

Minority-carrier injection into semiconductors containing traps

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The paper presents an analysis of minority-carrier injection into lifetime and relaxation semiconductors containing traps, linearized within the framework of a "small-signal theory." In that sense it parallels a previous paper on the trap-free case. The restrictive assumptions customarily made in the literature as regards injection ratio, mobility ratio, carrier lifetime, dielectric relaxation time, doping levels, and specific characteristics of the trapping centers, zero recombination rate $np = n_i^2$, and neglect of diffusion current, are avoided here. Explicit solutions are obtained of the carrier concentration and field profiles, and plotted for a series of interesting cases, some designed to illustrate the nature of the phenomena, some to facilitate experimental verification. The results show that the establishment of lifetime and relaxation regimes depends in a complex manner on the parameters of the system. Minority-carrier injection can result in the appearance of a field maximum and a total resistance increase, not only in the trap-free case as previously reported, but to a greatly augmented extent in the presence of traps in suitable concentrations and energetic positions. The results have a potential bearing on the interpretation of many types of electrical measurements on semiconductors and semi-insulators. The equations themselves are general (except for the restriction to small currents) and can be extended to a variety of other nonequilibrium transport effects in solids.

INTRODUCTION

In a previous paper,¹ the authors discussed solutions to the linearized transport equations (small-signal theory) for the case of minority-carrier injection into a homogeneous trap-free medium. The conclusion was that, in the right circumstances, a field maximum could arise near the injecting boundary, leading to an overall resistance increase. Such an increase would be expected, for instance in an n -type "lifetime" semiconductor ($\tau_D < \tau_0$), with $\mu_n > \mu_p$. It arises essentially from a diffusion of majority carriers in a direction opposed to the current. In order to keep the total current constant, the local field must increase. At higher current densities, the resistance increase is expected to disappear; the system then conforms to the conventional expectation of a resistance decrease due to injected carriers. The present paper is concerned with corresponding cases in the presence of traps. In practice, trap density and carrier lifetime cannot, of course, be controlled independently, but for present purposes, it is assumed that the two parameters can be arbitrarily adjusted. In small-signal theory, the diffusion-length lifetime τ_0 is taken as constant,² i.e., independent of excitation level. It will be shown on the basis of Shockley-Read recombination mechanism³ (again for n -type material) that inasmuch as the traps capture additional majority carriers, they diminish the above field maximum; inasmuch as they capture minority carriers, they increase it. This is in accordance with the expectations from the sign

of the corresponding space charges. For traps of sufficient density and appropriate level, the resistance increase can be very much higher than in the corresponding lifetime case without traps. Popescu and Henisch⁴ showed that minority-carrier trapping is capable of driving what would otherwise be a relaxation-controlled system into the lifetime regime, and thereby produce the resistance increase. They did so (again on the assumption of independently controlled parameters) for unit injection ratio and $b (= \mu_n/\mu_p) = 1$ only. The present paper is not limited to these conditions.

TRANSPORT EQUATIONS

For the small-signal theory, the transport equations applicable to this case are the same as Eqs. (13)–(17), of the previous paper,¹ except for the modification arising from the presence of traps. In principle, traps modify the system in two ways: (a) by modifying the recombination term, and (b) by modifying Poisson's equation. Within the limitations of a "small-signal theory," (a) is of no significance, because as long as the Shockley-Read parameters τ_{n0} and τ_{p0} and n_1 and p_1 are themselves constant, we have

$$R = \frac{\Delta np_e + \Delta p n_e}{\tau_{n0}(p_e + p_1) + \tau_{p0}(n_e + n_1)} = \frac{\Delta np_e + \Delta p n_e}{\tau_0(n_e + p_e)}, \quad (1)$$

where τ_0 is the "diffusion-length lifetime" defined by van Roosbroeck.² On the other hand, (b) is always important, as will be shown below. It is here assumed, as in Shockley-Read theory, that the recombination center has only two occupation

states, differing by one electronic charge (single, monovalent trapping level). For an n -type material in equilibrium, the traps will have a certain occupancy, at the expense of the donors, so that

$$N_D^* = n_e + N_{te}^-, \quad (2)$$

where N_D is the concentration of donors (assumed to be completely ionized throughout), and N_{te}^- is the concentration of negatively charged traps. In the steady state, we have equality between the net rates of electron and hole capture ($dN_t^-/dt = 0$). Using the same normalization procedures as Popescu and Henisch⁴ this yields

$$M_e = \frac{N_{te}^-}{n_e} = M_0 \frac{1 + P_1 \tau}{1 + N_1 + \tau(P_e + P_1)}. \quad (3)$$

For nonequilibrium, we have

$$M = M_0 \frac{N + P_1 \tau}{N + N_1 + \tau(P + P_1)}, \quad (4)$$

where $M_0 = N_t^-/n_e$, the normalized trap density and $\tau = \tau_{n0}/\tau_{p0}$. Also, in normalized Shockley-Read terms,

$$P_1 N_1 = N_t^2 = P_e N_e = P_e. \quad (5)$$

The space charge arising from nonequilibrium occupation of the recombination centers is given by $\Delta Q_t = M_e - M$.

Accordingly, Poisson's equation becomes

$$\frac{dE}{dX} = \frac{1}{1 + P_e} (\Delta P - \Delta N + \Delta Q_t). \quad (6)$$

Using the small-signal approximation ($\Delta P < P_e$ and $\Delta N < N_e$) this becomes, via Eqs. (3)–(5) above,

$$\Delta Q_t = \alpha \Delta P - \beta \Delta N, \quad (7)$$

$$\alpha = [M_0 \tau / (1 + N_1)] / [1 + N_1 + \tau(P_e + P_1)], \quad (8)$$

$$\beta = [M_0 P_e / (P_e + P_1)] / [1 + N_1 + \tau(P_e + P_1)],$$

and the remaining transport equations are

$$J_n = \frac{b}{1 + P_e} \left(E + \frac{d\Delta N}{dX} \right), \quad (9)$$

$$J_p = \frac{1}{1 + P_e} \left(P_e E - \frac{d\Delta P}{dX} \right), \quad (10)$$

$$\frac{d^2 \Delta N}{dX^2} + \frac{dE}{dX} - \frac{1}{1 + P_e} (\Delta N P_e + \Delta P) A_n = 0, \quad (11)$$

$$\frac{d^2 \Delta P}{dX^2} - P_e \frac{dE}{dX} - \frac{1}{1 + P_e} (\Delta N P_e + \Delta P) A_p = 0, \quad (12)$$

where the normalizations are¹

$$x = XL_D, \quad E_{\text{real}} = E(kT/qL_D), \quad j = J\mu_p kT(n_e + p_e)/L_D,$$

$$p = Pn_e, \quad n = Nn_e, \quad \Delta N = N - N_e = N - 1,$$

$$\Delta P = P - P_e,$$

and

$$L_D = [\epsilon kT/q^2(n_e + p_e)]^{1/2}, \quad (13)$$

effective Debye length

$$A_n = \frac{\tau_{Dn}}{\tau_0(1 + P_e)} = \frac{\tau_{Dn}}{\tau_{p0}[(1 + N_1) + \tau(P_e + P_1)]}, \quad (14)$$

$$A_p = bA_n, \quad (15)$$

$$\tau_{Dn} = \epsilon/q\mu_n n_e.$$

For the present case, the boundary conditions are

$$N = N_e \quad \text{and} \quad P = P_e \quad \text{at} \quad X = \infty, \quad (16)$$

$$J = \frac{1}{1 + P_e} \left(b \frac{d\Delta N(0)}{dX} - \frac{d\Delta P(0)}{dX} \right), \quad (17)$$

and

$$\gamma = \frac{d\Delta P(0)}{dX} \left/ \left(\frac{d\Delta P(0)}{dX} - b \frac{d\Delta N(0)}{dX} \right) \right. \quad (18)$$

at the injecting boundary ($X = 0$).

CARRIER AND FIELD PROFILES; EXPLICIT SOLUTIONS

By manipulations very similar to those used in the previous paper, the solutions of the above equations can be shown to be as follows:

$$\Delta N = \frac{J}{b(\eta - \alpha)} \left(\frac{[\gamma(b + P_e) - P_e](1 + \alpha - A_n)}{\alpha^{1/2}} \exp(-\alpha^{1/2} X) - \frac{(1 + \beta)(1 - \gamma) + b[\gamma(1 + \alpha) - A_n]}{\eta^{1/2}} \exp(-\eta^{1/2} X) \right), \quad (19)$$

$$\Delta P = \frac{J}{b(\eta - \alpha)} \left(\frac{[\gamma(b + P_e) - P_e](1 + \beta - A_p)}{\alpha^{1/2}} \exp(-\alpha^{1/2} X) + \frac{P_e[(1 + \beta)(1 - \gamma) + b[\gamma(1 + \alpha) - A_n]]}{\eta^{1/2}} \exp(-\eta^{1/2} X) \right), \quad (20)$$

where

$$\alpha = \frac{b + P_e}{1 + P_e} A_n = \frac{1 + b^{-1} P_e A_p}{1 + P_e},$$

and

$$\eta = \frac{1 + \beta + P_e(1 + \alpha)}{1 + P_e}. \quad (21)$$

These equations may be compared with Eqs. (33) and (34) in the previous paper.

It will be seen that (apart from the coefficient) the first exponential terms, $\exp(-X\sqrt{\alpha})$, arising from the diffusion of carriers, are the same, but the second, $\exp(-X\sqrt{\eta})$, arising from space-charge considerations are modified by the presence of traps.

It can be shown that the constant in this exponent

$$E = \frac{(1+P_e)J}{b+P_e} \left(1 + \frac{1}{b(1+P_e)(\eta-\alpha)} ([\gamma(b+P_e) - P_e][b(1+\alpha) - (1+\beta)] \exp(-\alpha^{1/2}X) - (P_e + b)[(1+\beta)(1-\gamma) + b[\gamma(1+\alpha) - A_n]] \exp(-\eta^{1/2}X) \right) \quad (23)$$

Again the traps influence only the second term. ΔQ_t is given by Eq. (7).

Inspection of Eq. (23) shows that there is, as in the trap-free case, a value of X , namely, X_m , for which the field has a maximum value E_m , implying a resistance increase:

$$\exp[(\eta^{1/2} - \alpha^{1/2})X_m] = \frac{(\alpha\eta)^{1/2}(1+P_e)[(1+\beta)(1-\gamma) + b[\gamma(1+\alpha) - A_n]]}{[\gamma(b+P_e) - P_e][b(1+\alpha) - (1+\beta)]A_n} \quad (24)$$

It can be shown that for $\gamma > \gamma_0 = P_e/(b+P_e)$, (where γ_0 is the injection ratio in the unperturbed bulk) this equation has a solution for

$$b > (1+\beta)/(1+\alpha), \quad (25)$$

and

$$A_n < \{\gamma[b(1+\alpha) - (1+\beta)] + (1+\beta)\}/b.$$

This may be simply compared with the trap-free case by putting $\alpha = \beta = 0$, ($M_0 = 0$). Depending on the position and density of the traps, α and β vary, which means that X_m and E_m vary. Compared with the trap-free case (and other things being equal) E_m may thus be increased or decreased by the presence of traps.

CARRIER AND FIELD PROFILES; TYPICAL SITUATIONS

For the sake of comparison, concentration and field contours were calculated for the semiconductor parameter used by Popescu and Henisch,⁴ Fig. 2(a). This was possible, because that case, though computed by means of the full equations falls into the range of the "small-signal" approximation. The agreement with the solutions here derived from the linearized equations is excellent. The plotted solutions which follow refer to two groups of problems. Group A deals with a hypothetical (compensated) intrinsic semiconductor with varying trap parameters; group B deals with

can be written [using Eqs. (5) and (8)] as

$$\eta = 1 + M_0 N_1 / (1 + P_e)(1 + N_1)^2, \quad (22)$$

and is thus independent of τ , and dependent only on the static characteristics of the recombination center.

In the same way we obtain

compensated intrinsic germanium, before and after the introduction of traps.

Figures 1-7 represent a series of results all for compensated intrinsic material ($P_e = N_e = 1$), and unit injection ratio ($\gamma = 1$), as well as $b = 10$, $J = 10^{-2}$. The intrinsic case is interesting, because it extends the range of validity of the small-signal approximation without affecting the principal characteristics of the analysis in any way. The assumption $\gamma = 1$ represents the maximum effects obtainable for minority-carrier injection. In particular cases γ may be less than unity; this reduces the departures from equilibrium but, likewise, does not alter the situation in any fundamental way. The values of α , M_0 , τ , N_1 are varied; in all these cases $P_1 = 1/N_1$ [Eq. (5)].

For comparison, Fig. 1 represents $M_0 = 0$, the trap-free case, for which conditions (25) are fulfilled thus leading to a field maximum.

Figure 2 refers to a case in which the recombination centers are in the middle of the band gap: $N_1 = P_1 + 1$ (we assume here, for simplicity that the densities of states in the conduction and valence band are equal, $N_v = N_c$). Moreover $\tau = 1$ means

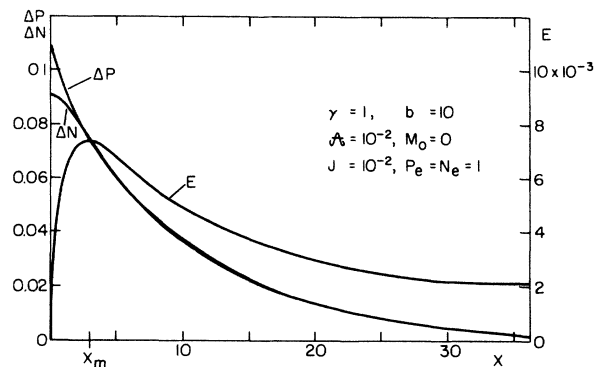


FIG. 1. Concentration and field contours for an intrinsic semiconductor characterized by unit injection ratio, no traps. Lifetime regime; $\alpha = 10^{-2}$.

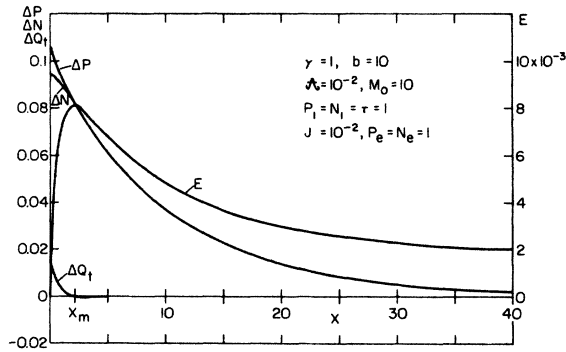


FIG. 2. Concentration and field contours for the semiconductor of Fig. 1, with recombination centers in the middle of the band gap. Lifetime regime; $G = 10^{-2}$.

that the capture cross sections for electrons and holes are the same (as long as the two types of carriers have the same thermal velocity, $\tau = \tau_{n_0} / \tau_{p_0} \sim \sigma_p / \sigma_n$). While the material is in thermal equilibrium, local neutrality is achieved by compensation, which in this special case of Fig. 2, implies $N_D \approx \frac{1}{2}N_t$. The donors play no other role. Figure 2 shows that the field maximum is slightly increased for $M_0 = 10$, as compared with $M_0 = 0$ of Fig. 1. This field increase is related to the positive space charge in traps, which attracts an additional concentration of free electrons into the trapping region.

Figures 3 and 4 show the same situation, but for different positions of the recombination level, relative to the Fermi level, which remains in midgap. In Fig. 3 the recombination levels are above the Fermi level and therefore mostly empty in equilibrium; in Fig. 4 they are mostly full. In the former case the centers preferentially capture electrons in the latter holes. As one could, accordingly, expect, the field maximum is reduced

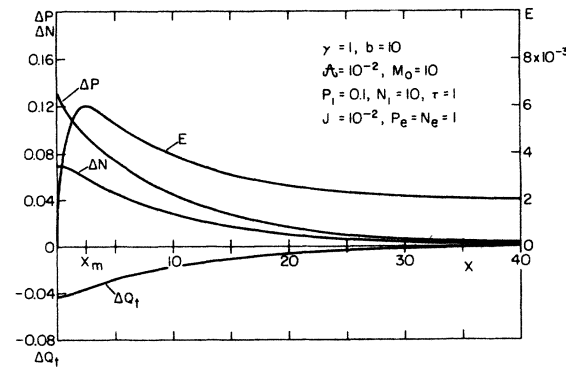


FIG. 3. Concentration and field contours for the semiconductor of Fig. 1, with recombination centers above the midgap. Lifetime regime; $G = 10^{-2}$.

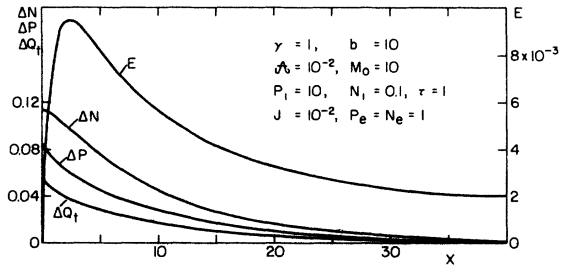


FIG. 4. Concentration and field contours for the semiconductor of Fig. 1, with recombination centers below the midgap. Lifetime regime; $G = 10^{-2}$.

in Fig. 3 ($\Delta Q_t < 0$), and appreciably augmented in Fig. 4 ($\Delta Q_t > 0$), compared with the results in Fig. 2. With increasing values of M_0 , E_m also increases, and moves towards smaller values of x . For $M_0 = 100$ and the same parameters as in Fig. 4 we have $E_m/E_\infty \approx 7$.

Figure 5 refers to the same hypothetical material, but in a relaxation case, characterized by $G = 10^2$, $A_n \approx 18.2$. With these parameters, the conditions for a field maximum [relations (25)] are not satisfied. Accordingly Fig. 5 shows no E_m , but demonstrates majority-carrier depletion, as in the trap-free case. However, the depletion is much smaller than in the trap-free case, because most of the positive space charge now resides in the traps. If the traps were above the middle of the bands (in the presence of an appropriate number of donors, smaller than in the previous case) the same concentration would lead to a larger depletion of majority carriers. Calculations show that ΔQ_t would remain positive as distinct from the lifetime case, for which it can be negative (Fig. 3) or positive (Fig. 4).

Figure 6 shows an interesting case in which, by a suitable choice of parameters, a material which

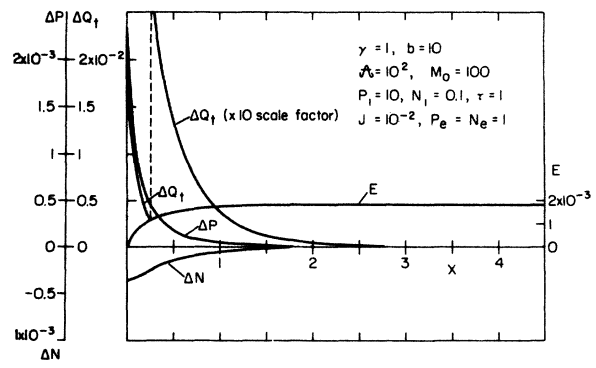


FIG. 5. Concentration and field contours for the semiconductor of Fig. 1 with recombination centers below the midgap. Relaxation regime; $G = 10^2$.

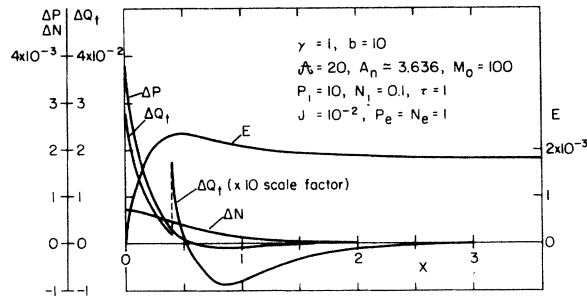


FIG. 6. Concentration and field contours for the semiconductor of Fig. 1, a boundary case. Injection of minority carriers into the trap-free material ($M_0 = 0$) leads to a relaxation regime. Injection into the same material with traps ($M_0 = 100$) leads to a lifetime regime; $\alpha = 20$.

is a relaxation semiconductor for $M_0 = 0$ because $A_n = 3.64 > 1$ for $\gamma = 1$, is brought in the lifetime regime by the addition of traps $M_0 = 100$. In that sense only, the case is similar to that described by Popescu and Henisch⁴ (Fig. 3), but Fig. 6 demonstrates a reversal in the sign of ΔP not previously encountered. For $\gamma = 1$, a field maximum is expected [by reference to the relationships (25)] if $A_n < 1 + \alpha \approx 8.51$, and this inequality is obviously satisfied in the present case.

The trap-free lifetime case is not associated with a field maximum as long as the mobility ratio $b \leq 1$. However, when $b > 1$ the field maximum must appear.¹ When traps are present, E_m could appear even for $b < 1$; see inequalities (25).

A case is shown on Fig. 7 for which $b = 1$, but it is only of academic interest because $b = 1$ is not ordinarily encountered.

Figures 8 and 9 refer to compensated intrinsic germanium, with a single trap level in mid gap. This corresponds to doping with iron; and also to traps created by neutron bombardment. As regards iron, Fig. 8, the literature⁵ yields the fol-

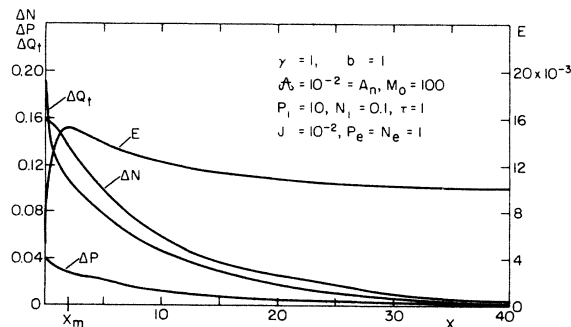


FIG. 7. Concentration and field contours for a case characterized by unit mobility ratio and injection ratio. Lifetime regime; $\alpha = 10^{-2}$.

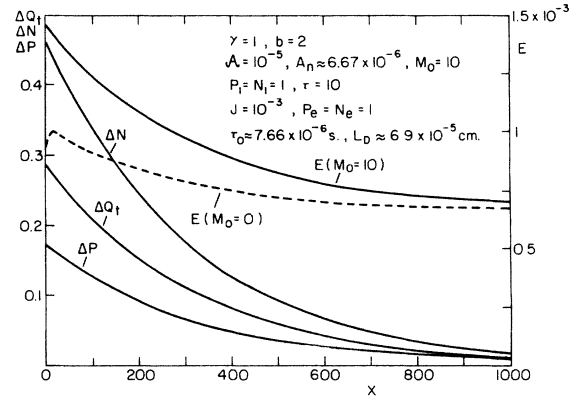


FIG. 8. Concentration and field contours for hole injection into intrinsic (compensated) germanium. Iron recombination centers. Lifetime regime; $\alpha = 10^{-5}$.

lowing estimates of the capture crosssections for the first acceptorlike level, $E_C - E_T \approx 0.35 \text{ eV}$, at $T = 300 \text{ K}$:

$$\sigma_p^* \approx 0.6 \times 10^{-15} \text{ cm}^2, \quad \sigma_n^0 \approx 0.6 \times 10^{-16} \text{ cm}^2.$$

Assuming a thermal velocity of 10^7 cm sec^{-1} , this leads to $\tau_{n0} \sim 7 \times 10^{-6} \text{ sec}$, $\tau_{p0} \sim 7 \times 10^{-7} \text{ sec}$, and $\tau = 10$. Via Eq. (1), these values enable τ_0 to be calculated $\tau_0 \approx 7.7 \times 10^{-6} \text{ sec}$ and this, in turn, enters into $\alpha \approx 10^{-5}$ [Eqs. (14) and (21)].

In Fig. 8, the broken line shows the field maximum of the trap-free case for comparison. The full E line refers to the case with traps, as here envisaged ($N_t = 2.4 \times 10^{14} \text{ cm}^{-3}$, $M_0 = 10$). It is evident that the field maximum is greatly enhanced, and though its location is now closer to $X = 0$, the extent of the disturbance reaches farther into the semiconductor than before, e.g., to a depth of $1000 L_D$. With $L_D = 6.9 \times 10^{-5} \text{ cm}$ this gives $690 \mu\text{m}$. ΔQ_t is positive everywhere, because the

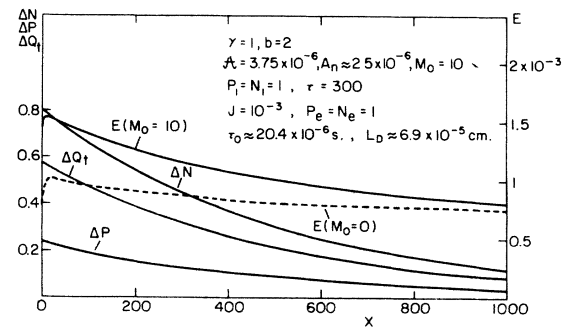


FIG. 9. Concentration and field contours for hole injection into intrinsic (compensated) germanium. Recombination centers created by 14-MeV neutron bombardment. Lifetime regime; $\alpha = 3.75 \times 10^{-6}$.

centers are mostly hole acceptors. Space-charge neutrality is almost complete everywhere ($\Delta P + \Delta Q_t = \Delta N$), but there *are* residual departures from neutrality, too small to be visible on the figure, and those are responsible for the field contour. The normalized current density $J = 10^{-3}$ corresponds to a current $j \approx 2.4 \text{ mA cm}^{-2}$.

Figure 9 corresponds to the case of germanium originally *n* type and bombarded by neutrons, as described by Curtis,⁶ until $N_t \approx 2N_D$ so as to maintain the intrinsic character of the semiconductor. The centers so created are 0.32 eV above the top of the valence band,⁷ at $T \approx 300 \text{ K}$:

$$\sigma_p^- \approx 6 \times 10^{-15} \text{ cm}^2, \quad \sigma_n^0 \approx 2 \times 10^{-17} \text{ cm}^2.$$

This makes $\tau \approx 300$, much higher than in the previous case. The same calculation as before leads to: $\tau_0 \sim \tau_{n_0} \sim 20 \times 10^{-6} \text{ sec}$ and thus $G \approx 3.7 \times 10^{-6}$. As Fig. 9 shows, the field maximum is again enhanced by approximately the same ratio as in the previous case.

DISCUSSION AND SUMMARY

(i) *Concentration and field contours.* Reference to Eqs. (7), (19), (20), and (23) shows that the first exponential term is dominant in a pronounced lifetime semiconductor ($A_n, G \ll 1$), and the second in a pronounced relaxation semiconductor ($A_n, G \gg 1$). Only the second exponent is affected by traps (although both coefficients are trap dependent). This difference affects the shape of the concentration contours for large values of X bordering the equilibrium region. Other things being equal, and within the framework of the present small-signal theory (only), this means that the nonequilibrium effects decay more quickly in relaxation than in lifetime semiconductors as X increases. This different behavior is easily seen by comparing Figs. 5 and 6 with Figs. 1–4. This is so because in the pronounced relaxation case and for sufficiently high values of X the second exponential term dominates and the exponent factor $\{[1 + \beta + P_e(1 + \alpha)]/(1 + P_e)\}^{1/2}$ is always greater than one. The conditions near the boundary between the lifetime and relaxation regimes are discussed in (iii) below.

(ii) *Space-charge and neutrality considerations.* The simplest neutrality assumption is $\Delta P - \Delta N = 0$ and its lack of justification was already discussed for the case without traps.¹ In the presence of traps, the neutrality assumption takes the form given by Eq. (7): $\Delta P(1 + \alpha) = \Delta n(1 + \beta)$. If this assumption were used as an *a priori* postulate, it would eliminate dE/dX in Eqs. (11) and (12). However, since this is obviously wrong, conventional

practice has been to allow the dE/dX terms to stand in these equations. If this were done here, and dE/dX eliminated between Eqs. (11) and (12), one would obtain a solution in the form

$$\Delta P \sim \Delta N \sim \exp(-G^{1/2}X) = \exp[-x/(D_a\tau_0)^{1/2}],$$

$D_a = [kT(n_e + p_e)\mu_n\mu_p/q(\mu_n n_e + \mu_p p_e)]$ being the ambipolar diffusion constant which is equivalent to eliminating the second exponential term in Eqs. (19) and (20). No resistance increase would then be expected, nor any majority-carrier depletion. For sufficiently high values of X , the neutrality assumption is indeed almost satisfied (e.g., see Figs. 8 and 9) but the residual departures from neutrality, small as they are, lead to the field contour as calculated. All this shows that the neutrality assumptions, no matter how mathematically attractive they may appear, are highly misleading in the case with traps, as also in the case without.

(iii) *Boundaries between operating regimes.* Operating regimes under minority-carrier injection ($\Delta P > 0$) can be defined in different ways: (a) on the basis of whether ΔN is positive or negative, (b) whether there is or is not a field maximum E_m , or (c) whether the total resistance of the system has increased or decreased. Moreover, any of the three criteria could be applied to the trap-free case, or to the multitude of possible cases with traps. Classification (a) has been the basis of the distinction between a lifetime regime ($\Delta N > 0$) and a relaxation regime ($\Delta N < 0$), whether traps are present or not.^{4,8} However, there is no simple and necessary connection between this criterion and the remaining two. Thus, for instance, even in a perfectly well-defined lifetime regime, there may be no field maximum (and thus no resistance increase), because the mobility ratio may not be sufficiently high [see Eq. (25)]. Criteria (b) and (c), though not identical, are more closely related to one another than either are to (a). This will be clear by reference to Fig. 10. Without a field maximum there is, of course, no possibility of a resistance increase, but even with a field maximum the resistance increase is not inevitable. It appears only if area A is larger than area B on the diagram.

The classification of effects on the basis of criterion (a) is possible only when the sign of ΔN is independent of X , but previous results¹ have shown that sign changes can occur, and these blur the boundary between the lifetime and relaxation regimes. It is nevertheless possible to define situations in which ΔN is zero irrespective of X , but only as $\gamma = 1$.¹ In the presence of traps and $\gamma = 1$, the conditions $\Delta N = 0$ everywhere is satisfied again as long as $A_n = 1 + \alpha$ [relations (25)], where α de-

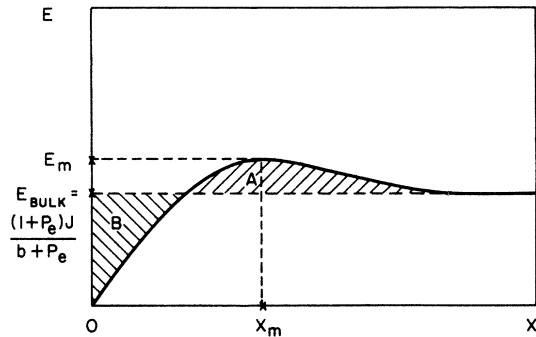


FIG. 10. Field contours when relations (25) are satisfied.

depends on the nature and concentration of the traps.

Within the framework of a "small-signal theory," the present work extends previously available results. As predicted by van Roosbroeck, majority-carrier depletion ($\Delta N < 0$) is expected for very low lifetime material (relaxation regime), but as stated by Kiess and Rose,⁹ Stöckmann¹⁰ and by Popescu and Henisch,^{4,11} this cannot lead to an increase of total resistance. Hybrid cases are possible, in which a relaxation semiconductor is driven into the lifetime regime ($\Delta N > 0$) through minority-carrier trapping near the injecting boundary. This leads to a majority-carrier concentration gradient opposed to the current, and thus to a field maximum. The present equations show, that this maximum must be expected, not only under the restrictive conditions previously imposed⁴ but in a variety of other circumstances,

depending on injecting ratio, mobility ratio, diffusion-length lifetime, dielectric relaxation time, and trap characteristics. When areas $A > B$ in Fig. 10, the presence of a maximum field also ensures a total resistance increase; otherwise it does not. A field maximum is therefore a necessary but not sufficient criterion for such a resistance increase. Though the present results are obviously limited to low currents, they are free from the restrictive assumptions usually made, e.g., zero recombination,^{8,12,13} unit mobility ratio,^{4,8,11,12} unit injection ratio,^{4,11} zero trapping,¹² "conductivity-locked" mode of transport ($\sigma_n/\sigma_p \approx \text{constant}$,¹³⁻¹⁶ negligible diffusion in the high-field region,⁸ etc. It should be remembered, however, that they refer to semi-infinite systems, and neglect contributions to the resistance which may arise from the contact itself. Experimental observations by Ilegems and Queisser¹⁷ showed a resistance increase, and could be due to a relaxation case brought into the lifetime regime by injection.⁴ Experimental work on *a priori* lifetime cases is still in progress at University Park, Pa. and Montpellier, France.

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