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Paraconductivity of three-dimensional amorphous superconductors—evidence for a short-wavelength cutoff in the fluctuation spectrum

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Measurements of the temperature dependence and magnetic-field dependence of the paraconductivity of a three-dimensional amorphous superconductor are presented. The data are analyzed in terms of several current theories and are found to give good agreement for low fields and temperatures near T_c. Strong pair-breaking effects due to thermal phonons are believed to account for the observed absence of anomalous Maki-Thompson contributions to the paraconductivity. The paraconductivity falls well below predicted theoretical values in the high-temperature and high-field limits. This is attributed to the reduced role of high-wave-vector contributions to the paraconductivity. It is shown that the introduction of a short-wavelength cutoff in the theoretical fluctuation spectrum provides a phenomenological account of the discrepancy between theory and experiment.

I. INTRODUCTION

Over the past ten years, there has been considerable interest in the study of the effects of thermal fluctuations on the properties of superconductors. In particular, the paraconductivity has been studied experimentally[1–4] in the case of thin superconducting films, and also in bulk type-II superconductors.[5] Much theoretical work has been devoted to predicting the temperature and magnetic-field dependences of the paraconductivity beginning with the early work of Aslamazov and Larkin[6] (AL) and followed by several others.[7–10] Recently, two of the authors reported measurements of the temperature dependences of the paraconductivity of bulk amorphous superconductors.[11] Fluctuation effects in *bulk* amorphous superconductors are strongly enhanced by the short electron mean free path and can be measured over a broad range of reduced temperature owing to the nearly-temperature-independent normal-state conductivity. A universal temperature dependence of the paraconductivity was found based on several samples studied. These results were found to be in good accord with theory for temperatures close to T_c but exhibit strong deviations from theory at higher temperature.

In the present paper, we present measurements of both the temperature and magnetic-field dependences of the paraconductivity σ' of amorphous superconductors. The temperature dependence follows that previously reported. The observed magnitude of σ' suggests a general absence of Maki-Thompson contributions.[7] This absence is believed to arise from strong pair-breaking effects due to thermal phonons. Both the temperature and field dependences show strong deviations from theoretical predictions at high temperature and high field. The paraconductivity falls much more rapidly than predicted in these limits. It is proposed that this behavior arises from the strongly reduced role of short-wavelength fluctuations. The effect can be taken into account by introducing a short-wavelength cutoff in the fluctuation spectrum. The cutoff can be determined by fitting the data and is found, to within experimental uncertainty, to be the reciprocal of the zero-temperature coherence length $\xi(0)$. A detailed theory of this effect would have to take into account the internal structure of a bound pair exactly. These effects have been studied previously in calculations of the diamagnetic susceptibility of superconductors above T_c.[12–14] The present results for the fluctuation conductivity show very similar features when compared with experimental results for the diamagnetic susceptibility.[15] Taken together, these results imply that the theory of fluctuation phenomena in superconductors is, with suitable extension, capable of providing a consistent account of available experimental data.

II. EXPERIMENTAL

Measurements of the fluctuation conductivity were performed on several speciments of amorphous $Zr_{75}Rh_{25}$ and $La_{78}Au_{22}$ prepared by rapid quenching from the liquid state.[16] Measurements of T_c for $Zr_{75}Rh_{25}$ were found to give slightly higher ($T_c \cong 4.5$ K) values than those previously reported for this amorphous system.[17] In addition, it was found that the width of the superconducting transition was much less ($\Delta T_c = 7 \times 10^{-3}$ K as measured by the 10%–90% points on the resistance curve) than that reported in Ref. 17. This is attributed to differences in the two quenching apparatus. The alloys were prepared by induction melting of the constituents on a silver boat under an argon atmosphere. The ingots were remelted several times to ensure homogeneity, and were subsequently broken into small fragments which were used to quench foils. The foils have a thickness of $\sim 45 \mu m$ and an area of several cm^2. The structure of each foil was checked by x-ray diffraction scans using a Norelco diffractometer (Cu $K\alpha$ radiation). This method allows detection of crystalline precipitates in the amorphous matrix when the volume fraction of such precipitates is of the order of a few percent of the sample volume. All foils showing evidence of crystalline phase precipitation were rejected for use in this study. The possibility that crystalline phase precipitates exist which are not detected by x-ray diffraction was considered. Such precipitates could effect the conductivity in the region of interest, particularly if they are superconducting with a T_c higher than that of the amorphous matrix. Several facts suggest that such an effect is not significant. First, the temperature dependence of the excess conductivity σ' has been found to be nearly identical for several amorphous alloys prepared by more than one technique.[11] The techniques include both liquid quenching and sputtering. It is highly unlikely that crystal phase precipitation could produce an identical effect on σ' in all samples studied since the character and extent of such precipitation would differ from one alloy to another and from one method of preparation to another. Second, rejected La-Au and Zr-Rh samples showing substantial precipitation of crystallites in the x-ray diffraction scans following quenching have been studied. Resistivity and ac induction measurements indicate that in both cases the precipitated phase has a lower T_c than the amorphous matrix. Specific-heat measurements on the La-Au system[18] confirm that $T_c \sim 2$ K for the precipitate phase. For Zr-Rh, the precipitate phase is cubic and has $T_c \sim 2.7$ K. Since neither of these phases is superconducting in the temperature range where σ' is measured, it is presumed that their presence below limits of detection by x-ray diffraction will not produce a significant effect on σ'.

The sample resistivities were measured for temperatures ranging from 1.9 to 10 K and magnetic fields ranging up to 75 kG produced by a superconducting solenoid. The current densities used in the measurement were $\lesssim 10$ A/cm^2. The sample is located in an exchange gas container the temperature of which can be stabilized to better than 1 mK. The temperature is measured using a carbon resistor with the calibration checked below 4.2 K against the vapor pressure of He I during each run. Above 4.2 K the calibration was extrapolated and checked against a Au-Fe thermocouple and later against a standard carbon glass thermometer (Lakeshore Cryotronics). The absolute accuracy of the temperature measurement is ± 0.1 K (over the entire temperature range) while the relative accuracy (near T_c) is ± 1 mK.

The specimens used in the measurements were long strips with width ~ 1 mm and length $\sim 1-2$ cm. The sample resistance was determined to an accuracy of better than 1 part in 10^4. The error in the absolute resistivity is $\pm 10\%$ due mainly to uncertainty in the sample geometry. The normal-state resistivity was also measured up to room temperature for reference in a separate experimental station. The normal-state resistance is constant to one part in 10^4 in the temperature range from 3 to 5 T_c. The variation of σ_0, the normal state conductivity, with H has been implicitly taken into account in the data analysis. This is accomplished by using the value of $\sigma_0(H)$ as measured in the applied field at the same temperature where σ_0 is defined in zero field. Thus, $\sigma_0(H)$ implicitly includes a small contribution (several parts in 10^4 for the highest field used) arising from the magnetoresistivity of the matrix.

In all measurements reported which involve an applied field H, the specimen was aligned perpendicular to the field using a mounting fixture. The relative angle between the sample current and H is 90° with an error of about 1°. Two perpendicular orientations are possible since the sample cross section has dimensions of 45 μm × 1 mm. Stronger effects arising from surface superconductivity were observed when the larger of these dimensions was parallel to H. This is discussed later in the text.

III. EXPERIMENTAL RESULTS AND COMPARISON WITH THEORY

To begin, a summary of relevant parameters characterizing the superconducting state of amorphous $Zr_{75}Rh_{25}$ is given for reference. The transition temperature in zero field, T_{c0}, varied by ± 0.05 K for the several samples studied. The value $T_{c0} = 4.562$ K given in the table refers to the most homogeneous sample (as evidenced by the width of the resistive transition) and is defined as the highest temperature for which the measured sample resistance $R = 0$. The width of the superconducting transition for this sample was $\Delta T_{c0} = 7$ mK defined by the 10% and 90% points on the resistive transition curve. The resistive transition is shown in Fig. 1. This transition is somewhat sharper than those observed in the most homogeneous samples used in the study off Ref. 11, permitting analysis of the fluctuation conductivity extending closer to T_c. The temperature dependence of the upper critical field of the sample $H_{c2}(t)$ ($t = T/T_{c0}$) was measured for reference in comparing the field dependence of the fluctuation conductivity to that predicted by theory. The value of $H_{c2}(t)$ was determined from the $R = 0$ criteria. A small but measurable flux flow resistance of order $10^{-3}R_0$ was ignored and $H_{c2}(t)$ was taken by extrapolating the R-vs-H data (with $R/R_0 > 0.10$) to $R = 0$. The results are shown in Fig. 1 and compared to the predictions of the Maki theory[19] (with no paramagnetic limiting or spin-orbit effects taken into account) for $H_{c2}(t)$. Effects of surface superconductivity were also observed. Between H_{c2} and roughly $1.5H_{c2}$, a "tail" in the R-vs-H curve which varied in magnitude among specimens was observed. The "tail" consists of a roughly linear variation of R above H_{c2} which terminates at a well-defined field. The total change is typically a few percent or less of R_0 and depends on the sample orientation. The point at which this tail terminates was taken to be H_{c3}. In the analysis of the fluctuation conductivity, care was taken to compare the fluctuation conductivity with theoretical predictions only for $H > H_{c3}(t)$ in order to avoid confusing it with the effects of surface superconductivity.

The measurement of the field-dependent fluctuation conductivity was carried out in two ways. In the first, the field is held constant and the sample resistance measured as a function of temperature. In the second, the temperature is fixed and the field is varied

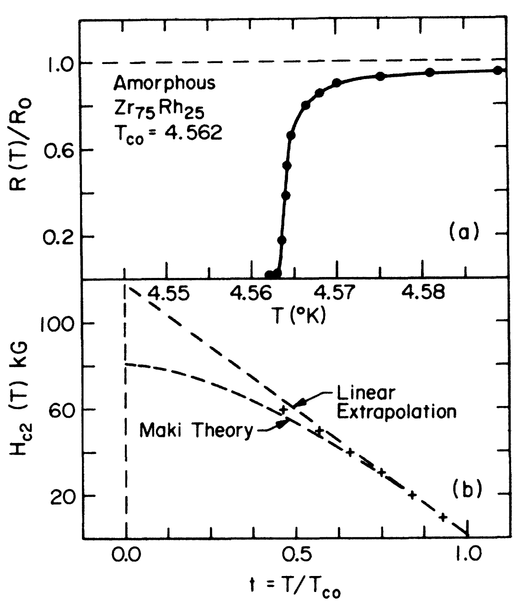

FIG. 1. (a) Normalized resistance vs temperature and (b) upper critical field H_{c2} vs reduced temperature $t = T/T_c$ for amorphous $Zr_{75}Rh_{25}$. The data for $H_{c2}(t)$ are compared with the predictions of Maki (Ref. 18) assuming no paramagnetic limiting and no spin-orbit effect.

from $H = 0$ up to $H = 75$ kG. The normal-state conductivity was taken to be independent of temperature over the range of interest, but was corrected at each field by taking into account a small coefficient of magnetoresistivity which was evaluated at high temperature ($3T_c$) and equal to 2.9×10^{-6} ($\mu\Omega$ cm/G). The results of the first set of measurements are shown in Fig. 2. It is clear that the width ΔT_c of the superconducting transition increases with increasing field. This effect can to a large extent be attributed to surface superconductivity as previously mentioned. For $H = 0$, the dependence of the excess conductivity σ' on temperature is illustrated by plotting $\ln(\sigma'/\sigma_0)$ vs $\ln t^*$ [where $t^* = (T - T_c)/T_c$ and σ_0 is the normal-state conductivity at $3T_c$]. This is shown in Fig. 3 along with data for amorphous $La_{78}Au_{22}$ and compared with the prediction of the AL theory. The value of σ_{AL} was determined from the formula

$$\sigma_{AL}(3D) = \tfrac{1}{32} e^2/\xi t^{*1/2} \ , \qquad (1)$$

with ξ determined from $H_{c2}(T)$ data using the relation $H_{c2} = |t^*|/2e\xi^2$. In practical units $e^2 = 2.43 \times 10^{-4}\ \Omega^{-1}$. Since both H_{c2} and σ_0 have been measured from amorphous $Zr_{75}Rh_{25}$ and $La_{78}Au_{22}$, one can determine $\sigma_{Al}(3D)$ completely from measured quantities. As found previously,[11] the AL prediction agrees well with the experimental data near T_c both in magnitude and temperature dependence. The apparent absence of the Maki-Thompson[7] (MT) contribution to σ' deserves further comment. First, it should be mentioned that the theoretical MT contribution has the same temperature dependence as the AL contribution for the three dimensional case; the MT contribution to σ' alters only the magnitude of σ' such that[7]

$$(\sigma_{MT} + \sigma_{AL}) = 5\sigma_{AL} \ .$$

Experimentally, σ' has been found to be very close to σ_{AL} (in the temperature range close to T_c) in all cases where the relevant parameters have been measured. For the two alloys of this study, a summary is given in Table I. Recent data[20] on an amorphous $Mo_{48}Ru_{32}P_{20}$ alloy for which σ', σ_0, and $H_{c2}(t)$ have been measured is also given in the table. In all cases, it is found that $\sigma'/\sigma_{AL} = 1.0 \pm 0.2$. The absence of the MT terms seems to be a general property of this class of materials. Keck and Schmid[10] have pointed out that pair breaking by thermal phonons can lead to suppression of MT terms in amorphous superconductors. Their estimate of pair breaking by thermal phonons in amorphous Pb and Bi and Ga films gave values of $\epsilon_{ph} = 1-3$, where ϵ_{ph} is the pair breaking parameter defined as

$$\epsilon_{ph} = \pi\hbar/8K_B T_c \tau_{ph} \ , \qquad (2)$$

with

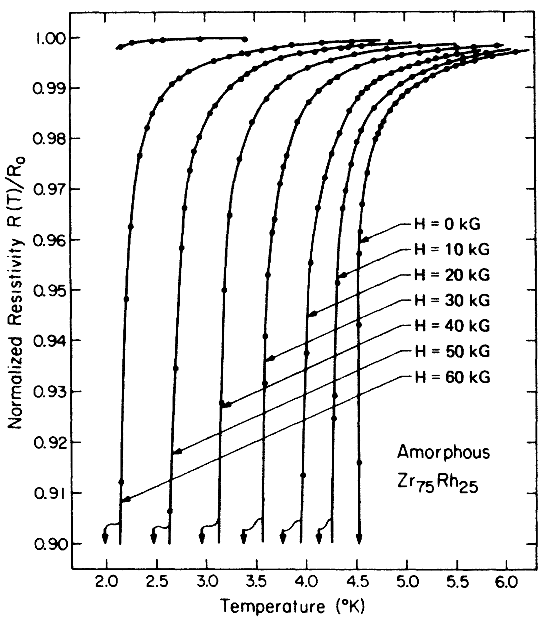

FIG. 2. Normalized resistance vs temperature with various applied magnetic fields. The arrow in the lower part of the graph indicates the temperature for which $R=0$ in each applied field.

$$\frac{1}{\tau_{\mathrm{ph}}}=4\pi\int_0^\infty d\omega\,\frac{\alpha^2(\omega)F(\omega)}{\sinh(\hbar\omega/K_BT)} \quad . \tag{3}$$

Such large values of ϵ_{ph} can be attributed to the strong enhancement of the Eliashberg function $\alpha^2(\omega)F(\omega)$ at small ω experimentally observed in amorphous superconductors by tunneling.[21–23] Recent tunneling experiments on amorphous transition-metal superconductors,[24] and specific-heat measurements on amorphous $La_{78}Au_{22}$,[22] suggest that the electron-phonon coupling constant λ is not as large as for these materials as reported for amorphous Pb, Bi, and Ga. It would seem to follow that the enhancement of $\alpha^2(\omega)F(\omega)$ at low frequencies should not be as pronounced for amorphous alloys of the type studied here. On the other hand, $2/\tau_{\mathrm{ph}}$ is defined in terms of an integral over ω of $\alpha^2(\omega)F(\omega)$ weighted by the statistical factor $\sinh^{-1}(\hbar\omega/K_BT)$, whereas λ is defined as

$$\lambda=2\int\frac{\alpha^2(\omega)F(\omega)\,d\omega}{\omega} \quad , \tag{4}$$

with $\alpha^2(\omega)F(\omega)$ weighted by ω^{-1}. At temperatures of interest ($T_c<T<3T_c$), the contributions to $1/\tau_{\mathrm{ph}}$ involve only very low-frequency phonons since

$$\sinh^{-1}(\hbar\omega/K_BT)\rightarrow e^{\hbar\omega/K_BT}$$

for $\hbar\omega>K_BT$, while λ involves essentially all phonons. The presence of a significant number of low-frequency contributions to $\alpha^2(\omega)F(\omega)$ can yield a rather large $1/\tau_{\mathrm{ph}}$ while λ remains of rather moderate size. Recent heat-capacity measurements suggest the presence of very low-frequency localized phonons in amorphous $Zr_{70}Pd_{30}$ alloys.[25] Thus, the apparent strong pair-breaking effects observed in amorphous transition-metal alloys are not inconsistent with a weak to intermediate value of λ. Such strong pair breaking is likely associated with an enhancement of $\alpha^2(\omega)F(\omega)$ at small ω and subsequent thermal phonon pair breaking. The universal absence of MT contributions to σ' in amorphous superconductors can then be understood.

Returning to Fig. 3 and comparing the experimental results with the predictions of AL shows that the temperature dependence of σ' is well described near T_c ($-6\lesssim\ln t^*\lesssim-3$) whereas the data fall below the AL predictions for higher temperature. The main difference as compared to the results of Ref. 11 is the extended range of agreement between experiment and theory for $t^*\rightarrow0$. This, as previously noted, is attributed to improved sample homogeneity.

The resistive transition as a function of applied field as measured at constant temperature with the applied field perpendicular to the direction of current flow is shown in Fig. 4. These results can be compared with the predictions of Maki[7] or with those of Usadel.[8] From Eq. (16) of Ref. 7(c), the Maki result is expressed in the following form for the case of H perpendicular to the direction of current flow:

TABLE I. Comparison of the observed paraconductivity σ' with the prediction of the AL theory σ_{AL}, in the temperature range near T_c ($t^*<0.1$). Values of the normal state resistivity ρ_0, and the zero temperature coherence length $\xi(0)$ based on linear extrapolation of the H_{c2} vs T data (as discussed in the text) are given for reference.

Alloy	T_c (K)	$(dH_{c2}/dT)_{T=T_c}$ (kG/K)	$\xi(0)$ (Å)	$\rho_0(\mu\Omega\,\mathrm{cm})$	$(\sigma'/\sigma_{\mathrm{AL}})$
$La_{78}Au_{22}$	3.75	22	63	240 (±50)	0.87 (±0.2)
$Zr_{75}Rh_{25}$	4.56	26	52	220 (±50)	115 (±0.2)
$Mo_{48}Ru_{32}P_{20}$	6.17	26	44	300 (±50)	0.93 (±0.2)

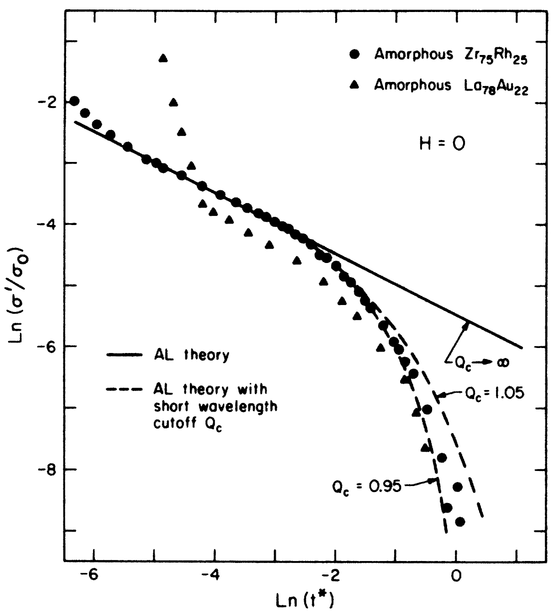

FIG. 3. Temperature dependence of the fluctuation conductivity ($\ln\sigma'$ vs $\ln t^*$) for $H=0$. The solid line is the prediction of the AL theory for amorphous $Zr_{75}Rh_{25}$. The dashed lines refer to the modified AL theory with a short-wavelength cutoff Q_c as discussed in the text.

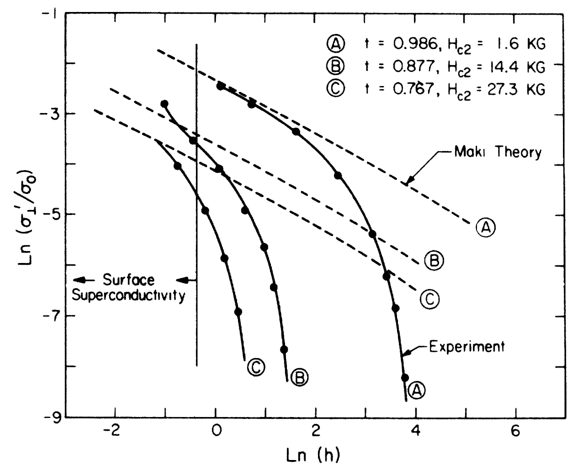

FIG. 4. Magnetic field dependence of the fluctuation conductivity $\sigma_\perp'$ at several reduced temperatures. Results are compared to the Maki theory (dashed lines) computed from Eq. (1) of the text.

$$\sigma_\perp' = e^2/h\,(2\pi)^2(4\pi K_B T/hD)[X(H,T)]^{-1/2}\ , \quad (5)$$

in cgs units with e the electron charge, D the electronic diffusivity, and X a function of field and temperature given by

$$X(H,T)=\frac{\ln(T/T_{c0})+\psi[\frac{1}{2}+(eDH/2\pi K_B T)]-\psi(\frac{1}{2})}{\psi^{(1)}[\frac{1}{2}+(eDH/2\pi K_B T)]}\ , \quad (6)$$

where T_{c0} is the transition temperature in zero field and ψ and $\psi^{(1)}$ are, respectively, the digamma and trigamma functions. The electronic diffusivity is experimentally determined from the gradient of the upper critical field

$$[dH_{c2}(T)/dT]_{T=T_{c0}}\ .$$

The remaining parameters are all fixed leaving no free parameters. Equation (5) was evaluated as a function of field at several fixed temperatures and then compared with the experimental data for $Zr_{75}Rh_{25}$. The data and theoretical predictions were compared by plotting $\ln\sigma_\perp'$ as a function of

$$\ln[H-H_{c2}(t)]/H_{c2}(t)=\ln h\ .$$

For the experimental data, the reduced field h is defined in terms of the experimentally measured $H_{c2}(t)$ at various fixed temperatures. Disagreement between the theoretical (Maki) $H_{c2}(t)$ curve (Fig. 1) with no paramagnetic or spin-orbit effects included and the experimental curve $H_{c2}(t)$ is taken into account by this procedure in a manner noted in Ref. 5. In analyzing these data, care was taken to avoid confusing the effects of fluctuation conductivity with those of surface superconductivity. Theoretically, it is known that $H_{c3}(t) \lesssim 1.69H_{c2}(t)$. By plotting the experimental data for low temperature where H_{c2} is fairly large, one can observe a clear break in the σ-vs-H curves typically for $H \simeq 1.5H_{c2}$. This was interpreted as surface superconductivity. In comparing the experimental field dependence of σ' with theory, such regions were avoided. In Fig. 4 regions of possible surface superconductivity are indicated by a vertical line and arrows. It can be seen in Fig. 4 that at low fields $H \ll H_{c2}(0)$ the theory and experiment are in good agreement, whereas for $H \lesssim H_{c2}(0)$ the experimental values of $\sigma_\perp'$ fall considerably below those predicted by theory. It is worthwhile to point out that the deviations from theory occur at roughly the same absolute field H for all fixed temperatures. Since the reduced field h is defined in terms of $H_{c2}(t)$, the deviations occur at different values of h.

The field dependence of the fluctuation conductivity of amorphous $La_{78}Au_{22}$ was also measured. A plot similar to that in Fig. 4 shows that the experimental data follow very closely the behavior observed for amorphous $Zr_{75}Rh_{25}$. This result is taken to confirm that the field dependence observed is general, just as the temperature dependence already found in Ref. 11 appears to be general for three-dimensional amorphous materials.

The temperature dependence of σ' in a constant nonzero applied field can be compared with theory by plotting $\ln(\sigma'/\sigma_0)$ as a function of

$$\ln\{[T-T_c(H)]/T_c(H)\}=\ln t^*(H) \ ,$$

where $T_c(H)$ is the transition temperature of the superconductor in the field H. The value of $T_c(H)$ was determined from experimental data using the $R=0$ criteria. A small, but detectable flux flow resistance which is particularly observable in high fields was found to be present for $T<T_c(H)$. This small resistance is of order 10^{-3} of the normal-state resistance as mentioned previously, and was ignored in defining $T_c(H)$ which was determined by linear extrapolation of the R-vs-T curve [with $(R/R_0)>0.10$] to $R=0$. Results of the above comparisons are shown in Fig. 5. The effect of surface superconductivity can be clearly observed as a large increase in σ' for $\ln[t^*(H)]\lesssim -2$. For $\ln[t^*(H)]>-2$, the temperature dependence of σ' for $H\neq 0$ is very similar to the temperature dependence of $H=0$ and falls well below the temperature dependence predicted by the Maki theory (shown in Fig. 5 for $H=10$ kG). It appears that in the limit where $t^*(H)\rightarrow 0$ the theory and experiment will agree closely (for $H\neq 0$) although the existence of surface superconductivity prevents measurement of σ' in this region. In this limit the Maki theory predicts $\sigma'\sim[t^*(H)]^{-1/2}$ similar to the AL theory for $H=0$. Finally, the magnitude of σ'/σ_0 for fixed $t^*(H)$ is found to increase progressively with increasing field. The Maki theory does not account for this increase as the calculation magnitude of σ'/σ_0 at constant $t^*(H)$ is nearly independent of H when calculated based on Eq. (1). It should be explicitly mentioned that in the limit of $H\rightarrow 0$, the Maki expression reduces to the AL expression in Eq. (1). This is accounted for if one notes that Eq. (4) takes into account the regular (AL) contribution to σ' but not the anomalous (Maki-Thompson) contribution. As previously pointed out, both contributions have the same temperature dependence in three dimensions but the MT term is four times larger than the AL term.

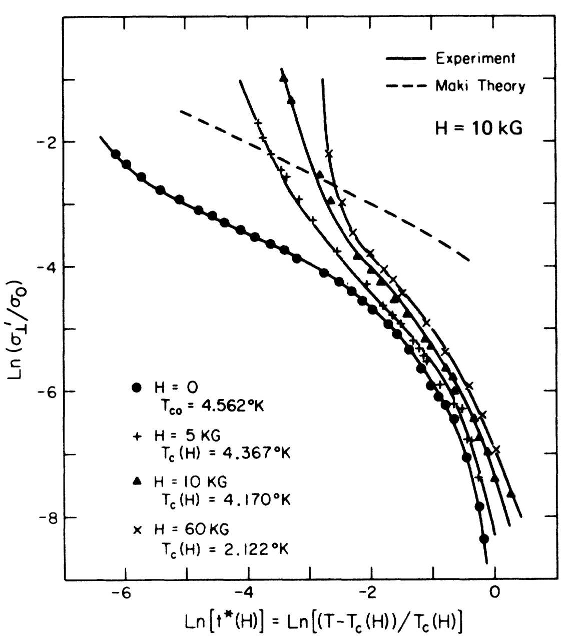

FIG. 5. Temperature dependence of the fluctuation conductivity for amorphous $Zr_{75}Rh_{25}$ in constant applied magnetic field. The data are compared to the Maki theory illustrated in Fig. 5 for the case of $H=10$ kG. The T_c of the sample in each applied field is given in the lower left-hand corner.

IV. BREAKDOWN OF THEORY AT HIGH FIELD AND HIGH TEMPERATURE

As previously suggested in Ref. 11, the breakdown of the Ginzburg-Landau theory in the high-temperature and high-field limit is most likely associated with the breakdown of the slow-variations approximation and the effect of nonlocal electrodynamical corrections. The AL theory approximately takes into account higher-order terms in the free-energy expansion. The Maki theory also takes all terms into account within an approximation and should be valid at all temperatures as opposed to being valid only near T_c. For the case of fluctuation diamagnetism, this problem has been considered in detail by several authors. Gollub *et al.*[15] give a rather complete discussion of both the experimental and theoretical results pertaining to this problem. On the experimental side, they demonstrate a universal field dependence of $M'/\sqrt{H}T$, where M' is the excess diamagnetization, on the reduced variable H/H_s. The field H_s is a scaling field empirically determined for each metal and alloy studied. Empirically, H_s is defined as the field for which $M'/\sqrt{H}T$ falls to one-half the value predicted by the Ginzburg-Landau theory.

In the present study, we consider the manner in which short-wave-length fluctuations contribute to the field dependence and the temperature dependence of the paraconductivity. A simple phenomenological modification of the AL theory is first considered which suggests a reduced role of short-wavelength fluctuations as compared to the AL theory.

For a three-dimensional superconductor, AL express[6] the supercurrent density for fluctuations as

$$j_\omega=i_\omega\left(\frac{\pi^3}{6}\right)C^2\frac{1}{m\rho_0^2}\int_0^\infty\frac{q^4A_\omega\,dq}{(t^*+\eta q^2)^3} \ , \qquad (7)$$

where C is a constant, ρ_0 the Fermi momentum, A_ω the vector potential, and η reduces to the square of the zero-temperature coherence length $\xi(0)$ in the dirty limit where the electron mean free path $l << \xi_0$. Thompson[26] first pointed out that for one- and two-dimensional superconductors, it is necessary to introduce a long-wavelength cutoff in the q integration to avoid unphysical singularities. Maki,[7] Patton,[9] and others have discussed the physical significance of the long-wavelength cutoff which can be interpreted in terms of a pair-breaking parameter.

It is clear that an effective short-wavelength cutoff also applies to the fluctuation conductivity in one, two, or three dimensions. For a dirty superconductor the higher-order correction terms in the Ginzburg-Landau formalism are important for the case where $q \sim \xi(0)^{-1}$. The contributions to (7) for such q vectors become important in considering the high-temperature and high-field limits of fluctuation phenomena. A detailed theory for the role of such high q fluctuations require that one exactly take into account the internal structure and dynamics of the bound pair. As a convenient means of phenomenologically treating the problem, we can simply introduce a short-wavelength cutoff Q_c into the q integration of Eq. (7) to approximate a reduced role of high q fluctuations. This modification of the AL theory is strictly phenomenological.

We have experimentally observed a universal temperature dependence for σ'. By using Q_c as an adjustable parameter, one can attempt to fit the experimental data. Equation (7) was evaluated numerically as a function of Q_c and the results used to fit the experimental temperature dependence of σ'. The temperature dependence predicted by Eq. (7) was found to be rather sensitive to the choice of Q_c. It was found that the data are fit extremely well for 0.95 $\lesssim 1.05$, where Q_c is expressed in units of $\xi(0)^{-1}$. A typical set of curves used to fit the data for amorphous $Zr_{75}Rh_{25}$ are shown in Fig. 3. To within experimental error, it is clear that a $Q_c \approx \xi(0)^{-1}$ describes the data very well. This result is not surprising since the break down of the slow variations approximation is expected in just this range of q. A similar fit to the data for the $La_{80}Au_{20}$, $Mo_{30}Re_{70}$, and Nb_3Ge amorphous samples of Ref. 11 gave a nearly identical result. We conclude that a cutoff wave vector $Q_c = [\xi(0)]^{-1}$ provides an adequate mathematical description of the reduced role of high q fluctuations as regards the temperature dependence of σ'.

An analysis of the field dependence of the fluctuation conductivity in terms of a cutoff wave vector is more difficult. We can follow Gollub[15] in attempting to describe the field dependence for $\sigma_\perp'$ by determining the characteristic field for which significant deviations from theory occur. The data shown in Fig. 4 exhibit a very similar behavior as a function of reduced field h as the data of Fig. 3 as a function of reduced temperature. The natural scale for significant deviations in the field dependent behavior from theory is expected to be of similar magnitude to that found in the diamagnetic case. A simple argument[27] which considers only the leading correction to the Ginzburg-Landau theory suggests that the slow variations approximation should break down for fields

$$H \sim \Phi_0/2\pi\xi^2(0) \sim H_{c2}(0) \quad . \tag{8}$$

The characteristic field at which the field dependence of the experimental data departs from the Maki theory can be estimated by first noticing that the experimental curves of Fig. 4 can be brought into coincidence if the reduced field variable

$$h = [H - H_{c2}(t)]/H_{c2}(t)$$

is redefined as

$$h^* = [H - H_{c2}(t)]/H_{c2}(0) \quad ,$$

which amounts to measuring $[H - H_{c2}(t)]$ in units of $H_{c2}(0)$. Comparison of data taken at different values of fixed t shows that to within experimental error, all data are described rather well by a single universal curve when $\ln(\sigma'/\sigma_0)$ is plotted against $\ln h^*$. One can then define a characteristic value of h^* for which data deviate from the Maki theory. The Maki theory predicts that σ' behaves like $[H - H_{c2}(t)]^{-1/2}$ for $H > H_{c2}(t)$ and $[H - H_{c2}(t)]$ small. We can estimate the deviation from the theory by determining the behavior of the slopes of the $\ln(\sigma'/\sigma_0)$-vs-$\ln(h^*)$ curve $[s = d(\ln\sigma')/d\ln(h^*)]$ as a function of h^*. For $h^* \rightarrow 0$, the data approach $s = -\frac{1}{2}$ as predicted by theory. If we estimate the value of h^* for which s deviates from $-\frac{1}{2}$ significantly, we find for example that $s \approx 1$ for $h^* \approx 0.15$ and $s \approx 2$ for $h^* \approx 0.30$ [where $H_{c2}(0)$ was taken to be the Maki extrapolated value in Fig. 1]. Thus, significant deviations from theory occur for fields of order of a few tenths of $H_{c2}(0)$. The general features of the field dependence of σ' in the amorphous La alloy are similar to those just discussed. It is concluded that the observed dependence of σ' on h and the characteristic value of h^* for breakdown of the theory are general features of three-dimensional amorphous superconductors.

In view of the present data, it would seem to be worthwhile to extend the theoretical results of AL, Maki, Patton, and those mentioned in Ref. 15 for the case of the diamagnetic fluctuations to the case of fluctuation conductivity in high field and at high temperature. It seems likely that a suitable extension of the microscopic theory can provide a complete and consistent picture for understanding the experimental data. Such an extension would provide a detailed test of the microscopic theory well beyond the limits of validity of the approximations made in the Ginzburg-Landau theory.

V. SUMMARY

Measurements of the paraconductivity σ' as a function of temperature and applied magnetic field for a three-dimensional amorphous superconductor have been presented. The data have been compared with several current theories. For temperatures close to T_c and small fields, the dependence of σ' on T and H is well described by the theories of Aslamasov and Larkin, Maki, Thompson, and Usadel. It is pointed out that for amorphous superconductors, for high temperature and large fields, the experimental data give values of σ' which fall well below those predicted by theory. This discrepancy is attributed to the manner in which the current theories treat short-wavelength fluctuations. A convenient phenomenological description of the temperature dependence of σ' is obtained by assuming a short-wavelength cutoff in the AL theory in order to account for the reduced role of high q fluctuations. A good fit to the data is obtained by taking this cutoff wave vector to be $Q_c \approx [\xi(0)]^{-1}$. A complete theory must exactly take into account the higher-order terms in a theoretical expansion of the free energy and in so doing properly account for the internal structure of a bound pair. The problem of treating the field dependence of σ' similarly must be treated by properly accounting for high q contributions. Experimentally, it has been observed that the field dependence of σ' deviates from that predicted by theory for fields exceeding a few tenths of $H_{c2}(0)$. Finally, it has been pointed out that the general absence of anomalous Maki-Thompson contributions to σ' in amorphous superconductors can be related to enhancement of the Eliashberg function $\alpha^2(\omega)F(\omega)$ at low frequencies in this class of materials.

It is hoped that these results will stimulate theoretical interest in appropriately extending the theory of paraconductivity to account for the remaining discrepancies between theory and experiment. Such an extension has apparently been undertaken already.[28]

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