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Hall effect in finite specimens of arbitrary lifetime and trap content

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This paper presents a linearized Hall-effect theory, applicable to specimens of arbitrary size, carrier concentration, lifetime, dielectric relaxation time, concentration of (single level) recombination centers, and surface properties. Along the lines of Van Roosbroeck and Casey, the distinction is made between the behavior of lifetime and relaxation semiconductors. By analogy with Popescu and Henisch, who dealt with injection, the Hall effect also involves "lifetime regimes" and "relaxation regime," depending on whether carriers are augmented or depleted in certain regions. The present analysis, which avoids the *a priori* assumption of space-charge neutrality, is one-dimensional and limited to small magnetic fields. The boundary conditions at the Hall electrodes are assumed to depend on effective surface recombination velocities for electrons and holes of arbitrary value. The results show that the corrective terms involving surface properties and sample size can be of either sign and of magnitude comparable to the value of the Hall voltage calculated by conventional method. As expected, this is most important when the specimen dimensions in the direction of the Hall field are of the same order as the ambipolar diffusion length. The typical examples calculated include high-lifetime germanium.

I. INTRODUCTION

The analysis of problems involving the simultaneous transport of electrons and holes by drift and diffusion is handicapped by the fact that the transport equations defy explicit solution for the general case. The choice has therefore to be made between computer-derived numerical solutions of the complete equations for particular parameters and boundary conditions or else explicit solution of the equations with sufficient *ad hoc* assumptions (e.g., bulk neutrality, zero recombination, freedom from traps, etc.) to permit their solution. The former course is cumbersome and excessively specific for many purposes; the latter dangerous and often misleading, because the precise consequences of *ad hoc* assumptions cannot be easily foreseen. There are, however, instances where apparently innocent "simplifying" assumptions distort the entire physical picture. A third possibility has recently been explored, namely, explicit solutions of the equations in linearized form, but without any other simplifying assumptions.[1] The significance of the results is then limited to the realm of low currents ("small signal theory") but the conclusions are otherwise general.

In the ordinary way, Hall-effect theory treats the specimen as infinite in the direction of the Hall field. An equivalent simplification is to assume that the carrier concentrations are everywhere in equilibrium (lifetime $\tau_0=0$).[2] An alternative simplification is to assume $\tau_0=\infty$.[3] Some attempts are on record[4,5] in which the presence of surfaces (characterized by variable surface recombination velocities) has been taken into account. However, none of the hitherto available solutions has been free from arbitrary assumptions, e.g., concerning neutrality, recombination conditions, dielectric relaxation time, etc. Linearized transport theory, on the other hand, lends itself very well to the analysis of this problem. It can be shown that $E_H=E_{H0}+\Delta E$, where E_{H0} is the Hall field calculated in the conventional way ($\tau_0=0$ or specimen assumed infinite) and ΔE a correction term dependent on specimen size and the nature of the surfaces and the prevailing recombination statistics. The results demonstrate that ΔE can be comparable with E_{H0} and can even exceed it in magnitude. Moreover, ΔE can be of either sign, which means that geometrical, lifetime, and surface considerations can bring about a highly misleading *reversal* of Hall polarity. A series of concentration and field contours is presented, corresponding to cases of special interest, some relating to materials of arbitrary parameters and some to germanium and gallium arsenide. The development of this "low-signal theory" is analogous to the linearized treatment of minority-carrier injection through a boundary, as previously discussed,[1,6,7] along lines based on earlier work by Van Roosbroeck[8,9] and Casey.[9] As far as the concentration contours are concerned, here too the consequences of finite specimen size can be divided into lifetime and relaxation regimes. But for the Hall voltage the corrective terms are most important for the high-lifetime case. The important parameter is the relative width of the specimen (in the Hall-field direction) as compared with the ambipolar diffusion length.

II. HALL EFFECT FOR FINITE SPECIMEN SIZE; SMALL-SIGNAL THEORY

In the one-dimensional analysis which follows, it has been assumed in the first instance that trapping is absent, that the specimen is isotropic, that the electron and hole concentrations are maintained by a bimolecular recombination mechanism which permits a constant (excitation independent) diffusion-length lifetime τ_0 to be defined, and that the Hall mobilities and conductivity mobilities of the carriers are equal. Under the heading of "small-signal theory," the phenomena are further assumed to be linear with magnetic fields. We concern ourselves only with the case of small applied magnetic field. The standard transport relationships for the steady state involve Poisson's equation, two equations for the electron and hole currents, and two continuity relationships. Assuming complete ionization of the impurities levels

$$\frac{\partial E_H(z)}{\partial z}=\frac{q}{\epsilon}\,(p-p_e-n+n_e)\,, \tag{1}$$

$$j_n(z)=q\mu_n nE_H-q\mu_n^2 nBE_x+\mu_n kT\,\frac{\partial n}{\partial z}\;, \tag{2}$$

$$j_p(z)=q\mu_p pE_H+q\mu_p^2 pBE_x-\mu_p kT\,\frac{\partial p}{\partial z}\;, \tag{3}$$

$$\frac{\partial n}{\partial t}=-\frac{np-n_e p_e}{\tau_0(n_e+p_e)}+\left(\mu_n\frac{\partial(nE_H)}{\partial z}-\mu_n^2 BE_x\frac{\partial n}{\partial z}+\frac{\mu_n kT}{q}\frac{\partial^2 n}{\partial z^2}\right)=0\,, \tag{4}$$

$$\frac{\partial p}{\partial t}=-\frac{np-n_e p_e}{\tau_0(n_e+p_e)}-\left(\mu_p\frac{\partial(pE_H)}{\partial z}+\mu_p^2 BE_x\frac{\partial p}{\partial z}-\frac{\mu_p kT}{q}\frac{\partial^2 p}{\partial z^2}\right)=0\,. \tag{5}$$

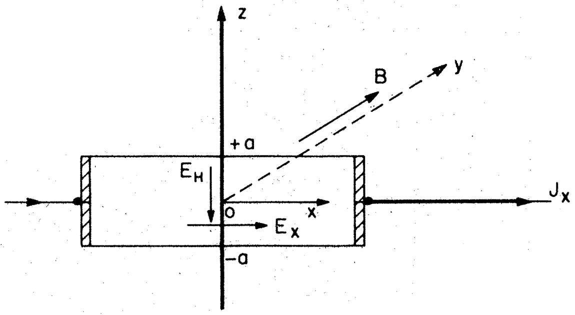

FIG. 1. Schematic sketch of the sample, E_x, B, E_H are, respectively, the longitudinal electric field, the magnetic field, and the resultant Hall field.

E_x is the longitudinal electric field, B the magnetic field, $E_H(z)$ the Hall field, and the remaining symbols have their usual meanings (see also Fig. 1). Writing the departures from the equilibrium concentrations n_e and p_e as $\Delta n=n-n_e$ and $\Delta p=p-p_e$, and neglecting the products which are of the second order in the magnetic field, namely: $\Delta n E_H$, $\Delta p E_H$, ΔnB, and ΔpB and the product $\Delta n\Delta p$ in the recombination term, we obtain

$$\frac{dE_H(z)}{dz}=\frac{q}{\epsilon}(\Delta p-\Delta n)\,, \tag{6}$$

$$j_n(z)=q\mu_n n_e E_H-q\mu_n^2 n_e BE_x+\mu_n kT\,\frac{d\Delta n}{dz}\,, \tag{7}$$

$$j_p(z)=q\mu_p p_e E_H+q\mu_p^2 p_e BE_x-\mu_p kT\,\frac{d\Delta p}{dz}\;, \tag{8}$$

$$\frac{n_e\Delta p+p_e\Delta n}{\tau_0(n_e+p_e)}=\mu_n n_e\frac{dE_H}{dz}+\frac{\mu_n kT}{q}\;\frac{d^2\Delta n}{dz^2}\;, \tag{9}$$

$$\frac{n_e\Delta p+p_e\Delta n}{\tau_0(n_e+p_e)}=-\;\mu_p p_e\frac{dE_H}{dz}+\frac{\mu_p kT}{q}\;\frac{d^2\Delta p}{dz^2}\;. \tag{10}$$

Equations (6) and (10), and equations (6) and (9) give

$$\frac{d^2\Delta p}{dz^2}-\Delta p\left(\frac{q^2p_e}{\epsilon kT}+\frac{qn_e}{kT\mu_p\tau_0(n_e+p_e)}\right)+\Delta n\left(\frac{q^2p_e}{\epsilon kT}-\frac{qp_e}{kT\mu_p\tau_0(n_e+p_e)}\right)=0\,, \tag{11}$$

$$\frac{d^2\Delta n}{dz^2}-\Delta n\left(\frac{q^2n_e}{\epsilon kT}+\frac{qp_e}{kT\mu_n\tau_0(n_e+p_e)}\right)+\Delta p\left(\frac{q^2n_e}{\epsilon kT}-\frac{qn_e}{kT\mu_n\tau_0(n_e+p_e)}\right)=0\,. \tag{12}$$

It will be seen that the last two equations (as a result of linearization) are independent of both the magnetic and longitudinal electric fields. The total transverse current density $j_{T(z)}$ must be zero, which yields

$$q(\mu_n n_e+\mu_p p_e)E_H+qBE_x(\mu_p^2 p_e-\mu_n^2 n_e)+kT\left(\mu_n\frac{d\Delta n}{dz}-\mu_p\frac{d\Delta p}{dz}\right)=0\,. \tag{13}$$

This gives the Hall field $E_{H(z)}$, after solving Eqs. (11) and (12) for $n(z)$ and $p(z)$ and the corresponding gradients. These solutions are

$$\Delta n=-\frac{Q}{p_e}\exp\!\left(\frac{z}{L_D}\right)-\frac{C}{p_e}\exp\!\left(\frac{-z}{L_D}\right)+R(1-A_n)\exp\!\left(\frac{z}{(\tau_0D_a)^{1/2}}\right)+S(1-A_n)\exp\!\left(\frac{-z}{(\tau_0D_a)^{1/2}}\right)\;, \tag{14}$$

$$\Delta p = Q\exp\left(\frac{z}{L_D}\right) + C\exp\left(\frac{-z}{L_D}\right) + R(1-A_p)\exp\left(\frac{z}{(\tau_0 D_a)^{1/2}}\right) + S(1-A_p)\exp\left(\frac{-z}{(\tau_0 D_a)^{1/2}}\right)\ . \tag{15}$$

Q, C, R, and S are integration constants which must be determined by reference to the boundary conditions, and A_n and A_p are defined as follows:

$$A_n = \epsilon/q\mu_n\tau_0(n_e+p_e) = A_p/b\ , \tag{16}$$

$$L_D = [\epsilon kT/q^2(n_e+p_e)]^{1/2}\ , \tag{17}$$

the Debye length, $\epsilon = \epsilon_r\epsilon_0$, ϵ_r being the dielectric constant, and

$$D_a = kT(n_e+p_e)\mu_n\mu_p/q(\mu_n n_e + \mu_p p_e)\ , \tag{18}$$

the ambipolar diffusion constant.

III. BOUNDARY CONDITIONS

At the two surfaces $z=\pm a$, we have the effective surface recombination velocities s_n and s_p. A deeper analysis of surface recombination as such would have to take account of the surface traps, as well as any departures from neutrality at the surface and in its immediate vicinity. However, the concept used here is that of the "*effective* recombination velocity," related to the excess concentrations and the current through Eqs. (19) and (20):

$$j_p(a) = qs_p\Delta p(a) = qs_n\Delta n(a) = -j_n(a)\ , \tag{19}$$

$$j_p(-a) = -qs_p\Delta p(-a) = -qs_n\Delta n(-a) = -j_n(-a)\ . \tag{20}$$

This also means that $\Delta n(a)$ and $\Delta p(a)$ are here the extra carrier concentrations notionally at $z=\pm a$, but in fact just inside the surface by a very small distance. This is the conventional approach, but the contribution (if any) to the total Hall voltage made by that "very small distance" is not included in the considerations. Recombination on other faces is neglected. These approximations are id only for small departures from equilibrium, i.e., for $\Delta p < p_e$ and $\Delta n < n_e$, as already postulated. It should be noted here that the case of symmetric surface recombination dealt with in this paper ensures antisymmetric solutions for $\Delta n(z)$ and $\Delta p(z)$.

Neutrality over the Hall cross section is then obtained

$$\int_{-a}^{+a} (\Delta p - \Delta n)dz = 0\ , \tag{21}$$

leading to

$$E_H(a) = E_H(-a)\ . \tag{22}$$

IV. SOLUTIONS

Application of the above boundary conditions shows at once that $Q=-C$ and $R=-S$. When Q and C are evaluated, we obtain

$$\Delta p = BE_xC_1\left[(1-A_p)\frac{\sinh[z/(\tau_0 D_a)^{1/2}]}{\sinh[a/(\tau_0 D_a)^{1/2}]} + p_e\frac{(1-A_n)s_n-(1-A_p)s_p}{s_n n_e + s_p p_e}\,\frac{\sinh(z/L_D)}{\sinh(a/L_D)}\right]\ , \tag{23}$$

$$\Delta n = BE_xC_1\left[(1-A_n)\frac{\sinh[z/(\tau_0 D_a)^{1/2}]}{\sinh[a/(\tau_0 D_a)^{1/2}]} - n_e\frac{(1-A_n)s_n-(1-A_p)s_p}{s_n n_e + s_p p_e}\,\frac{\sinh(z/L_D)}{\sinh(a/L_D)}\right]\ . \tag{24}$$

Equation (13) gives the Hall field E_H,

$$E_H^{(z)} = E_{H0} + \frac{kT}{q}BE_xC_1\left[\frac{(1-A_n)s_n-(1-A_p)s_p}{(s_n n_e+s_p p_e)L_D}\,\frac{\cosh(z/L_D)}{\sinh(a/L_D)} + \frac{\mu_p-\mu_n}{(\mu_n n_e+\mu_p p_e)(\tau_0 D_a)^{1/2}}\,\frac{\cosh[z/(\tau_0 D_a)^{1/2}]}{\sinh[a/(\tau_0 D_a)^{1/2}]}\right]\ , \tag{25}$$

in which E_{H0} is the standard Hall field, defined for the case of zero lifetime

$$E_{H0} = [(\mu_n^2 n_e - \mu_p^2 p_e)/(\mu_n n_e + \mu_p p_e)]BE_x\ . \tag{26}$$

Taking the zero of potential at $z=0$ we also have $V_H(z) = -\int_0^z E_H(z')dz'$, which gives

$$V_H(z) = -E_{H0}z - \frac{kT}{q}BE_xC_1\left(\frac{(1-A_n)s_n-(1-A_p)s_p}{(s_n n_e+s_p p_e)}\,\frac{\sinh(z/L_D)}{\sinh(a/L_D)} + \frac{\mu_p-\mu_n}{\mu_n n_e+\mu_p p_e} + \frac{\sinh[z/(\tau_0 D_a)^{1/2}]}{\sinh[a/(\tau_0 D_a)^{1/2}]}\right)\ . \tag{27}$$

In these relations, C_1 is given by

$$C_1 = \frac{(\mu_n+\mu_p)\mu_n\mu_p n_e p_e/(\mu_n n_e+\mu_p p_e)(1-\mathfrak{a})}{s_n s_p(n_e+p_e)/(s_n n_e + s_p p_e) + D_a/(\tau_0 D_a)^{1/2}\coth a/(\tau_0 D_a)^{1/2}}\ , \tag{28}$$

with

$$\mathcal{A}=\frac{L_D^2}{\tau_0 D_a}=\frac{(b+p_e)A_n}{(1+p_e)}=\frac{(b+p_e)A_p}{(b+bp_e)} \quad . \tag{29}$$

In the Appendix, we give details of the procedure for the case with traps, but it can, of course, be applied to the trap free case also. The following comments refer to these equations: (i) The carrier concentration $\Delta n(z)$ and $\Delta p(z)$ and the potential $V_H(z)$ are odd functions of z. The Hall field $E_H(z)$ is an even function. (ii) The equations suggest that there is a discontinuity for $\mathcal{A}=1$, but this is not so, as numerator and denominator both go to zero. (iii) It will be seen from Eqs. (23) and (24) that in the region where the carriers are expected to accumulate under the action of the magnetic force, $(z>0)$, Δn and Δp can in fact be either positive or negative, depending on the values of $\mathcal{A}$, A_n, A_p, s_n, and s_p. There is therefore an analogy between the present situation and the case of minority carrier injection through a boundary.[1,6,7] Here, for n-type material $\Delta p>0$, $\Delta n>0$ is analogous to a "lifetime regime," and $\Delta p>0$, $\Delta n<0$ to a "relaxation regime" (see Fig. 2). It will be seen from Eq. (23) that the condition $A_p=1$ together with $s_n=0$ represent a boundary, for which Δp is zero, irrespective of z and Δn. Correspondingly, $A_n=1$, and $s_p=0$ is the boundary for $\Delta n=0$, irrespective of z and Δp. Similar boundary conditions can be devised for the case of minority carrier injection through a boundary[1]. (iv) For the special case where the surface densities are equal $\Delta n(a)=\Delta p(a)$, corresponding to $s_n/s_p=s=1$, one can show that neutrality prevails everywhere if $\mu_n/\mu_p=b=1$ leading to $E_H(z)=E_{H0}$ for any value of z. Thus, $s=b=1$, a very special case, is equivalent to making the specimen infinite in the z direction. (v) For $a\gg(\tau_0 D_a)^{1/2}$ and L_D, the consequences of the finite sample width are important only in the vicinity of $z=\pm a$. In these cases simplifications can occur, as the terms involving hyperbolic functions in Eqs. (23)–(25) and (27) become, respectively, $\exp[(z-a)/(\tau_0 D_a)^{1/2}]$ and $\exp[(z-a)/L_D]$.

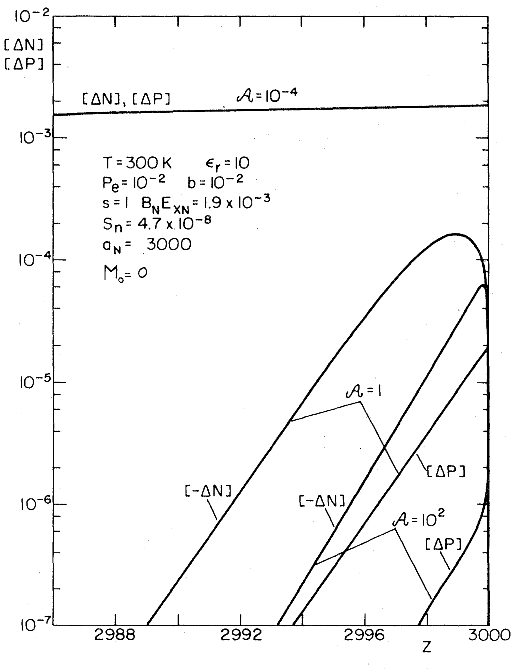

FIG. 2. Concentration contours near the plane $Z=a_N$. n-type semiconductor; $n_e=10^{11}\text{cm}^{-3}$, $P_e=10^9\ \text{cm}^{-3}$. The three values of $\mathcal{A}$, 10^{-4}, 1, 10^2 correspond to lifetimes of $\tau_0\simeq 1.08\times10^{-2}$, 1.08×10^{-6}, and 1.08×10^{-8} sec respectively.

V. HALL VOLTAGE

We have

$$V_H=V_H(a)-V_H(-a)\,, \tag{30}$$

where $V_H(a)$ and $V_H(-a)$ come from Eq. (27). The effects here discussed can best be presented as the ratio of the real Hall voltage V_H to the standard Hall voltage V_{H0} (for zero lifetime), namely

$$V_{H0}=-2aBE_x\left(\frac{\mu_n^2 n_e-\mu_p^2 p_e}{\mu_n n_e+\mu_p p_e}\right) \quad . \tag{31}$$

Thus,

$$\frac{V_H}{V_{H0}}=1+\frac{n_e p_e(\mu_n+\mu_p)(s_n\mu_p-s_p\mu_n)}{(p_e s_p+n_e s_n)(\mu_n^2 n_e-\mu_p^2 p_e)} \times\left\{\frac{qa}{kT}\left(\frac{s_n s_p(\mu_n n_e+\mu_p p_e)}{\mu_n\mu_p(p_e s_p+n_e s_n)}\right) + \frac{a}{(\tau_0 D_a)^{1/2}}\left[\coth\left(\frac{a}{(\tau_0 D_a)^{1/2}}\right)\right]\right\}^{-1} \quad . \tag{32}$$

The following conclusions arise from this equation.

(a) It will be clear by inspection, that the correction term is important when one is dealing with material, for which the normal Hall effect is small, by virtue of $\mu_n^2 n_e-\mu_p^2 p_e\approx 0$. In particular, this would be true for near-intrinsic materials for which the mobility ratio happens to be close to unity.

(b) The sign of the correction term depends on $(s_n\mu_p-s_p\mu_n)$. If $s_n\mu_p-s_p\mu_n=0$, and only then, we have $V_H=V_{H0}$, which means that the finite size of the semiconductor does not enter into the considerations. This is not in conflict with the statement under (iv) above, since we are here concerned with the field integral. Even when E_H is not everywhere equal to E_{H0} (case corresponding to $s=b\neq 1$) the integrals can be the same.

(c) For a large effect, the denominator in Eq. (32) must be small. Of its two terms, the second is the more sensitively variable; that term must therefore be small. It is easy to show that $[a/(\tau_0 D_a)^{1/2}]\coth[a/(\tau_0 D_a)^{1/2}]$ is always greater than one and its smallest value is in fact unity. For that term to be close to unity, it is necessary that the length a be smaller than or of the same order as $(\tau_0 D_a)^{1/2}$, i.e., the width of the sample in the direction of the Hall field should be comparable with the ambipolar diffusion length. To obtain this with a sample of convenient operational size, the diffusion length lifetime should be high. (Alternatively, the carrier mobilities should have high values.)

(d) It will be shown in the Appendix, that trapping considerations do not change the Hall voltage V_H, everything else being equal. Concentrations and field contours are modified, but not the integral between $z=-a$ and $z=+a$ of the Hall field V_H.

VI. NORMALIZATION

The complexity of the above equations makes it desirable to use them in normalized form. Normalized versions are presented below, covering in fact the more general case of a material with a single set of recombination centers. They correspond to the above equations by putting the normalized recombination center density $M_0(=N_t/n_e)$ equal to zero. The symbols are defined as follows:

$$P_e=\frac{p_e}{n_e},\quad N_e=\frac{n_e}{n_e}=1,\quad a_N=\frac{a}{L_D},$$
$$V_N=\frac{qV}{kT},\quad Z=\frac{z}{L_D},\quad E_N=E\frac{qL_D}{kT},$$
$$J=j\frac{L_D}{\mu_p kT(n_e+p_e)},\quad B_N=B\mu_p, \tag{33}$$
$$S_{n,p}=\frac{s_{n,p}L_D n_e}{(n_e+p_e)\frac{\mu_p kT}{q}},\quad s=\frac{s_n}{s_p}=\frac{S_n}{S_p}.$$

Quantities whose real values are represented by lower case letters are given in their normalized form either as capitals, or else with the subscript N. With these the equations become(see Appendix):

$$\Delta P(Z)=C_2\left[(1+\beta-A_p)\frac{\sinh(\mathcal{A}^{1/2}Z)}{\sinh(\mathcal{A}^{1/2}a_N)}-P_e\frac{[(1+\beta-A_p)-(1+\alpha-A_n)s]}{P_e+s}\frac{\sinh(\eta^{1/2}Z)}{\sinh(\eta^{1/2}a_N)}\right], \tag{34}$$

$$\Delta N(Z)=C_2\left[(1+\alpha-A_n)\frac{\sinh(\mathcal{A}^{1/2}Z)}{\sinh(\mathcal{A}^{1/2}a_N)}+\frac{[(1+\beta-A_p)-(1+\alpha-A_n)s]}{P_e+s}\frac{\sinh(\eta^{1/2}Z)}{\sinh(\eta^{1/2}a_N)}\right], \tag{35}$$

$$E_{HN}^{(Z)}=E_{HON}+C_2\left[\mathcal{A}^{1/2}\frac{(1+\beta)-b(1+\alpha)}{b+P_e}\frac{\cosh(\mathcal{A}^{1/2}Z)}{\sinh(\mathcal{A}^{1/2}a_N)}-\eta^{1/2}\frac{(1+\beta-A_p)-(1+\alpha-A_n)s}{P_e+s}\frac{\cosh(\eta^{1/2}Z)}{\sinh(\eta^{1/2}a_N)}\right], \tag{36}$$

$$V_{HN}(Z)=-E_{HON}Z-C_2\left[\frac{(1+\beta)-b(1+\alpha)}{b+P_e}\frac{\sinh(\mathcal{A}^{1/2}Z)}{\sinh(\mathcal{A}^{1/2}a_N)}-\frac{(1+\beta-A_p)-(1+\alpha-A_n)s}{P_e+s}\frac{\sinh(\mathcal{A}^{1/2}Z)}{\sinh(\eta^{1/2}a_N)}\right], \tag{37}$$

$$\frac{V_{HN}}{V_{HON}}=1+\frac{[P_e(1+b)(s-b)]/[(P_e+s)(b^2-P_e)]}{a_N(1+P_e)(b+P_e)S_n/b(P_e+s)+a_N\mathcal{A}^{1/2}\coth(\mathcal{A}^{1/2}a_N)}, \tag{38}$$

$$E_{HON}+B_N E_{XN}\frac{(b^2-P_e)}{(b+P_e)}, \tag{39}$$

being the normalized Hall field, for zero lifetime material, and (see Appendix for-the definition of the traps parameter N_1, P_1, τ)

$$\alpha=M_0\frac{\tau/(1+N_1)}{1+N_1+\tau(P_e+P_1)},\quad \beta=M_0\frac{P_e/(P_e+P_1)}{1+N_1+\tau(P_e+P_1)},\quad \eta=\frac{1+\beta+P_e(1+\alpha)}{1+P_e},$$
$$C_2=B_N E_{XN}\frac{P_e(1+b)/(\eta-\mathcal{A})(1+P_e)}{[(1+P_e)(b+P_e)/b][S_N/(P_e+s)]+\mathcal{A}^{1/2}\coth(\mathcal{A}^{1/2}a_N)}. \tag{40}$$

VII. COMMENTS

A. Zero lifetime limit

When $\tau_0 \to 0$, the three quantities $\mathcal{A}$, A_n, $A_p \to \infty$. It can be easily shown that Eq. (36) then gives

$$E_{HN}(z) \approx E_{HON} + \text{const}/\mathcal{A}^{1/2},$$

and so $E_{HN} \approx E_{HON}$ and $V_{HN} \approx V_{HON}$. We obtain thus the well known relationship (39) corresponding to a zero lifetime semiconductor. Carrier equilibrium prevails everywhere ($\Delta N = \Delta P = 0$).

B. Infinite lifetime limit

If $\tau_0 \to \infty$, then $\mathcal{A}$, A_n, $A_p \to 0$. In practice this case implies $M_0 = 0$, i.e., $\eta = 1$, $\alpha = \beta = 0$ [cf. Eqs. (40)], and $S_n = S_p \to 0$. Using appropriate asymptotic limits for the hyperbolic functions, we obtain via Eq. (36),

$$E_{HN} = B_N E_{XN} [(b - P_e)/(1 + P_e)], \tag{41}$$

or in unnormalized form:

$$E_H = B E_x [(\mu_n n_e - \mu_p P_e)/(n_e + P_e)]$$

This equation was first derived (using a different method) by Fowler.[3]

VIII. RESULTS AND DISCUSSION

A number of cases have been quantitatively evaluated for illustrative purposes: Figure 2 shows the variation of the carrier densities as a function of Z, close to $Z = a_N$, when the parameter $\mathcal{A}$ is varied between 10^{-4} and 10^2. The three values correspond to lifetimes of 1.08×10^{-2}, 1.08×10^{-6}, and 1.08×10^{-8} sec, respectively. The Debye length is 1.19×10^{-3} cm, and the surface recombination is assumed to be very low. The semiconductor is n-type, ΔN and ΔP are even functions of Z [cf. Eqs. (34) and (35)]. It will be seen that $\mathcal{A} = 10^{-4}$ corresponds to a lifetime regime; ΔN and ΔP are nearly equal right to the boundary $Z = a_N$. Nevertheless (though this cannot be seen on Fig. 2), the corrective term $\Delta P - \Delta N$ is here more important in influencing the Hall voltage than it is for the other cases, characterized by much smaller concentration increments. Conversely, for $\mathcal{A} = 10^2$, i.e., low carrier lifetimes, we have a relaxation regime, i.e., majority carrier depletion. This occurs everywhere in the bulk, except in the immediate surface region where the boundary conditions demand $\Delta P(a) = \Delta N(a)$. The carrier concentrations are governed by "zero recombination," i.e., $P_e \Delta N + N_e \Delta P = 0$, and this is again analogous to the injection of minority carriers into a relaxation semiconductor.[1,6,9]

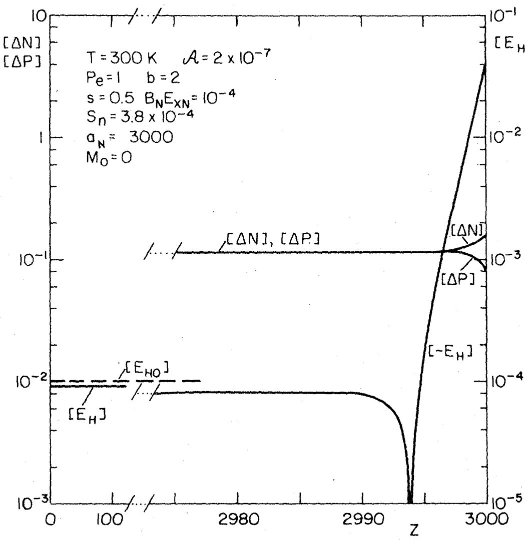

FIG. 3. Concentration and field contours near the plane $Z = a_N$. Intrinsic high lifetime germanium; diffusion length lifetime $\tau_0 \simeq 380$ μsec; ambipolar diffusion length $(\tau_0 D_a)^{1/2} \simeq 0.154$ cm, $s_p > s_n$.

Figures 3–6 correspond to intrinsic germanium of high lifetime. We have $\mathcal{A} = 2 \times 10^{-7}$ in all cases, with a lifetime of $\tau_0 = 380$ μsec. The Debye length is $L_D \approx 6.9 \times 10^{-5}$ cm and the ambipolar diffusion length $(\tau_0 D_a)^{1/2} \simeq 0.154$ cm. The sample width in the z direction, $2a = 0.415$ cm, is of the same order as the ambipolar diffusion length, to illustrate the

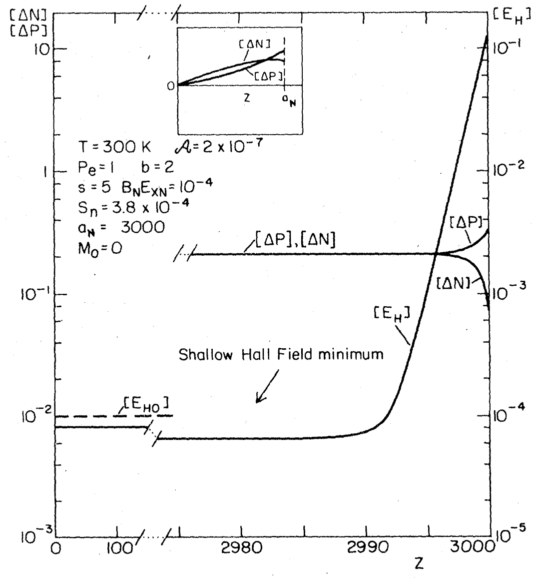

FIG. 4. Concentration and field contours near the plane $Z = a_N$. Intrinsic high lifetime germanium; $\tau_0 \simeq 380$ μsec $(\tau_0 D_a)^{1/2} \simeq 0.154$ cm, $s_p < s_n$. Inset: Schematic carrier concentration showing a crossover corresponding to a Hall-field E_H minimum.

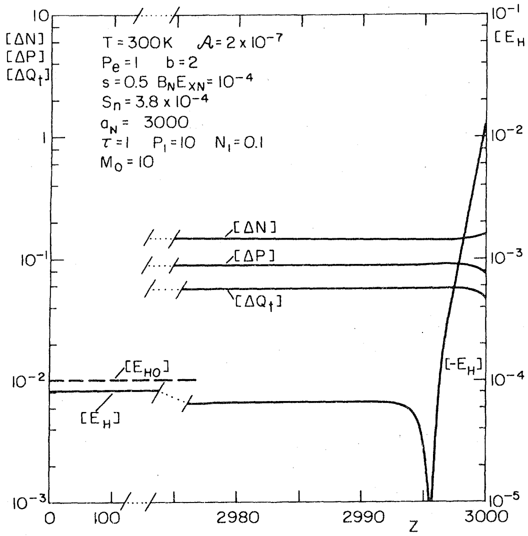

FIG. 5. Concentration and field contours near the plane $Z = a_N$. Intrinsic *compensated* germanium; high lifetime. Single level of recombination centers below the Fermi level, $N_t = 2.4 \times 10^{14}$ cm^{-3}. $\tau_0 \simeq 380$ μsec, $(\tau_0 D_a)^{1/2} \simeq 0.154$ cm, $s_p > s_n$.

importance of the correction effects in that situation.

Figures 3 and 4 relate to a case without traps, for varying ratios of the surface recombination velocities. The effective surface recombination velocities themselves are assumed to be high; $s_p = 1000$ cm sec^{-1} and $s_n = 500$ cm sec^{-1} (cf. Fig. 3) but entirely plausible, e.g., for grounded Ge surfaces. In this case, $V_{HN}/V_{HON} \simeq 0.74$, with $V_{HON} = -0.6$ as evaluated using Eq. (38). This gives $V_{HO} = -15$ mV. The measured Hall voltage is thus about 74% of the value which would correspond to zero lifetime. Figure 3 shows the carrier concentrations and the field contour. It will be seen that a field inversion occurs within a few Debye lengths of the surface. Nevertheless, the contribution which the immediate surface region makes to the field integral (the Hall voltage) is by no means negligible.

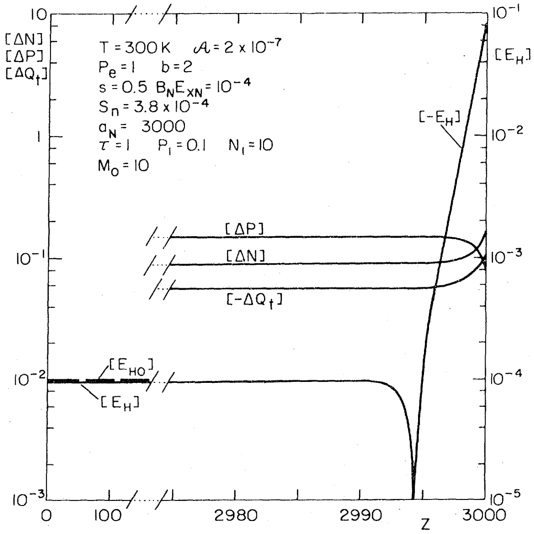

FIG. 6. Concentration and field contours near the plane $Z = a_N$. Intrinsic *compensated* germanium, high lifetime. Single level of recombination centers above the Fermi level, $N_t = 2.4 \times 10^{14}$ cm^{-3}. $\tau_0 \simeq 380$ μsec, $(\tau_0 D_a)^{1/2} \simeq 0.154$ cm, $s_p > s_n$.

Figure 4 represents essentially the same case, but with the ratio of surface recombination velocities reversed: $s_n = 500$ cm sec^{-1}, $s_p = 100$ cm sec^{-1}. For the measured Hall voltage, this situation yields $V_{HN}/V_{HON} \approx 1.24$, and the same value of $V_{HO} = -15$ mV. Comparison with Fig. 3 shows that close to the surface the relative magnitudes of ΔN and ΔP are now reversed. Over most of the specimen ΔN is nearly equal to ΔP, but actually greater, to an extent which cannot be seen on the scale of the representation in Fig. 4. The insert shows this situation schematically. There is a crossover in the concentration contours, corresponding to a field minimum.

Figures 5 and 6 deal also with a case similar to that in Fig. 3, except that traps are now envisaged, in Fig. 5 below the Fermi level, in Fig. 6, above. Trap density: $N_t = 2.4 \times 10^{14}$ cm^{-3}. The quantities N_1, P_1, and τ which characterize the traps according to the Shockley-Read model,[10] are further discussed in the Appendix. An adequate concentration of donor or acceptor levels is assumed to be present, so as to keep the material intrinsic. The concentration contours are now no longer coincident in the bulk, because quasi-neutrality is maintained through ΔQ_t, the charge in traps. As between Figs. 5 and 6, the relative magnitudes of ΔN and ΔP are again reversed. When the recombination centers are located below the Fermi level, they fill preferentially with holes $\Delta Q_t > 0$ (see Fig. 5). The reverse situation prevails when the recombination center is above the Fermi level. It is important to note that, in both cases, the measured Hall voltage would be

$$V_{HN} \simeq 0.74 V_{HON},$$

i.e., equal to the value without traps (Fig. 3). This reflects the properties of Eq. (32) or (38).

Figure 7 corresponds to the case of intrinsic gallium arsenide, for a sample width $2a \simeq 0.84$ cm in the z direction. The Debye length is here $L_D \simeq 8.4 \times 10^{-2}$ cm. This means that the specimen is only a few Debye lengths wide, instead of thousands, as above. The value of 𝒜 has been chosen to be 10^2, corresponding to a lifetime of 3.6 μsec. The two surface recombination velocities are $s_p = 1000$ cm sec^{-1} and $s_n = 500$ cm sec^{-1}. The calculated value of $V_{HN}/V_{HON} = 0.99$ for these parameters. $[V_{HO} = 0.51$ V]. The correction term is therefore very weak. This is so because the low lifetime makes the ambipolar diffusion length small and, in this case, much smaller than the specimen dimensions. It will be seen that ΔN and ΔP are nevertheless highly nonuniform. The relative magnitudes of ΔN and ΔP are again governed by the "zero net recombination" condition, i.e., $P_e \Delta N + N_e \Delta P = 0$, that is, $\Delta N = -\Delta P$ for an intrinsic sample. Near the boundary the conditions for surface recombination, Eqs. (19) and (20), lead to a reversal of sign for ΔP. Figure 8 will serve as a schematic summary of these relationships, in terms of V_{HN}/V_{HON}. This ratio is plotted as a function of $a_N \mathcal{A}^{1/2} = a/(\tau_0 D_a)^{1/2}$ for several values of s and b. As discussed in connection with Eq. (32) or (38), $a/(\tau_0 D_a)^{1/2}$ is the important quantity here. We have seen that for very low lifetime (high value of $a/(\tau_0 D_a)^{1/2}$), $V_{HN} \simeq V_{HON}$. This is as expected, because the real width of the sample is then much greater than the ambipolar diffusion length. The correction terms become unimportant. On the other hand, when the lifetime is high, the correction terms, which may be positive or negative, are high also. As an example: for $n_e = p_e = 7 \times 10^8$ cm^{-3}, $T = 300$ K, $\epsilon_r = 10$, L_D is equal to 10^{-2} cm. For $\mu_p = 2000$ cm^{2}V^{-1}sec^{-1}, $b = 2$, $a_N = 20$; $a_N \mathcal{A}^{1/2} = 1$ corresponds to $\tau_0 \simeq 590$ μsec and $(\tau_0 D_a)^{1/2} \simeq 2 \times 10^{-1}$ cm and $a_N \mathcal{A}^{1/2} = 100$ to $\tau_0 \simeq 5.9 \times 10^{-8}$ sec and $(\tau_0 D_a)^{1/2} \simeq 2 \times 10^{-3}$ cm. For the curve on Fig. 8 which relates to $b = 1.1$ and $s = 0.5$, (other parameters constant), the measured Hall effect should reverse when the lifetime goes up from a few tenths of a microsecond to a few hundred microseconds. The measured Hall voltage V_{HN} as such is then low ($V_{HO} \simeq -1$ mV) but the corrective factor is substantial, and the total result highly misleading.

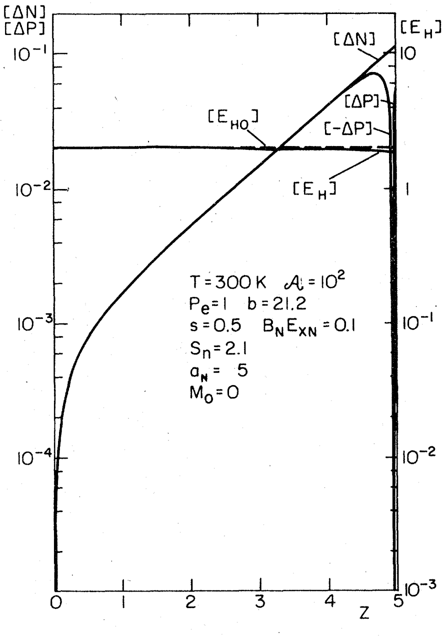

FIG. 7. Concentration and field contours along an intrinsic Ga-As sample, $\tau_0 \simeq 3.6$ μsec, $(\tau_0 D_a)^{1/2} \simeq 8.4 \times 10^{-3}$ cm, $s_n < s_p$.

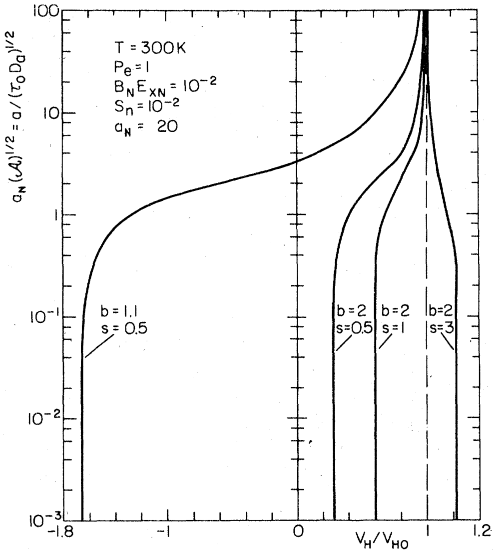

FIG. 8. The ratio V_{HN}/V_{HON} as a function of $a_N \mathcal{A}^{1/2} = a/(\tau_0 D_a)^{1/2}$, for several values of the parameters s and b. $b = 2$; $s = 3$, 1, 0.5; $V_{HON} = -4 \times 10^{-1}$, i.e., $V_{HO} = -10$ mV. $b = 1.1$; $s = 0.5$; $V_{HON} = -4 \times 10^{-2}$, i.e., $V_{HO} = -1$ mV.

IV. CONCLUSIONS

The results show that, as one would expect, the corrective terms arising from finite specimen size and surface properties are important when the specimen dimensions in the direction of the Hall field are comparable to the ambipolar diffusion length. In the ordinary way, this could be so only in high lifetime materials and this is as far as the present model goes. However, the ultimate range of the theory may be wider. Thus it is conceivable that the n-type Hall effect observed in high resistivity glasses[11] (which are otherwise p-type conducting) might also be interpreted along the present lines. This does not follow immediately from the above, because the model, in its present state, does not distinguish between recombination centers and trapping centers. However, such a distinction could be introduced and corresponds, indeed, to observational necessities. Its consequences cannot be intuitively predicted with high reliability, but a more elaborate model which includes trapping as well as recombination will also multiply the circumstances under which Hall effect reversals can occur.

ACKNOWLEDGMENTS

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APPENDIX: SMALL SIGNAL THEORY IN THE PRESENCE OF A SINGLE MONOVALENT RECOMBINATION LEVEL

The presence of a recombination center modifies the transport equations through the recombination term and Poisson's equation. For Shockley-Read recombination,[10] the recombination rate is of the form:

$$R=(np-n_e p_e)/[\tau_{no}(p+p_1)+\tau_{po}(n+n_1)]\,, \tag{42}$$

where τ_{no}, τ_{po}, $\tau=\tau_{no}/\tau_{po}$, n_1, and p_1 have their usual meaning according to this model. The assumption of a fixed $\Delta n/\Delta P$ ratio through the operation of constant carrier lifetimes[12] is here avoided. Within the limitations of a "small signal theory" Eq. (42) reduces to

$$R=(\Delta n p_e+\Delta p\, n_e)/\tau_0(n_e+p_e)\,, \tag{43}$$

with

$$\tau_0(n_e+p_e)=\tau_{po}[(n_e+n_1)+\tau(p_e+p_1)]\,. \tag{44}$$

Equation (44) relates the value of τ_0: "diffusion length lifetime" defined by Van Roosbroeck[8] to the trap parameters. Following the lines discussed previously[1,7] and using the normalization relations defined by Eq. (33) it can be shown that Poisson's equation becomes

$$\frac{dE_{HN}(Z)}{dZ}=\frac{1}{1+P_e}(\Delta P-\Delta N+\Delta Q_t)\,, \tag{45}$$

in which the space charge arising from nonequilibrium occupation of the recombination centers is given by

$$\Delta Q_t=\alpha\Delta P-\beta\Delta N\,, \tag{46}$$

with α and β given by Eq. (40)

$$\frac{d^2\Delta N}{dZ^2}-\Delta N\frac{P_e A_n+1+\beta}{1+P_e}-\Delta P\frac{A_n-1-\alpha}{1+P_e}=0\,. \tag{47}$$

In a similar way

$$\frac{d^2\Delta P}{dZ^2}-\Delta N\frac{A_p-1-\beta}{1+P_e}P_e-\Delta P\frac{A_p+P_e+\alpha P_e}{1+P_e}=0\,. \tag{48}$$

These two differential equations are readily solved by differentiating equation (48) twice, and using Eq. (47). We obtain:

$$\frac{d^4\Delta P}{dZ^4}-(\eta+\mathcal{A})\frac{d^2\Delta P}{dZ^2}+\eta\mathcal{A}\Delta P=0\,. \tag{49}$$

In the above, use has also been made of relations (29) and (40). A similar expression involving ΔN can be obtained along the same lines. The solutions are

$$\begin{aligned}\Delta P={}&Q\exp(\eta^{1/2}Z)+C\exp(-\eta^{1/2}Z)\\&+R(1+\beta-A_p)\exp(\mathcal{A}^{1/2}Z)\\&+S(1+\beta-A_p)\exp(-\mathcal{A}^{1/2}Z),\end{aligned} \tag{50}$$

$$\begin{aligned}\Delta N={}&-\frac{Q}{p_e}\exp(\eta^{1/2}Z)-\frac{C}{p_e}\exp(-\eta^{1/2}Z)\\&+R(1+\alpha-A_n)\exp(\mathcal{A}^{1/2}Z)\\&+S(1+\alpha-A_n)\exp(-\mathcal{A}^{1/2}Z)\,.\end{aligned} \tag{51}$$

The total transverse current density $J_T(Z)$ must be zero, leading to the Hall field $E_{HN}(Z)$:

$$\begin{aligned}E_{HN}(Z)=\frac{1}{b+P_e}\bigg(&\frac{d\Delta P}{dZ}-b\frac{d\Delta N}{dZ}\\&+B_N E_{XN}(b^2-P_e)\bigg)\,.\end{aligned} \tag{52}$$

Using boundary conditions similar to Eqs. (19) and (20), Eqs. (34)–(38) in the text are obtained.

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