\nu_n \nu_p$.

(b) For infinite current, $E_0 \to \pm\infty$, the secondary solution goes to infinity according to the expression

$$\lim_{E_0 \to \pm\infty} [\alpha_2(E_0)/E_0] = \pm 1 , \tag{23}$$

while the main solution goes to zero for any finite τ_D/τ_0 values according to the expression

$$\lim_{E_0 \to \pm\infty} [E_0 \alpha_1(E_0)] = \tfrac{1}{2}[a \pm (a^2 + 4\lambda_1^2)^{1/2}]. \tag{24}$$

This behavior implies that, however great (but finite) τ_D/τ_0 is, one can safely neglect carrier diffusion at high-enough currents in the forward or reverse direction ($|J|, |E_0| \to \infty$). We shall come back to this later in more quantitative terms.

(c) The curves $\alpha(E_0)$ have quite distinct behaviors in various ranges of the parameter τ_D/τ_0. This can easily be seen if the characteristic equation (17) is regarded as a quadratic equation in $\alpha E_0(\alpha)$ (Appendix A), the solutions of which are real or complex.

As long as τ_D/τ_0 is outside the interval (ν_n, ν_p) (i.e., $\tau_D/\tau_0 \gtrless \nu_n, \nu_p$), there are distinct solutions α_1, α_2 for the whole range of currents $E_0 \in (-\infty, +\infty)$ (see Fig. 1). Both curves have extremes in α with respect to the field E_{01}, E_{02} determined by the condition

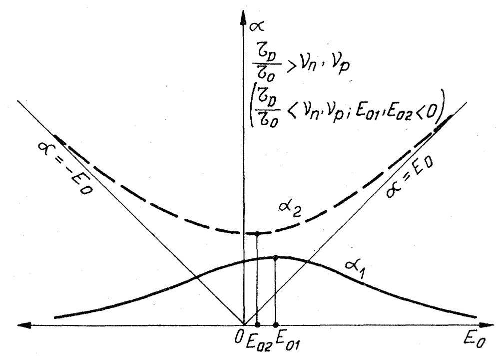

FIG. 1. Real distinct solutions α_1, α_2.

$$E_{01,2}\,\alpha(E_{01,2})=\tfrac{1}{2}a. \tag{25}$$

As α is positive, the fields E_{01}, E_{02} have the sign of the parameter a [Eq. (19)]. For an n-type material, when $\tau_D/\tau_0<\nu_n, \nu_p$ (i.e., ν_n or ν_p, whichever is smaller), the extremes correspond to a finite current in the reverse direction $J_{1,2}<0$, while for $\tau_D/\tau_0>\nu_n, \nu_p$ (i.e., ν_n or ν_p, whichever is greater) they correspond to a current in the forward direction $J_{1,2}>0$, which can go to infinity when $\tau_D/\tau_0\to\infty$ (with the exception of the peculiar case $N_0>P_0$ but $\sigma_{n0}\leq\sigma_{p0}$).

When the mobilities are unequal and τ_D/τ_0 is inside the interval (ν_n, ν_p) (i.e., $\min\nu_n, \nu_p<\tau_D/\tau_0<\max\nu_n, \nu_p$), there is a range of fields (E_{0m}, E_{0M}) within which the solutions $\alpha_{1,2}(E_0)$ are complex. They correspond to a behavior in which oscillations in space appear around the asymptotic unperturbed-bulk values. The fields which limit the range of oscillatory solutions E_{0m}, E_{0M} have the sign of the parameter a, positive for $\nu_n>\nu_p$ and negative for $\nu_n<\nu_p$. For these fields degeneracy appears [i.e., $\alpha_1(E_{0m,M})=\alpha_2(E_{0m,M})$] (Fig. 2).

(d) The particular cases $\tau_D/\tau_0=\nu_n, \nu_p$ correspond to significant boundary behaviors. There is now only one field value E_{0c} for which degeneracy appears [i.e., $\alpha_1(E_{0c})=\alpha_2(E_{0c})=\alpha_c$]. These cases are more easily dealt with by using the following alternate forms of the characteristic equation (Appendix A):

$$(\alpha^2-\alpha E_0-1)(\alpha^2+\alpha E_0-\lambda_1^2)=2\alpha E_0 P_0\left(\frac{1}{\nu_n}\frac{\tau_D}{\tau_0}-1\right), \tag{26}$$

$$(\alpha^2-\alpha E_0-\lambda_1^2)(\alpha^2+\alpha E_0-1)=-2\alpha E_0 N_0\left(\frac{1}{\nu_p}\frac{\tau_D}{\tau_0}-1\right); \tag{27}$$

as for the particular cases above one gets, respectively,

$$(\alpha^2-\alpha E_0-1)(\alpha^2+\alpha E_0-1/\nu_p)=0 \quad \text{for } \tau_D/\tau_0=\nu_n, \tag{28}$$

$$(\alpha^2-\alpha E_0-1/\nu_n)(\alpha^2+\alpha E_0-1)=0 \quad \text{for } \tau_D/\tau_0=\nu_p. \tag{29}$$

The comparison of Eqs. (28) and (29) with Eqs. (26) and (27) triggers the following remark: a given semiconductor, with a finite nonzero P_0/N_0 ratio, behaves under carrier perturbation in the asymptotic range like a unipolar n-type material ($P_0=0$) for $\tau_D/\tau_0=\nu_n$, and like a unipolar p-type material ($N_0=0$) for $\tau_D/\tau_0=\nu_p$.

Later, when we discuss equations of this type for a wider range of conditions, more details will be given concerning the pattern of the solutions. With reference to Eqs. (28), (29), and (21) we shall now only say that in both above-specified cases the main solution has a current range in which peculiar one-carrier perturbations appear:

(a) When $\tau_D/\tau_0=\nu_n$, for fields higher than E_{0c}, the main solution corresponds to an asymptotic behavior with perturbed holes and unperturbed electrons $\Delta P\neq0$, $\Delta N=0$ ($\beta_P=\infty$). It previously has been shown[3] that there are conditions under which this behavior extends itself outside the asymptotic range for any ΔP values; in Ref. 3, the equality $\tau_D/\tau_0=\nu_n$ corresponds to $A=1$.

(b) When $\tau_D/\tau_0=\nu_p$, for fields up to E_{0c}, the main solution corresponds to an asymptotic behavior with perturbed electrons and unperturbed holes $\Delta P=0$, $\Delta N\neq0$ ($\beta_P=0$). It can again be shown that this behavior may also extend itself outside the asymptotic range for any ΔN value. The equality $\tau_D/\tau_0=\nu_p$ corresponds to $A=\mu_p/\mu_n$, with notations in Ref. 3.

(c) For equal mobilities $\nu_n=\nu_p=1$, the condition $\tau_D/\tau_0=1$ corresponds to a very peculiar behavior of the main solution; there is an unperturbed hole conduction $\Delta N\neq0$, $\Delta P=0$ for all negative currents, and an unperturbed electron conduction $\Delta N=0$, $\Delta P\neq0$ for all positive currents ($E_{0c}=0$).

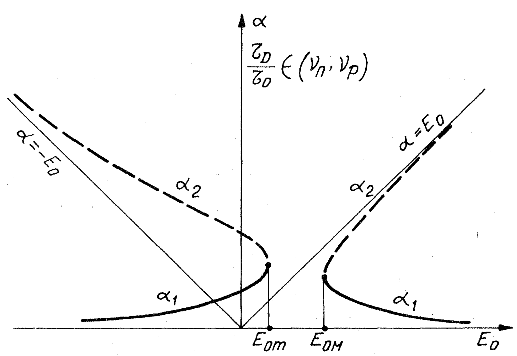

FIG. 2. Solutions α_1, α_2 when complex solutions appear.

In order to better interpret some cases presented above we shall make more use of Eqs. (20) and (21) for $\beta_P = \Delta P/\Delta N$. We shall evaluate this ratio for the main solution at various fields $\beta_P(E_0)$ and for different ratios τ_D/τ_0 (see also Appendix B).

(d) For zero current we already know that the α_1, α_2 values are 1 or λ_1, depending on the τ_D/τ_0 ratio.

When $\tau_D/\tau_0 < \nu_n, \nu_p$, $\alpha_1(0) = \lambda_1$ and

$$\beta_P^1(0) = \frac{1-(1/\nu_p)(\tau_D/\tau_0)}{1-(1/\nu_n)(\tau_D/\tau_0)} \gtrless 1 \text{ for } \nu_n \lessgtr \nu_p. \tag{30}$$

In the limit case $\tau_D/\tau_0 \to 0$, $\beta_1(0) \to 1$, i.e., *neutrality* is approached.

When $\tau_D/\tau_0 > \nu_n, \nu_p$, $\alpha_1(0) = 1$ and

$$\beta_P^1(0) = (1-\nu_n)/(1-\nu_p) = -P_0/N_0 \tag{31}$$

(by using $\nu_n N_0 + \nu_p P_0 = 1$, $n_0 + P_0 = 1$), i.e., the solution corresponds to a *zero recombination* condition ($NP = N_i^2$) in the asymptotic range.

We have already shown that $\alpha_1(0) = 1$ even for τ_D/τ_0 inside the interval (ν_n, ν_p), down to $\tau_D/\tau_0 = \nu_n/\nu_p$. This is the lowest τ_D/τ_0 value for which the main solution corresponds to zero recombination under no current. Now, even for lower τ_D/τ_0 values, for which the zero recombination condition is no longer the main solution, it can still be the physical one (i.e., $C_1 = 0$, $C_2 \neq 0$); this is the case of a lifetime p-n junction in thermal equilibrium, in which strong carrier perturbations ΔN, ΔP can appear close to the junction, while the zero-recombination condition $NP = N_i^2$ is everywhere conserved.

(e) For infinite current the expression for β_P along the main solution [see also Eq. (24)] becomes

$$\beta_P^1(\infty) = [1 + \nu_n(\alpha E_0)_\infty]/[1 - \nu_n(\alpha E_0)_\infty]. \tag{32}$$

If $\tau_D/\tau_0 \to 0$, for an n-type material one obtains

$$\lim_{\tau_D/\tau_0 \to 0} \beta_P^1(+\infty) = 1, \qquad \lim_{\tau_D/\tau_0 \to 0} \beta_P^1(-\infty) = P_0/N_0, \tag{33}$$

i.e., *neutrality* is approached at very high currents in the forward direction, and *equality recombination* for very high currents in the reverse direction.

If $\tau_D/\tau_0 \to \infty$, also for an n-type material, one gets

$$\lim_{\tau_D/\tau_0 \to \infty} \beta_P^1(+\infty) = -\nu_n/\nu_p, \qquad \lim_{\tau_D/\tau_0 \to \infty} \beta_P^1(-\infty) = -P_0/N_0. \tag{34}$$

For very high currents in the reverse direction the limit has the obvious significance of a *zero-recombination* condition. At the limit for very high currents in the forward direction the contributions of carrier perturbations in local conductivity cancel each other as $\nu_n \Delta N = -\nu_p \Delta P$; the resulting condition is an *unperturbed-conductivity* one, namely, $\Delta\sigma = 0$.

Up to now the equations give β_P values only for a few particular cases. General, closed expressions which should give β_P whatever the current and τ_D/τ_0 ratio cannot be written. But it can be shown (Appendix B) that for the main solution $\alpha_1(E_0)$,

$$\beta_P^1(E_0) \begin{array}{l} >0 \text{ for } \tau_D/\tau_0 < \nu_n, \nu_p, \\ <0 \text{ for } \tau_D/\tau_0 < \nu_n, \nu_p \end{array} \tag{35}$$

for the whole range of field or current values (see, e.g., Fig. 3).

For $\tau_D/\tau_0 \in (\nu_n, \nu_p)$ there is first of all a certain range of fields within which spatial oscillations appear, so that one cannot speak of a certain

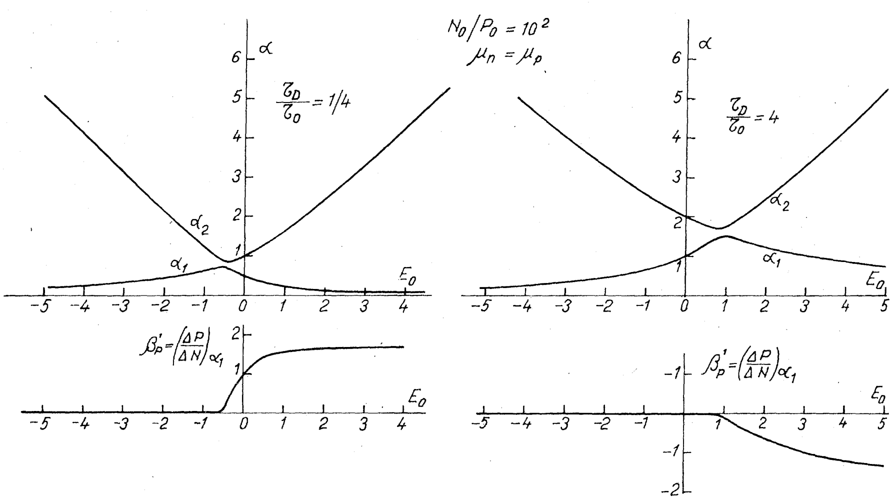

FIG. 3. Numerical solutions $\alpha_{1,2}(E_0)$ and $\beta_P^1(E_0)$ for lifetime and relaxation cases. Characteristic patterns for β_P^1 in the two cases should be noted. For zero net recombination $\beta_P^1 = -0.01$; for equality recombination $\beta_P^1 = 0.01$.

TABLE I. Lifetime, n type ($N_0 > P_0$).

τ_D/τ_0	$<\nu_n, \nu_p$	$\to 0$ ($\ll 1$, extreme case)		
E_0 or J	Any value in $(-\infty, +\infty)$	(Limit in the reverse direction) $-\infty$	0	(Limit in the forward direction) $+\infty$
$\beta_P = \frac{\Delta P}{\Delta N}$	>0	P_0/N_0 (equality recombination)	1 (neutrality)	1 (neutrality)

value and sign of β_P^1 for the whole asymptotic region; but it can also be shown that for fields below and above this range the values of β_P^1 are real but of opposite signs. Depending on whether ν_n is greater or smaller than ν_p, the crossing of the oscillatory range of fields changes β_P^1 from negative to positive, or the other way around (Appendix B).

The above presented results, summarized in Tables I and II, show that, except for the trap-free case with equal-mobility carriers, the condition $\tau_D/\tau_0 = 1$ does not represent a significant boundary between different injection-extraction behaviors of the semiconductors. But the conditions $\tau_D/\tau_0 < \nu_n, \nu_p$ and $\tau_D/\tau_0 > \nu_n, \nu_p$ do correspond to distinct behaviors; these can be called lifetime and relaxation ones within the spirit of the definition which compares neutralization and recombination tendencies.[1,3,4] Now, if the whole range of possible currents is considered, the tendencies to approach neutrality, or zero recombination through $NP = N_i^2$, do not represent essential features for lifetime or relaxation semiconductors, even in the extreme cases $\tau_D \ll \tau_0$ or $\tau_D \gg \tau_0$. Such tendencies do appear, but are restricted to limited current ranges.

The only characteristic feature for the injection-extraction behavior of a semiconductor seems to be the sign of the parameter $\beta_P = \Delta P/\Delta N$. We conclude that lifetime or relaxation regimes are characterized by the way in which departures from equilibrium of electron and hole concentrations are coupled, as regards sign, in the asymptotic range of a certain behavior. The lifetime case corresponds to a positive β_P ($\Delta N, \Delta P$ of the same sign), while the relaxation case corresponds to a negative β_P ($\Delta N, \Delta P$ of opposite signs). This conclusion is in complete agreement with that of previous evaluations of nonlinear behaviors for unity injection ratio.[3,4]

The general equations concerning asymptotic behavior presented above allow for numerical solutions to be obtained for any set of model parameters at any given current. An example of calculated curves is given in Fig. 3.

In order to visualize better the pattern of the solutions and allow for a fast quantitative assessment of significant current (or E_0) values in some usual cases, we shall deal below with approximations suitable to the particular case of pronounced extrinsic materials. We leave for a future presentation such interesting problems as the oscillatory regime, injection and extraction of carriers in intrinsic materials, or nearly intrinsic ones in which unequal mobilities ensure opposite carrier and conductivity characters (for instance $n_0 > p_0$ and simultaneously $\sigma_{n0} < \sigma_{p0}$).

IV. SOLUTIONS FOR PRONOUNCED EXTRINSIC MATERIALS

We explicitly deal with an n-type material $N_0 \gg P_0$, $\mu_n \gtrless \mu_p$, but still $\sigma_{n0} \gg \sigma_{p0}$. In the equations concerning such materials, P_0 or P_0/N_0 are to be

TABLE II. Relaxation, n type ($N_0 > P_0$).

τ_D/τ_0	$>\nu_n, \nu_p$	$\to \infty$ ($\gg 1$, extreme case)		
E_0 or J	Any value in $(-\infty, +\infty)$	(Limit in the reverse direction $-\infty$	0	(Limit in the forward direction) $+\infty$
$\beta_P = \frac{\Delta P}{\Delta N}$	<0	$-P_0/N_0$ (zero net recombination) $NP = N_i^2$	$-P_0/N_0$ (zero net recombination) $NP = N_i^2$	$-\nu_n/\nu_p$ (unperturbed conductivity)

regarded as small parameters, for instance in Eq. (26). This equation shows that the solutions are satisfactorily approximated by the simpler form

$$(\alpha^2 - \alpha E_0 - 1)\left(\alpha^2 + \alpha E_0 - \frac{1}{\nu_p}\,\frac{\tau_D}{\tau_0}\right) = 0\ , \qquad (36)$$

which corresponds to the unipolar case $P_0 = 0$, $(N_0 = 1)$, $\lambda^2 = (1/\nu_p)(\tau_0/\tau_D) = a + 1$.

It can be shown (Appendix C) that the first parentheses correspond to the equation for electron perturbation ΔN, in which recombination and hole contribution to space charge are neglected, while the second parentheses correspond to the equation for hole perturbation ΔP, in which total space charge and electron contribution to recombination are neglected.

The equation for β_P corresponding to Eq. (36) is

$$\beta_P = \frac{\nu_n}{\nu_p}\,\frac{\alpha^2 - \alpha E_0 - 1}{\alpha^2 + \alpha E_0 - \nu_n/\nu_p}\ . \qquad (37)$$

The following discussion concerning the pattern of the solutions of Eq. (36) is also valid for the similar kind of equations describing the particular cases $\tau_D/\tau_0 = \nu_n, \nu_p$, Eqs. (28) and (29).

The solutions of Eq. (36) are intersecting curves and the main solution is

$$\alpha_1(E_0) = \min[\alpha_i(E_0), \alpha_d(E_0)]\ . \qquad (38)$$

α_i is a positive, increasing solution, resulting from the first parentheses in (36),

$$\alpha_i(E_0) = \tfrac{1}{2}[\,E_0 + (E_0^2 + 1)^{1/2}] \qquad (39)$$

while α_d is a positive, decreasing solution, resulting from the second parentheses in (36),

$$\alpha_d(E_0) = \tfrac{1}{2}\{-E_0 + [E_0^2 + (1/\nu_p)(\tau_D/\tau_0)]^{1/2}\}\ . \qquad (40)$$

The most significant departures of the exact solutions of Eq. (26) from the approximate ones of Eqs. (39) and (40) appear close to the intersection of the latter ones,

$$\alpha_c = \alpha_i(E_{0c}) = \alpha_d(E_{0c}) = \left[\frac{1}{2}\left(\frac{1}{\nu_p}\,\frac{\tau_D}{\tau_0} + 1\right)\right]^{1/2}, \qquad (41)$$

where

$$E_{0c} = \left(\frac{1}{\nu_p}\,\frac{\tau_0}{\tau_0} - 1\right) \Big/ \left[2\left(\frac{1}{\nu_p}\,\frac{\tau_0}{\tau_0} + 1\right)\right]^{1/2}. \qquad (42)$$

For $\tau_D/\tau_0 \gtrless \nu_n, \nu_p$ (lifetime and relaxation cases) these departures do not result in spectacular changes of the curves α_1, α_2, but only in a split in α of the order of $\sqrt{P_0}$, which precludes their intersection at E_{0c}.

For $\tau_D/\tau_0 \in (\nu_n, \nu_p)$ (the intermediate regime) the above-mentioned departures result in qualitative changes in the α_1, α_2 curves which become discontinuous due to a split in E_0 values of the order of $\sqrt{P_0}$, which precludes the intersection at α_0; a gap of fields, or currents, appears where α values become complex, and to which spatial oscillations correspond. The above-discussed features are illustrated in the sketch of the solutions for the extrinsic case and various τ_D/τ_0 given in Fig. 4.

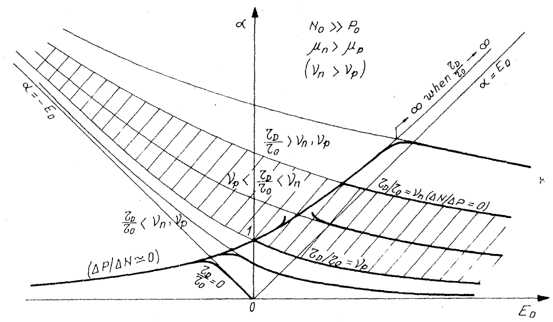

FIG. 4. Pattern of $\alpha_1(E_0)$ solutions for the pronounced-extrinsic case $N_0 \gg P_0$, and various τ_D/τ_0, when $\nu_n > \nu_p$.

One may see that two distinct effects govern the injection-extraction behavior of a pronounced n-type material in different current ranges, corresponding to the transition of the main solution from α_i to α_d, beyond the intersection field E_{0c}.

The first, approximately described by $\alpha_i(E_0)$ with $\beta_P = 0$, consists in a quasiunipolar space-charge controlled conduction of electrons (majority carriers), in which recombination has only little effect. When τ_D/τ_0 changes from zero to infinity, the material goes from equality recombination to zero recombination, but changes in α are negligible and changes in β_P are very small, from P_0/N_0 to $-P_0/N_0$ (see Tables I and II.) The α_i effect governs conduction from infinite currents in the reverse direction up to $J(E_{0c})$.

The second effect, described by $\alpha_d(E_0)$, consists of a bipolar conduction controlled by hole (minority carrier) perturbation. The spatial distribution of the latter is in turn controlled by recombination, while space charge has only little effect. The electron (majority-carrier) response to the hole perturbation depends on the balance between neutralization and recombination, controlled by the ratio τ_D/τ_0. The α_d effect governs conduction from $J(E_{0c})$ up to infinite currents in the forward direction. As hole recombination is essential in the α_d effect, the latter has a strong dependence on τ_D/τ_0; it is this dependence which controls the critical current $J_c = J(E_{0c})$ above which minority carrier perturbation begins to be prominent in the asymptotic range and in the general behavior [see Fig. 4 and Eq. (42)].

The current J_c represents also a quantitative limit beyond which, in the relaxation case, the conduction starts departing from a nearly-zero-recombination condition $NP = N_i^2$ towards a condition in which $NP = N_i^2$ can no longer be maintained (Figs. 3 and 4); beyond the same limit, diffusion currents gradually become negligible and a quasi-unperturbed conductivity condition is finally reached (Table II).

In terms of β_P, the critical current J_c marks the beginning of departure from small absolute values corresponding to quasiequality, or zero recombination, towards those corresponding to quasineutrality or unperturbed conductivity (Fig. 3, Tables I and II).

We shall now say a few more words about the conduction for currents greater than J_c, in which the α_d effect prevails. As we have already said, in this solution the space-charge term, i.e., dE/dX, is neglected. This reminds us the "quasi-neutrality" approximation widely made use of in the theory of ambipolar injection in insulators, e.g., in Ref. 9, or in the theory of high injection levels in semiconductors and semiconductor devices, e.g., in Ref. 10, where dE/dX is also neglected. But, at least for our asymptotic solution, the above-mentioned neglect is not just in accordance with quasineutrality, as it can be fulfilled not only in materials with fast dielectric relaxation $\tau_D/\tau_0 \simeq 0$, but also up to $\tau_D/\tau_0 \gg 1$. A less-restrictive condition is in fact necessary, namely, that above J_c the hole current divergence due to field variations plays a negligible part in the recombination process whatever the τ_D/τ_0 ratio. In strongly extrinsic materials and for not excessively high hole perturbations the validity of such a "quasiconstant field" condition for the hole (minority carrier) perturbation may extend itself well outside the asymptotic range. This could explain the success of "quasineutrality" calculations in some situations in which quasineutrality itself is improbable. This also accounts for the practically identical $\Delta P(X)$ dependences, obtained in Ref. 3 through numerical solutions of the nonlinear equations, from lifetime to the relaxation boundary (see Figs. 2 and 3 in Ref. 3), and even beyond it, up to $\tau_D/\tau_0 = 10$.

We have discussed up to now asymptotic behaviors for a wide range of conditions. One can start with an appropriate one among them, and numerically evaluate strong perturbation behaviors which appear close to the injection-extraction surface of the investigated semiconductor, under the given current conditions. Such numerical evaluations, and the way in which it is ensured that evaluated contours match the contact properties, are outside the scope of this paper. But we will qualitatively illustrate how behaviors within asymptotic quasi-

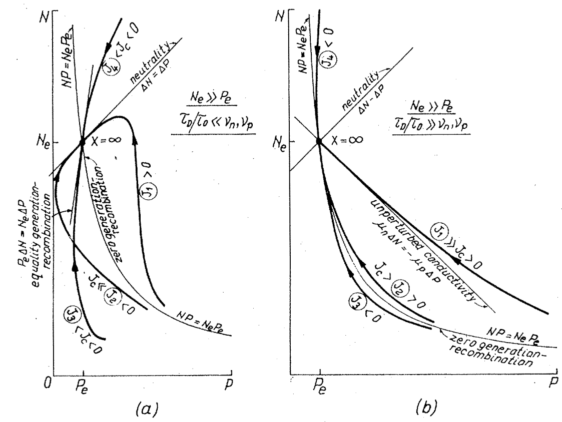

FIG. 5. Concentration contours in the N-P plane for extreme lifetime (a) and relaxation (b) semiconductors (pronounced-extrinsic, n type). The effect of current-density value on asymptotic and overall behavior is evidenced; $J_c = J(E_{0c})$, E_{0c} being the critical field given by Eq. (42).

unperturbed region have a significant bearing on behaviors in the region with strong perturbations.

Figure 5 shows qualitatively some concentration contours in the P-N plane illustrating changes in boundary asymptotic conditions at various currents, for pronounced lifetime and relaxation n-type materials. Contours corresponding to p-n junctions, J_1, J_2, J_3, approach the $NP = N_0P_0$ hyperbola towards the transition region of the junction, and have generation or recombination maxima; behavior in the bulk beyond these maxima does not qualitatively differ from the asymptotic one in the quasiunperturbed bulk. The change in asymptotic slope between lifetime and relaxation cases is immediately obvious.

For moderate positive and negative currents the lifetime (J_1, J_2) and relaxation (J_2, J_3) p-n junctions have "normal" behaviors. In contrast to the lifetime case, relaxation carrier contours for positive, zero, and negative currents are very close to each other.

In the "normal" relaxation case, even if holes are injected (J_2) or extracted (J_3) at the contact, in the asymptotic region practically only the electrons are perturbed, and only their perturbations have a significant contribution in the current. In other words, generation or recombination maxima have the character of "fronts" of generation or recombination.

Even if this "front" character does not exist in "normal" lifetime behaviors, it is not a general and exclusive property of the relaxation behavior. As previous analysis shows, this character depends on a very low P_0/N_0 ratio (pronounced extrinsic materials) and a current below J_c. It does not appear in the relaxation case at high positive currents [Fig. 5(b), curve J_1]. It can appear in principle in lifetime p-n junctions far in the negative current range; there are such solutions of the equations [Fig. 5(a), curve J_3], which correspond to an extraction of holes from a generation front, which in turn injects electrons into the asymptotic region under a strong field which diminishes towards the bulk. Physical realizability of such solutions with generation front is limited by junction breakdown. But asymptotic conductions at currents in the reverse direction beyond J_c do appear in lifetime materials through a space-charge controlled injection of majority carriers, [Fig. 5(a), curve J_4]. Curve J_4 in Fig. 5(b) also corresponds to a space-charge controlled injection of majority carriers, but for the relaxation case.

V. RESISTANCE ENHANCEMENT THROUGH MINORITY-CARRIER INJECTION

At least for trap-free relaxation materials the problem is a subject of controversy. Van Roosbroeck[1] predicted this enhancement, but his proof rested on simultaneously assuming zero recombination and negligible diffusion in the depletion region, while Kiess and Rose[11] and Döhler and Heyszenau[2] had sound objections against that. Numerical solutions for injection in semi-infinite trap-free materials[3] could not reveal the enhancement either.

In order to get more quantitative insight into this matter we analyze a necessary condition for resistance enhancement, namely, the possibility of having fields higher in the injection region than in bulk; asymptotic analysis can offer good suggestion concerning the conditions where such behaviors are to be looked for.

Of course, steady states with oscillations in space allow for fields higher than in bulk. What we are mainly concerned with now is whether such fields are possible in aperiodic steady states (with field contours which are monotonous or with one overshoot). Thus, we are looking for aperiodic asymptotic behaviors in which the field approaches the bulk limit value by decreasing. The space charge which ensures that is a negative one, for a positive current in an n-type material, i.e., $\Delta P - \Delta N < 0$.

Among the asymptotic contours for a positive current only those with

$$-P_0/N_0 < \beta_p < 1 \tag{43}$$

satisfy the above-mentioned condition.

The relaxation regime does not satisfy this condition in its asymptotic range. Consequently it allows for resistance enhancement only in the rather strange injection conditions, if any, in which aperiodic asymptotic behavior is compatible with a finite number of oscillations in the strong injection range.

The above-mentioned condition is fulfilled, and resistance enhancement through injection may appear, in a lifetime material in which electrons are more mobile than the injected holes, $\nu_n > \nu_p$ [see Eq. (30)]: in fact the upper limit for τ_D/τ_0 is inside the interval (ν_n, ν_p). On the other hand, the increase in resistance is not possible up to an infinite current in the forward direction, for which $\beta_P > 1$, Eq. (32). It can be shown that the upper limit in current corresponds to

$$E_0 \big|_{\beta_p = 1} = \tfrac{1}{2}(\nu_n - \nu_p)\left(\frac{1}{\nu_n \nu_p}\,\frac{\tau_D}{\tau_0}\right)^{1/2} . \tag{44}$$

The conclusions reached here are in agreement with those of previous numerical calculations, which could not reveal resistance increase through injection in trap-free materials with equal mobility carriers,[3] but proved it possible in materials with strong trapping of the injected minority carriers and a τ_D/τ_0 value in the lifetime range[4]; the strongly trapped holes can be regarded as having a lower effective mobility, in physical consistency with the condition $\mu_n > \mu_p$ (ν_n, ν_p) outlined here for a trap-free material. A greater space charge per mobility unit is ensured for every injected minority carrier, and this seems to be important in the physics of resistance enhancement through minority-carrier injection.

VI. CONCLUSIONS

The analysis of carrier-perturbation behavior in the asymptotic range allows for a clear distinction to be made between lifetime and relaxation behavior. It provides new significant information concerning the overall carrier-perturbation behavior.

For the relaxation regime (excepting $\tau_0 = 0$) the analysis of the asymptotic range allows one to draw the following conclusions expected to be valid also well outside this range, in agreement with results of previous numerical calculations for the nonlinear range: (a) There is an upper limit for the current in the forward direction up to which quasi-zero-recombination $NP = N_i^2$ is possible, in agreement with Ref. 12 (see Fig. 1 there). (b) There is no current in the forward direction for which a negligible diffusion is compatible with quasi-zero-recombination, in agreement with Ref. 3.

Previous work[1,3,4] has suggested that the value of the τ_D/τ_0 ratio allows for a distinction between two different regimes of semiconductor behavior. Van Roosbroeck and Casey[1] have first ascribed distinct regimes of semiconductor behavior to situations $\tau_D \gg \tau_0$ and $\tau_D \ll \tau_0$, and called them the relaxation case and the lifetime case.

The present analysis shows that the whole range of τ_D/τ_0 values $(0,\infty)$, is divided in three regions by qualitative changes in the asymptotic behavior; these appear at the values $\tau_D/\tau_0 = \nu_n, \nu_p$, where ν_n and ν_p are the ratios between carrier mobilities, μ_n and μ_p, respectively, and the average mobility.

The ranges $\tau_D/\tau_0 > \nu_n, \nu_p$ and $\tau_D/\tau_0 < \nu_n, \nu_p$ can be called *relaxation* and *lifetime* regimes, respectively, in general agreement with the previous definition. Within the range $\tau_D/\tau_0 \in (\nu_n, \nu_p)$ the semiconductor has, at different currents, either lifetime or relaxation behavior, *or even a new type of behavior* in which oscillations around the bulk values occur in space. For equally mobile carriers $(\nu_n = \nu_p = 1)$ there is no intermediate interval in τ_D/τ_0; lifetime and relaxation regimes have a common boundary $\tau_D/\tau_0 = 1$.

The value $\tau_D/\tau_0 = \nu_n \nu_p$ $(\lambda_1 = 1)$ within the interval (ν_n, ν_p) has an interesting significance for the zero-current case. A zero-current carrier perturbation is a superposition of two distinct perturbations (Appendix A): (i) the first involves a neutrality perturbation which relaxes in space with the Debye length L_D, generation, and recombination canceling each other $(NP = N_i^2)$; (ii) The second involves a recombination perturbation and relaxes in space with the ambipolar diffusion length L_a; when $\nu_n = \nu_p$, or τ_D/τ_0 approaches zero, the second effect corresponds to space-charge neutrality. When $\tau_D/\tau_0 > \nu_n, \nu_p$ then $L_D > L_a$, while $L_D < L_a$ in the opposite case.

As regards lifetime and relaxation regimes, even in their limit cases, charge neutrality and zero net-recombination, respectively, cannot be regarded as essential features of a certain behavior, when the whole range of currents is considered. In an n-type lifetime semiconductor with $\tau_D/\tau_0 \ll 1$, for high-enough currents in the reverse direction, in the asymptotic range *charge-neutrality* cannot be maintained and an *equality-recombination* condition is approached; it corresponds to equal contributions of electron and hole perturbations to the net-recombination rate. To round off the picture, in the relaxation semiconductors with $\tau_D/\tau_0 \gg 1$ at high-enough positive currents in the forward direction, in the relaxation range the *zero net-recombination* condition is replaced by an *unperturbed conductivity* condition; electron and hole contributions in the net-recombination rate no longer cancel, but do so in the local conductivity.

However, there is a characteristic feature of the lifetime or relaxation cases, valid throughout the whole current range. It is the way the perturbations of opposite-sign carriers are coupled in the asymptotic range, as indicated by the sign of the $\Delta P/\Delta N$ ratio; this ratio is always positive in the lifetime regime and negative in the relaxation regime.

Some interesting features are revealed for the particular case of asymptotic carrier-perturbation behavior in *pronounced extrinsic* materials. There are two physical effects, which compete in controlling the asymptotic behavior at various currents. From infinite currents in the reverse direction up to a certain critical value, a quasi-unipolar space-charge controlled conduction of majority carriers prevails. Above this critical current the conduction is bipolar and controlled by the minority-carrier perturbation. In the spatial distribution of the latter the recombination process is essential, while space charge has a negligible effect, however high the current level is.

The critical current, around which the conduction changes from one kind to the other, is in the reverse direction for the lifetime case $\tau_D/\tau_0 < \nu_n, \nu_p$, and in the forward direction for the relaxation case $\tau_D/\tau_0 > \nu_n, \nu_p$. In the deep-relaxation case the critical current increases as $(\tau_D/\tau_0)^{1/2}$. It is this critical current which represents in the deep-relaxation case the limit beyond which the zero-recombination condition cannot be maintained, beyond which diffusion becomes gradually negligible, and conduction changes towards an unperturbed-conductivity condition.

As for the possibility of increasing the resistance through a certain minority-carrier injection, which results in aperiodic steady states, it is crucial whether in the asymptotic range the field decreases or not towards its bulk value. This is never possible in a trap-free relaxation material $\tau_D/\tau_0 > \nu_n, \nu_p$. Such resistance increase through injection may appear in lifetime materials $\tau_D/\tau_0 < \nu_n, \nu_p$ and even in some materials with $\tau_D/\tau_0 \in (\nu_n, \nu_p)$, provided the mobility of the injected minority carriers is lower than that of the majority ones. These calculations, drawn for trap-free materials, are in physical consistency with those of a previous numerical analysis[4] concerning injection in materials with significant trapping. It was shown that, in order to obtain a resistance increase in such materials, they have to be in a lifetime regime defined by a positive $\Delta P/\Delta N$, and ensure that the injected minority carriers are strongly trapped, i.e., have a lower effective mobility.

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APPENDIX A

Second-order equations for electron and hole perturbations can be derived from Eqs. (12)–(15):

$$\frac{d^2\Delta N}{dX^2}+E_0\frac{d\Delta N}{dX}+N_0(\Delta P-\Delta N)=\frac{1}{\nu_n}\frac{\tau_D}{\tau_0}(P_0\Delta N+N_0\Delta P)\,, \tag{45}$$

$$\frac{d^2\Delta P}{dX^2}-E_0\frac{d\Delta P}{dX}-P_0(\Delta P-\Delta N)=\frac{1}{\nu_p}\frac{\tau_D}{\tau_0}(P_0\Delta N+N_0\Delta P)\,. \tag{46}$$

The three terms on the left-hand side give the contributions of diffusion, concentration variation, and field variation (space charge), in the net-combination rate on the right-hand side.

The above equations can be rewritten

$$\frac{d^2(P_0\Delta N+N_0\Delta P)}{dX^2}+E_0\frac{d(P_0\Delta N-N_0\Delta P)}{dX}=\lambda_1^2(P_0\Delta N+N_0\Delta P)\,, \tag{47}$$

$$\frac{d^2(\Delta P-\Delta N)}{dX^2}-E_0\frac{d(\Delta P+\Delta N)}{dX}=(\Delta P-\Delta N)+(P_0\Delta N+N_0\Delta P)\frac{\tau_D}{\tau_0}\frac{\nu_n-\nu_p}{\nu_n\nu_p}\,. \tag{48}$$

For the zero-field case $E_0=0$ the latter ones become equations for recombination perturbation $(P_0\Delta N+N_0\Delta P)$, and neutrality perturbation $(\Delta P-\Delta N)$ which have the following independent solutions: $-\Delta N,\ \Delta P,\ \Delta P-\Delta N\sim e^{-X}=e^{-X/L_D}$, and $P_0\Delta N+N_0\Delta P=0$; $-\Delta N,\ \Delta P,\ P_0\Delta N+N_0\Delta P\sim e^{-\lambda_1 X}=e^{-x/L_a}$, and $\Delta P-\Delta N=(\Delta\Sigma_p-\Delta\Sigma_n)\lambda_1^2$, where $\Delta\Sigma_n=\nu_n\Delta N$, $\Delta\Sigma_p=\nu_p\Delta P$, and L_a, L_D, and λ_1^2 have been previously defined.

The first solution corresponds to zero net-recombination and the second, when $\nu_n=\nu_p$ or $\tau_D/\tau_0\to 0$, to charge neutrality.

In the general case, by using Eqs. (16) and (20), the above equations become

$$\alpha^2-\alpha E_0-N_0-\frac{1}{\nu_n}\frac{\tau_D}{\tau_0}P_0+N_0\left(1-\frac{1}{\nu_n}\frac{\tau_D}{\tau_0}\right)\beta_P=0\,, \tag{49}$$

$$\alpha^2+\alpha E_0-P_0-\frac{1}{\nu_p}\frac{\tau_D}{\nu_p}N_0+P_0\left(1-\frac{1}{\nu_p}\frac{\tau_D}{\tau_0}\right)\frac{1}{\beta_p}=0\,. \tag{50}$$

By eliminating β_p from Eqs. (49) and (50) one obtains

$$(\alpha^2-\alpha E_0-1+l_p)(\alpha^2+\alpha E_0-1+l_n)=l_n l_p\,, \tag{51}$$

with

$$l_p=P_0\left(1-\frac{1}{\nu_n}\frac{\tau_D}{\tau_0}\right),\quad l_n=N_0\left(1-\frac{1}{\nu_p}\frac{\tau_D}{\tau_0}\right), \tag{52}$$

in which, with previous notation (19), $l_p-l_n=a$.

If Eq. (51) is reduced to its simplest form, Eq. (17) is obtained. If simple convenient manipulations are made Eq. (51) can also be written in the alternaltive forms given by Eqs. (26) and (27).

Equation (17) can be regarded as a quadratic equation for $\alpha E_0(\alpha)$, the solution of which is

$$\alpha E_0(\alpha)=\tfrac{1}{2}a\pm[\Delta(\alpha)]^{1/2}\,, \tag{53}$$

the parameter a being previously defined, and

$$\Delta(\alpha)=\alpha^4-\alpha^2(1+\lambda_1^2)+\lambda_1^2+\tfrac{1}{4}a^2 \tag{54}$$

is an expression which vanishes for

$$\alpha_{M,m}^2=\alpha_c^2\mp(l_p l_n)^{1/2}\,, \tag{55}$$

in which l_p, l_n have previously been defined, and

$$\alpha_c=[\tfrac{1}{2}(1+\lambda_1^2)]^{1/2}\,. \tag{56}$$

The values α_M, α_m correspond to extremes in $\alpha_1(E_0)$ and $\alpha_2(E_0)$, respectively. $\Delta(\alpha)<0$ when $\alpha\in(\alpha_M,\alpha_m)$. When $\tau_D/\tau_0\leq\nu_n,\nu_p$ or $l_p l_n>0$, then α_M,α_m are real (Fig. 1). When $\tau_D/\tau_0\in(\nu_n,\nu_p)$ or $l_p l_n<0$, there are no real α_M,α_m, but two distinct branches in $E_0(\alpha)$, or a complex-values gap in the dependence $\alpha(E_0)$, (Fig. 2). If diffusion had been neglected [second derivatives in Eqs. (45) and (46) or α^2 terms in Eqs. (49) and (50)], then the complex-values (oscillations) gap would not have appeared $[\Delta(\alpha)=\lambda_1^2+\frac{1}{4}a^2]$.

APPENDIX B

If β_P is extracted from Eqs. (49) and (50)

$$\beta_P=\frac{\alpha^2-\alpha E_0-N_0-(1/\nu_n)(\tau_D/\tau_0)P_0}{-N_0+1/\nu_n(\tau_D/\tau_0)N_0}=\frac{-P_0+(1/\nu_p)(\tau_D/\tau_0)P_0}{\alpha^2+\alpha E_0-P_0-(1/\nu_p)(\tau_D/\tau_0)N_0}\,, \tag{57}$$

one obtains through obvious manipulations

$$\beta_P=\frac{(\alpha^2-\alpha E_0)/\nu_p-N_0/\nu_p-P_0/\nu_n}{(\alpha^2+\alpha E_0)/\nu_n-N_0/\nu_p-P_0/\nu_n}=\frac{\nu_n}{\nu_p}\frac{\alpha^2-\alpha E_0-1/\nu_n}{\alpha^2+\alpha E_0-1/\nu_p}\,, \tag{58}$$

which is Eq. (21). It shows that the dependence of $\beta_P = \Delta P/\Delta N$ on τ_D/τ_0 is only an implicit one, $\beta_P(\tau_D/\tau_0) = \beta_P[\alpha(\tau_D/\tau_0)]$. In particular cases when α is known [Eqs. (22) – (24)] closed expressions can be deduced [Eqs. (30)–(34)].

General conclusions regarding β_P can be drawn from the following two equations, obtained by combining Eqs. (49) and (50):

$$2\alpha^2 = 1 + \lambda_1^2 + P_0\left(\frac{1}{\nu_p}\frac{\tau_D}{\tau_0} - 1\right)\frac{1}{\beta_P} + N_0\left(\frac{1}{\nu_n}\frac{\tau_D}{\tau_0} - 1\right)\beta_P, \tag{59}$$

$$2\alpha E_0 = a + P_0\left(\frac{1}{\nu_p}\frac{\tau_D}{\tau_0} - 1\right)\frac{1}{\beta_P} - N_0\left(\frac{1}{\nu_n}\frac{\tau_D}{\tau_0} - 1\right)\beta_P. \tag{60}$$

One immediately sees that $\beta_P = 0$ is a singular point, excepting the case $\tau_D/\tau_0 = \nu_p$. As a consequence β_P conserves sign along α_1, α_2 curves when $\tau_D/\tau_0 \notin (\nu_n, \nu_p)$. As the main solution α_1 is the smaller positive one, from Eq. (59) it is seen that β_P must be positive in the lifetime case $\tau_D/\tau_0 < \nu_n, \nu_p$, and negative in the relaxation case $\tau_D/\tau_0 > \nu_n, \nu_p$.

When $\tau_D/\tau_0 \in (\nu_n, \nu_p)$ the two terms containing β_P in Eq. (60) have the same sign. As their product is constant, the curve $E_0(\beta_P)$ has two disinct branches, which correspond to β_P either positive or negative, and which extend towards either $E_0 = -\infty$ or $E_0 = +\infty$. A short inspection of these curves shows that when the whole range of E_0 is covered $(-\infty, +\infty)$, β_P changes sign when crossing the oscillation gap (complex α, β_P) from negative to positive if $\nu_n > \nu_p$ and the other way around when $\nu_p < \nu_n$.

APPENDIX C

In the extreme extrinsic case $P_0 = 0$ $(N_0 = 1)$, the asymptotic equations for electron and hole perturbations (45) and (46), reduce to

$$\frac{d^2\Delta N}{dX^2} + E_0\frac{d\Delta N}{dX} + (\Delta P - \Delta N) = \frac{1}{\nu_n}\frac{\tau_D}{\tau_0}\Delta P, \tag{61}$$

$$\frac{d^2\Delta P}{dX^2} - E_0\frac{d\Delta P}{dX} = \frac{1}{\nu_p}\frac{\tau_D}{\tau_0}\Delta P. \tag{62}$$

The most important consequence is that the contribution of the space-charge term in the hole-current divergence, i.e., in the recombination rate, is negligible. The recombination rate itself reduces only to the hole-perturbation contribution.

In general, the two factors which control the penetration of carrier perturbation inside the semiconductor are carrier recombination and local space charge. In extreme extrinsic materials these effects correspond each to independent particular solutions of the system [Eqs. (61) and (62)].

One particular solution is a hole perturbation, the space distribution of which is determined by a hole-controlled recombination. The characteristic length of the exponential behavior is field dependent, but practically coincides with the hole-diffusion length L_p at fields lower than kT/eL_p. In this type of process the majority electrons may have the same exponential behavior, as they respond to the hole perturbation ensuring electrons for the hole-controlled recombination, while their space charge plays no role in controlling the hole distribution. Equation (62), which controls this type of process, corresponds to the vanishing second parentheses in Eq. (36).

Another particular solution of the system results from the fact that the trivial solution of Eq. (62), $\Delta P \equiv 0$, does not imply a trivial solution for the system as well. The process, whose spatial distribution is now described by Eq. (61) with $\Delta P \equiv 0$, is a one-carrier conduction consisting in a space-charge controlled conduction of majority electrons with no recombination. This particular form of Eq. (61) corresponds to the vanishing first parentheses in Eq. (36).

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