# Electron-electron scattering in transition metals

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Results of temperature-dependent parallel-field radio-frequency-size-effect measurements on rhenium and temperature-dependent resistance measurements on molybdenum are reported. The present data along with existing data on radio-frequency-size-effect and resistance measurements on molybdenum, tungsten, and rhenium show that a definite correlation between the two types of measurements exists. A discussion of the implication of these results on the observation of electron-electron scattering in transition metals is given.

For many years the low-temperature electrical resistivity of transition metals has been found to obey a $T^2$ law. While electron-electron ($e$-$e$) scattering should give rise to a $T^2$ dependence of resistance, simple theories of this scattering process do not readily lend themselves to an evaluation of the magnitude of the $e$-$e$ scattering frequency. Other mechanisms involving transport processes have been proposed to explain the observed $T^2$-dependent results.[1] A $T^2$ dependence has also been observed in the amplitude of the radio-frequency-size-effect (RFSE) signals[2] in W and Mo.[3] In the RFSE the temperature dependence of the amplitude of the signal arises from an isolated group, or orbit, of electrons on a particular sheet of the Fermi surface (FS) and is not affected by complicated transport effects.

We report the results of two measurements: (i) temperature-dependent measurements of the amplitude of parallel-field RFSE signals in rhenium and (ii) temperature-dependent measurements of the ideal electrical resistivity of molybdenum. In both of these measurements a $T^2$ dependence is observed in the temperature range 1–4 °K. While a $T^2$ dependence of the resistivity in Mo has previously been observed[4] no quantitative results were given. We demonstrate that in the case of molybdenum, tungsten, and rhenium the RFSE and resistance measurements correlate very well.

## I. RFSE IN RHENIUM

Radio-frequency-size-effect measurements were performed on a sample 0.231 mm thick ($R_{300}/R_{4.2} \simeq 20\,000$) and having $\hat{n} \| [11\bar{2}0]$, where $\hat{n}$ is a unit normal to the surface of the thin slab sample. Temperature-dependent data were taken on the signal arising from an extremal orbit centered around the $A$ point in the Brillouin zone of the hcp structure on the eighth-band electron sheet of the FS when the magnetic field was directed 22° from [0001] toward $[10\bar{1}0]$.[5] This was the strongest signal observed at any orientation of $\vec{H}$ in this sample and for this sample thickness it occurs at fields (400 G) above the superconducting critical field for all temperatures investigated. The measurement apparatus was that used previously in this laboratory.[6]

For mean free paths of the order of the sample thickness the parallel-field RFSE amplitude is given by

$$\mathcal{A} = \mathcal{A}_0 e^{-\pi\nu/\Omega}, \tag{1}$$

where $\nu$ is the scattering frequency for the carriers causing the signal and $\Omega$ is their cyclotron frequency. Since the results are reported for the thickest sample from our crystal on which the RFSE could be observed, the above single-pass-limit formula is appropriate.[7] The temperature dependence of $\nu$ is determined by a measurement

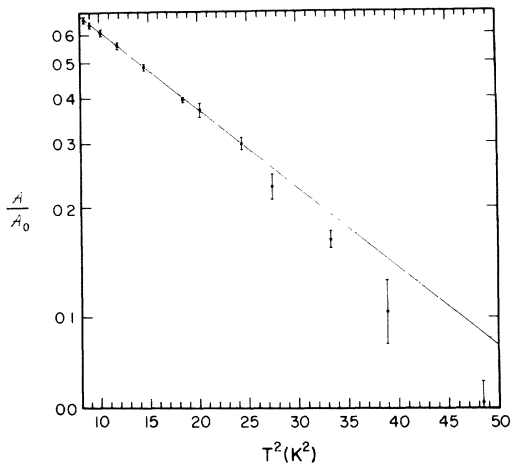


FIG. 1. $\mathcal{A}/\mathcal{A}_0$ vs $T^2$. Temperature dependence of the amplitude of a parallel field RFSE signal in rhenium. The data show a $T^2$ dependence of $\ln\mathcal{A}/\mathcal{A}_0$ up to 5 °K. Each point represents four measurements with the error bars representing one standard deviation. From ~5 to ~7 °K the data fits a $T^3$ law, indicative that electron-phonon scattering becomes the dominant factor in this temperature range. Above 7 °K no RFSE signals are observed.

of the temperature dependence of $\ln\mathcal{A}$. A plot of the logarithm of the signal amplitude versus $T^2$ is shown in Fig. 1. The equation $\ln\mathcal{A}/\mathcal{A}_0 = -(\pi/\Omega)\alpha T^2$ can be fit to this data below 5 °K. In order to determine the value of $\nu/T^2$ from this data, the cyclotron frequency for this orbit at the value of the applied field of the RFSE signal must be known.

We determined the effective mass for this orbit from Azbel-Kaner geometry cyclotron resonance measurements. For these measurements the same sample used in the RFSE measurements covered a small hole in the end of a $TE_{103}$, 35-GHz resonant cavity of a microwave spectrometer.[8] Six subharmonic resonant peaks for this mass were observed at 1.2 °K and were the strongest signals observed over the field range investigated (2.5–12.5 kG). The measured value of the effective mass for this orbit is $m/m_0 = 1.6$. Writing the scattering frequency for this orbit as $\nu(T) = \alpha T^2$, and incorporating the measured effective mass into Eq. (1), we obtain a value of $\alpha = 7.1 \times 10^{-7}/\text{sec K}^2$.

## II. RESISTANCE OF MOLYBDENUM

The resistances of two samples of Mo were measured between 1.2 and 4.2 °K. Both samples were rectangular parallelepipeds. Sample 1 had a cross-sectional area of $0.54 \times 0.85$ mm and sample 2 had a cross-sectional area of $0.91 \times 2.36$ mm. Each sample was about 3 cm long with the [111] crystallographic axis oriented along the length. The measurements of the ideal resistivity of Mo were made with a four-point technique using a superconducting chopper amplifier (Keithley Model 800 picovoltmeter) as the voltage sensor. Contact resistance was reduced by spot welding of the leads and was found to be unimportant from current versus voltage curves for the range of temperatures investigated. The results of the resistance of both of these samples versus $T^2$ are plotted in Fig. 2. As can be seen, a linear relationship is obtained with the difference in slopes being within experimental error (±5%).

The resistance ratio of these two samples is approximately the same, $R_{300}/R_{4.2} \approx 6500$. Since the coefficient of the $T^2$ dependence of these two samples, which vary widely in dimensions and therefore size-limiting mean free paths, is the same, the observed coefficient of the $T^2$ term in the resistivity cannot be due to sample size effects. In the temperature range of 1–4 °K the resistance of Mo obeys the law

$$\rho = \rho_0 + A T^2 , \tag{2}$$

with $A = 0.126 \times 10^{-11}\ \Omega\,\text{cm/K}^2$ independent of sample size. Volkenshtein *et al.*[4] have shown that $A$ does not change appreciably between samples having

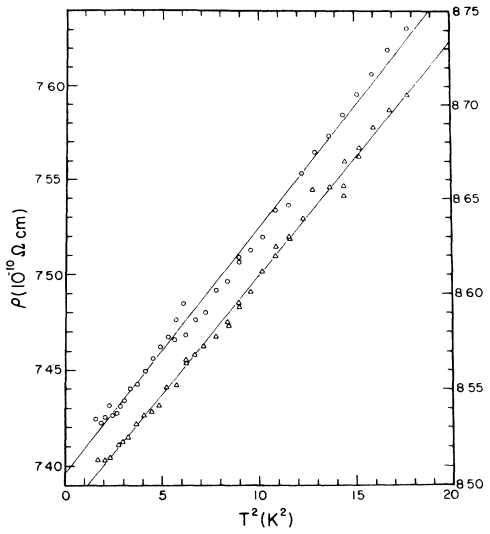


FIG. 2. $\rho$ vs $T^2$. Temperature dependence of the resistivities of two molybdenum samples. For the larger sample (denoted by ○, scale to left) $A = 0.129 \times 10^{-11}\ \Omega\,\text{cm/K}^2$ and for the smaller sample (denoted by △, scale to right) $A = 0.124 \times 10^{-11}\ \Omega\,\text{cm/K}^2$.

$R_{300}/R_{4.2} = 5000$ to $R_{300}/R_{4.2} = 20\,000$. Since our samples fall within this range and show no size dependence, the observed value of $A$ appears to be intrinsic to the $e$-$e$ scattering process.

## III. RESULTS

The results of these experiments and the results of RFSE measurements on tungsten and molybdenum as well as resistance measurements on tungsten and rhenium are shown in Table I. It can be seen that the correlation between the RFSE results ($\alpha$) and the resistance results ($A$) are good. The averaged values of $\alpha$ for tungsten are slightly less than for molybdenum and both are a factor of 3–4 less than for rhenium. The same holds true for the

TABLE I. Coefficients of the temperature-squared-dependent scattering frequencies and resistances for W, Mo, and Re.

| Metal | $10^7\sigma$ ($\text{K}^{-2}\,\text{sec}^{-1}$) | $10^{11}A$ ($\Omega\,\text{cm/K}^2$) |
|---|---|---|
| W | 2.0 [a] | 0.055 [b] |
| Mo | 2.4 [a] | 0.126 [c] |
| Re | 7.1 [c] | 0.45 [d] |

[a] From Ref. 3. These are average values for the eight orbits examined with unit weightings for each.

[b] E. L. Stone, M. D. Ewbank, J. Bass, and W. P. Pratt, Jr., Phys. Lett. A 59, 168 (1976).

[c] Present results.

[d] J. T. Shriempf, J. Phys. Chem. Solids 28, 2581 (1967).

coefficient of the $T^2$ term of the resistance of these three metals. It is seen that for these three metals the $T^2$ dependence of the resistance is a reasonable indicator of an intrinsic property of the scattering cross section for conduction electrons.

## IV. DISCUSSION

The present results suggest that when a $T^2$ dependence of the resistivity of a metal is observed, the slope of this dependence is indicative of the strength of the $e$-$e$ scattering. The coefficients of the $T^2$ dependence of the low-temperature resistities of the transition metals: V, Fe, Co, Zr, Ni, Nb, Ru, Pd, W, Re, Os, Ir, and Pt have been reported in the literature. These coefficients are shown in Table II. An important feature of this table is that a $T^2$ dependence has been observed in all transition metals which have been subjected to careful low-temperature resistance measurements. Measurements of the RFSE have not been made on any of the transition metals other than W, Mo, and Re. Outside of this group of metallic elements $e$-$e$ scattering has not been observed either in resistance or RFSE measurements in the temperature range of 1–4 °K.[9] In particular, when the $d$ bands are exactly filled (the noble metals) both the RFSE amplitude and the resistance are dominated by electron-phonon scattering in this temperature range.

Several mechanisms have been suggested to account for the enhanced $e$-$e$ scattering in transition metals. They are: the complicated Fermi-surface topology of transition metals, the wide variation in carrier effective masses in transition metals, and the high electron density of states combined with high Debye temperatures for this group of elements.

Ziman[10] shows that in a metal the normal $e$-$e$ scattering processes between electrons of like mass will not affect the resistivity. Scattering processes between electrons of like mass contribute to the resistivity only when umklapp processes are involved. Topologically complicated multiply connected Fermi surfaces make such umklapp processes much more favorable. The Fermi surfaces of the transition metals usually satisfy this condition and this could lead to an enhancement in the $e$-$e$ scattering rate. However, the Fermi surfaces of several nontransition metals such as the polyvalent hexagonal close-packed metals Mg, Zn, and Cd also penetrate the zone boundaries and are fully as complicated as those of the transition metals. Yet the resistivity and RFSE of each of these metals is dominated entirely by electron-phonon scattering in the temperature range above 1 °K.

TABLE II. Partial periodic table of transition metals, giving the value of $A$ in $\rho = \rho_0 + AT^2$ between 1 and 4 °K in units of $10^{-11}$ Ω cm/K$^2$. ($S$: superconducting in measurement range.)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3$d$ | Sc | Ti | V | Cr | Mn | Fe | Co | Ni | Cu |
| | | | $S$ | | | 3.1 | 0.98 | 3.4<br>0.95[a] | 0 |
| 4$d$ | Y | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag |
| | | | 0.23[a] $S$ | 0.126 | $S$ | 0.27 | | 3.1 | 0 |
| 5$d$ | La | Hf | Ta | W | Re | Os | Ir | Pt | Au |
| | $S$ | | $S$ | 0.1 | 0.45 | 0.2 | | 1.4 | 0 |

[a] Corrected for magnetoresistance:

Fe: J. G. Beitchman, C. W. Trussel, R. V. Coleman, Phys. Rev. Lett. 25, 1291 (1970). Note: This value is not corrected for magnetoresistance.

Co: D. Radhakrishma, Phys. Status Solidi 11, 111 (1965). Note: This value is not corrected for magnetoresistance.

Ni: G. K. White and R. J. Tanish, Phys. Rev. Lett. 19, 165 (1967). F. C. Schwerer and J. Silcox, Phys. Rev. Lett. 20, 101 (1968).[a]

Nb: G. W. Webb, Phys. Rev. 181, 1127 (1969).[a]

Mo: Present results (±0.003).

Ru: J. T. Schriempf and W. M. Macinnes, Phys. Lett. A 33, 511 (1970).

Pd: J. T. Schriempf, Phys. Rev. Lett. 20, 1034 (1968).

W: D. K. Wagner, J. C. Garland, and R. Bowers, Phys. Rev. B 3, 3141 (1971).

Re: J. T. Schriempf, Solid State Commun. 6, 813 (1968).

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Ir: N. V. Volkenshtein, V. A. Novoselov, and V. Ye. Startsev, Phys. Met. Metall. 31, 212 (1971).

Pt: C. Uher, C. W. Lee, and J. Bass, Phys. Lett. A 61, 344 (1977).

Normal electron-electron scattering processes can contribute to the resistivity if the electrons involved have significantly different masses. The magnitude of this effect is proportional to the magnitude of the variation of the masses in a particular metal. The effective masses of the charge carriers in Mo and W range from 0.40 to 2.50 and 0.27 to 1.83, respectively, on the orbits studied by Boiko *et al.* This is a variation of slightly over a factor of 6 in both cases. The effective masses of the carriers in In, Al, Tl, Cd, Zn, and Mg vary by an order of magnitude or more in each case.[11] In each of these nontransition metals the low-temperature resistivity is dominated by electron-phonon scattering.

Boiko *et al.* have suggested that the high electron density of states as measured by the electronic specific heat and the high Debye temperatures of the transition metals might yield information about the enhanced *e-e* scattering ratio. The relatively high electron concentrations and low phonon concentrations indicated by these parameters at low temperatures might provide for a greater predominance of *e-e* scattering over *e-p* scattering. Copper exhibits relatively high values for these quantities and yet is known to have a temperature-dependent resistivity dominated by *e-p* scattering down to 1 °K. The values for Al are even higher than for W but recent results[12] have shown that the low-temperature resistivity of this metal is also dominated by *e-p* scattering.

These considerations show that while large effective-mass differences, large densities of states, and complicated Fermi surfaces may be necessary for the observation of *e-e* scattering above 1 °K, they are not by themselves or in combination sufficient to explain the observed $T^2$ dependence of resistivity above 1 °K. Only when electron states near the Fermi level which are strongly hybridized by *d* states (3*d*, 4*d*, and 5*d* transition metals) is the $T^2$ dependence observed. The matrix element which determines the scattering probability for this effect must be strongly dependent on the orbital angular momentum of the electrons involved. Variations between transition metals may be explained by structural, density of states at the Fermi level, or Fermi-surface complications, but the overall magnitude of the effect must be due to an increased size of the scattering matrix element caused by the influence of *d* bands.

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