PHYSICAL REVIEW B VOLUME 17, NUMBER 6 15 MARCH 1978

Minority-carrier injection into semiconductors

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(Received 26 September 1977)

On the basis of the linearized transport equations (small-signal theory) it is shown that minority-carrier injection into a trap-free lifetime semiconductor leads to a local-field maximum, and hence to a hitherto unrecognized resistance *increase*, as long as the majority carriers have the greater mobility. This applies to any value of the injection ratio γ, where γ_0 is the fraction of current carried by minority carriers in the undisturbed bulk. The conventionally expected bulk resistance *decrease* makes itself felt only at relatively high current densities. It is also shown that the ratio $A_n = \tau_D/\tau_0$ (where τ_D is the dielectric relaxation time, and τ_0 the carrier lifetime) governs the boundary between the relaxation and lifetime regimes only when the injection ratio γ is unity. For $\tau_D/\tau_0 = 1$ we then have $\Delta N = 0$, irrespective of X. However, when $\gamma < 1$, there is no value of τ_D/τ_0 which gives $\Delta N = 0$ everywhere, hence no simple boundary between the two conduction regimes. The equations developed are general, and can be applied to a variety of other transport problems.

INTRODUCTION

In two previous papers, Popescu and Hensich[1,2] analyzed the problem of minority-carrier injection into lifetime and relaxation semiconductors, with and without traps. As a first approximation, a lifetime semiconductor is defined as one in which $\tau_D < \tau_0$, where τ_D is the dielectric relaxation time of the material and τ_0 the carrier lifetime. In a relaxation semiconductor $\tau_D > \tau_0$. The work involved computer generated numerical solutions of the standard transport equations which cannot be explicitly solved in their full generality. All solutions were based on the simplifying assumption of unit injection ratio ($\gamma = 1$), meaning that the entire current at $x=0$ is carried by minority carriers. In practice, the minority-carrier participation may be less, but the assumption of $\gamma = 1$ permitted an assessment of the *maximum effect* which might be expected in various circumstances. The calculations also assumed equal electron and hole mobilities ($\mu_n = \mu_p$), again for reasons of simplicity, but with some loss of generality, The numerical solutions showed (a) that there is indeed a majority-carrier depletion in the injection region, as first pointed out by van Roosbroeck and co-workers,[3,4] (b) that, contrary to previous predictions, such a majority-carrier depletion does *not* lead to a resistance increase in the absence of traps,[2] and (c) that injection *can* lead to a resistance increase when traps in sufficient concentration are present, but for reasons which have nothing to do with majority-carrier depletion. Indeed, when this phenomenon occurs, the majority-carrier concentration is locally augmented compared with the trap-free case, other things being equal. There are circumstances, actually much more favorable in the lifetime than in the relaxation case, in which this leads to a localized higher-than-equilibrium concentration of majority carriers which in turn creates an all important diffusion gradient in the direction opposing the current. It is this diffusion gradient which is responsible for the effective resistance increase.

It is the purpose of the present paper to show that (i) the transport equations can be explicitly solved when linearized within the framework of a "small-signal theory"; (ii) that the solutions are in agreement with the computer generated curves not only within the strict limits of the small-signal theory assumptions but in many cases over a surprisingly large range of circumstances outside it; (iii) that there are circumstances in which a resistance increase is associated with minority-carrier injection even in the absence of traps, provided one is dealing with the lifetime case. For an n-type semiconductor, these circumstances apply when $\mu_n > \mu_p$, where μ_n and μ_p are the mobilities of majority and minority carriers, respectively; and (iv) that no resistance increase can be expected from a trap-free relaxation case (always associated with majority-carrier depletion) as pointed out by Kiess and Rose,[5] no matter what the mobility ratio may be.

The linearization of the equations in the present form and with appropriate boundary conditions can, of course, be applied to other processes and phenomena, but only injection is dealt with in this paper. An application to Hall-effect analysis has also been demonstrated.[6]

Although contact resistances as such are not encompassed by the present (or previous) calculations, the results are believed to be of considerable importance in the interpretation of voltage-current characteristics.

Early calculations on the effect of injected minority carriers were carried out under a number of simplifying assumptions, which permitted the transport equations to be explicitly solved. Only recently has it been appreciated how much the solutions depended on those very assumptions, and that substantially different conclusions result when such assumptions are not made. Bardeen and Brattain[7] and Banbury,[8] for instance, assumed that the majority-carrier current *and* recombination can be neglected. In such circumstances, the injected minority carriers could do no other than to diminish the effective local resistivity everywhere. This expectation came to be regarded as a universal standard feature of all such systems. Even during subsequent work[9-11] it has been customary to assume local neutrality, on the ground that Δp, the departure from hole equilibrium, and Δn, the departure from electron equilibrium, are bound to be nearly equal. They are indeed, but the neglect of the small difference between them suppresses all consideration of field curvature and thus, indirectly, the suppression of some features arising from diffusion. It will be shown that the full equations, even in linearized form, yield local resistance decreases and increases, depending on distance from the injecting boundary. Within the limitations of the linearized theory (only), all these changes are independent of current density. Traps constitute yet another modifying factor,[2,12] but are not considered in this paper. On the other hand, whereas previous calculations were limited to an injection ratio of unity (for an assessment of the maximum effect), the present work is not resticted to this case.

LINEARIZED THEORY; GENERAL RELATIONSHIPS

Two equations for current, two continuity relationships, and Poisson's equation (together with $J=J_n+J_p$), as always, control the conduction mechanism and field distribution. For bimolecular recombination (trap-free case), the one-dimensional equations are

$$J_n = q\mu_n nE + \mu_n kT\frac{dn}{dx}\ , \tag{1}$$

$$J_p = q\mu_p pE - \mu_p kT\frac{dp}{dx}\ , \tag{2}$$

$$\frac{np-n_e p_e}{\tau_0(n_e+p_e)} - \mu_n\frac{d(nE)}{dx} - \frac{\mu_n kT}{q}\frac{d^2n}{dx^2} = 0\ , \tag{3}$$

$$\frac{np-n_e p_e}{\tau_0(n_e+p_e)} + \mu_p\frac{d(pE)}{dx} - \frac{\mu_p kT}{q}\frac{d^2p}{dx^2} = 0\ , \tag{4}$$

$$\frac{dE}{dx} = \frac{q}{\epsilon}(p-n+n_e-p_e)\ , \tag{5}$$

n_e and p_e being the equilibrium concentrations of electrons and holes, respectively. The carrier lifetime τ_0 is assumed to be independent of injection level.

For computation purposes, it is convenient to normalize the equations by putting

$$N=n/n_e\ ,\quad P=p/n_e$$

$$E=E_{\text{real}}/(kT/qL_D),\quad X=x/L_D\ ,$$

$$J=j/[\mu_p kT(n_e+p_e)/L_D]\ ,\quad V=v/(kT/q)\ ,$$

$$\Delta N=N-N_e=N-1\ ,\quad \Delta P=P-P_e\ ,$$

where $b=\mu_n/\mu_p$ and

$$L_D=[\epsilon kT/q^2(p_e+n_e)]^{1/2}, \tag{6}$$

this being the effective Debye length. The equations then become

$$J_n = \frac{b}{1+P_e}\left(NE+\frac{dN}{dx}\right), \tag{7}$$

$$J_p = \frac{1}{1+P_e}\left(PE-\frac{dP}{dx}\right), \tag{8}$$

$$\frac{d^2N}{dX^2}+\frac{d(NE)}{dX}-\frac{A_n}{1+p_e}(\Delta N\Delta P+\Delta P+P_e\Delta N)=0\ , \tag{9}$$

$$\frac{d^2P}{dX^2}-\frac{d(PE)}{dx}-\frac{A_p}{1+p_e}(\Delta N\Delta P+\Delta P+P_e\Delta N)=0\ , \tag{10}$$

$$\frac{dE}{dX}=\frac{1}{1+P_e}(\Delta P-\Delta N)\ , \tag{11}$$

with

$$A_n=\frac{\epsilon}{q\mu_n\tau_0(n_e+p_e)}$$

and (12)

$$A_p=bA_n=\frac{\epsilon}{q\mu_p\tau_0(n_e+p_e)}\ .$$

The parameter A_n (called A in the papers by Popescu and Henisch) is approximately equal to the ratio of the dielectric relaxation time to the carrier lifetime.

As long as consideration is limited to the regime of small currents, the transport equations can be linearized by assuming $\Delta N \ll 1$ and $\Delta P \ll P_e$ neglecting the cross products $E(dN/dx)$ and $E(dP/dx)$ which are of second order. The equations then reduce to

$$J_n=\frac{b}{1+P_e}\left(E+\frac{d\Delta N}{dX}\right), \tag{13}$$

$$J_p=\frac{1}{1+P_e}\left(P_eE-\frac{d\Delta P}{dX}\right), \tag{14}$$

$$\frac{d^2\Delta N}{dX^2}+\frac{dE}{dX}-\frac{A_n}{1+P_e}\,(\Delta NP_p+\Delta P)=0\,, \tag{15}$$

$$\frac{d^2\Delta P}{dX^2}-P_e\,\frac{dE}{dX}-\frac{A_p}{1+P_e}\,(\Delta NP_e+\Delta P)=0\,, \tag{16}$$

$$\frac{dE}{dX}=\frac{1}{1+P_e}\,(\Delta P-PN)\,. \tag{17}$$

Equations (15) with (17) and (16) with (17) give

$$\frac{d^2\Delta N}{dX^2}-\Delta N\left(\frac{1+P_eA_n}{1+P_e}\right)-\Delta P\left(\frac{A_n-1}{1+P_e}\right)=0\,, \tag{18}$$

$$\frac{d^2\Delta P}{dX^2}+\Delta N\left(\frac{P_e-P_eA_p}{1+P_e}\right)-\Delta P\left(\frac{A_p+P_e}{1+P_e}\right)=0\,. \tag{19}$$

These two differential equations can be readily solved in the general form

$$\Delta N=-\frac{B}{P_e}\,\exp(X)-\frac{C}{P_e}\,\exp(-X)$$
$$+R(1-A_n)\exp(X\sqrt{\mathcal{A}})+S(1-A_n)\exp(-X\sqrt{\mathcal{A}})\,, \tag{20}$$

$$\Delta P=B\exp(X)+C\exp(-X)$$
$$+R(1-A_p)\exp(X\sqrt{\mathcal{A}})+S(1-A_p)\exp(-X\sqrt{\mathcal{A}})\,, \tag{21}$$

where

$$\mathcal{A}=\frac{L_D^2}{\tau_0D_a}=\frac{1}{1+P_e}\,A_p+\frac{P_e}{1+P_e}\,A_n\,,$$
$$A_n=\frac{1+P_e}{b+P_e}\,\mathcal{A}=\frac{A_p}{b}\,, \tag{22}$$

$$D_a=\frac{kT(n_e+p_e)\mu_n\mu_p}{q(\mu_nn_e+\mu_pp_e)}\,, \tag{23}$$

the ambipolar diffusion constant. B, C. R, and S are constants of integration which remain to be fixed by reference to the boundary conditions.

FIELD MAXIMUM AND RESISTANCE INCREASE; THEORY

For the case here under consideration, minority-carrier injection through a boundary at $X=0$, the boundary conditions arise as follows.

(a) For a minority-carrier injecting contact, there is always a location, immediately at the end of the contact barrier, at which $E=0$. This location is defined as $X=0$. For different current levels this reference point need not be the same, but differences will be small. $E(0)=0$ leads at once to

$$J=\frac{1}{1+P_e}\left(b\,\frac{d\Delta N}{dX}\,(0)-\frac{d\Delta P}{dX}\,(0)\right). \tag{24}$$

(b) The injection ratio is defined as the fraction of the total current carried by minority carriers. Accordingly, the value γ_0 of this ratio in the undisturbed bulk is

$$\gamma_0=\frac{P_e}{b+P_e}\ . \tag{25}$$

At the injecting boundary ($X=0$) we have γ a variable parameter between γ_0 and unity. Under the above zero-field condition, we have

$$\gamma=\frac{d\Delta P}{dX}\,(0)\bigg/\left(\frac{d\Delta P}{dX}\,(0)-b\,\frac{d\Delta N}{dX}\,(0)\right), \tag{26}$$

(c) $N(\infty)=N_e$, (27)

(d) $P(\infty)=P_e$. (28)

Conditions (c) and (d) together imply

$$E(\infty)=J(1+P_e)/(b+P_e)\,. \tag{29}$$

Under these conditions, the integration constants can be evaluated as follows:

$$B=R=0\,, \tag{30}$$

$$S=J[\gamma(b+P_e)-P_e]/\sqrt{\mathcal{A}}\,b(1-\mathcal{A})\,, \tag{31}$$

$$C=JP_e[(1-A_p)-\gamma(1-b)]/b\,(1-\mathcal{A})\,. \tag{32}$$

In this form they can be substituted into Eqs. (20) and (21) to give $\Delta N(x)$ and $\Delta P(x)$,

$$\Delta N=\frac{J}{b(1-\mathcal{A})}\left(\frac{(1-A_n)[\gamma b-P_e(1-\gamma)]}{\sqrt{\mathcal{A}}}\,\exp(-X\sqrt{\mathcal{A}})\right.$$
$$\left.-[(1-\gamma)+b(\gamma-A_n)]\exp(-x)\right), \tag{33}$$

$$\Delta P=\frac{J}{b(1-\mathcal{A})}\left(\frac{1-\mathcal{A}_p)[\gamma b-P_e(1-\gamma)]}{\sqrt{\mathcal{A}}}\,\exp(-X\sqrt{\mathcal{A}})\right.$$
$$\left.+P_e[(1-\gamma)+b(\gamma-A_n)]\,\exp(-X)\right). \tag{34}$$

The field contour is obtained by adding Eqs. (13) and (14). In the present terms, this yields

$$E=\frac{J(1+P_e)}{b+P_e}\left(1+\frac{1}{b(1-\mathcal{A})(1+P_e)}\,\{(b-1)[\gamma b-P_e(1-\gamma)]\exp(-X\sqrt{\mathcal{A}})-[(1-\gamma)+b(\gamma-A_n)](b+P_e)\exp(-X)\}\right). \tag{35}$$

(Contrary to appearance, there is *no* discontinuity for $\mathcal{A}=1$, as can be shown by a limited series expansion of the exponential terms.)

Corresponding values of $V(X)$ are obtained by integration, taking $V=0$ at $X=0$, and the cumulative resistance R_X of the system between $X=0$ and X is given by

$$R_X = R_0\left(1+\frac{\{(b-1)[\gamma b - P_e(1-\gamma)]/\sqrt{\mathcal{A}}\}\,[1-\exp(-X\sqrt{\mathcal{A}})] - [(1-\gamma)+b(\gamma - A_n)](b+P_e)[1-\exp(-X)]}{b(1-\mathcal{A})(1+P_e)X}\right), \tag{36}$$

where R_0 $(0-X)$ is the resistance of the same length of material in the state of Ohmic conduction (zero injection, $\gamma=\gamma_0$).

Reference to Eq. (35) shows that E has a maximum at $X=X_m$: the crossover point between the ΔN and ΔP contours (ΔP=ΔN), we have

$$\exp[X_m(1-\sqrt{\mathcal{A}})] = \frac{(b+P_e)[(1-\gamma)+b(\gamma - A_n)]}{\sqrt{\mathcal{A}}(b-1)[\gamma b - P_e(1-\gamma)]} \tag{37a}$$

It can be shown that for $\gamma > \gamma_0$ this equation has a solution only for

$$b>1$$

and (37b)

$$A_n < [1+\gamma(b-1)]\,/b\,.$$

In the case dealt with by Popescu and Henisch,[1] $b=1$ and $\gamma=1$, there is no solution for a finite value of X, which explains why no field maximum was found, and hence no resistance increase, for the trap-free case. However, a resistance increase *is* here predicted in the low current range for all values of γ between γ_0 and unity, as long as the inequalities (37b) are satisfied.

FIELD MAXIMUM AND RESISTANCE INCREASE; RESULTS

Case $\gamma=1, b=1; A_n < 1$ [lifetime regime]

Figure 1 shows the results in terms of ΔN, ΔP, and E as functions of X. Because the normalizations are here different, the parameters used correspond exactly to those used in the computed curves, Fig. 2, of Popescu and Henisch[1] (PH); $J=0.0311$ in the present terms corresponds to the value $J=10$ with Popescu and Henisch normalization for J. To facilitate the comparison, Fig. 1 (only) is plotted with an abscissa of $X'=X(L_D/L_p)$ $=X\sqrt{\mathcal{A}}$, where X' refers to the PH normalization and X to that used in the present paper. Even though ΔP for small values of X is much greater than $P_e=0.01$ (upper limit for the small signal theory) the results of the linearized model are in remarkable agreement with those obtained by the use of the full equations. As long as $\Delta P < P_e$ and $\Delta N < N_e = 1$, the agreement between the two models extends in fact over a great variety of operational conditions including those in the relaxation regime. No field maximum and resistance increase is found in this case. The usefulness of the linearized model is confirmed by the high level of agreement with computer solutions derived from the complete equations.

Case $\gamma=1; b>1; A_n < [1+\gamma(b-1)]/b = 1$

Figure 2 shows that there is now a field maximum (corresponding to the crossover between ΔP and ΔN, and this implies a total resistance increase. It will be shown below that the conventional expectation of a resistance *decrease* is fulfilled only for a higher range of currents. A qualitative analysis made by Kiess and Rose[5] for the relaxation case leads to the conclusion that no resistance increase can arise from majority-carrier *depletion*. The present results do not, of course, conflict with this result in any way. There is, likewise, no resistance increase for the case $\gamma=1$, $A_n<1$ when $b<1$ (see discussion below).

Cases γ variable; $b>1; A_n < [1+\gamma(b-1)]/b$

Figure 3 shows the resistance increase between the injecting boundary and $X=X$, in correspondence with Eq. (36), for the same numerical values.
It will be seen that this increase is predicted for any injection ratio $\gamma > \gamma_0$.

Case of $\gamma=1$, for intrinsic germanium

Figure 4 deals with the same situation but with parameters corresponding to those of intrinsic germanium. The resistance of the sample is again increased as a result of minority-carrier injection. For example, in the case discussed, the

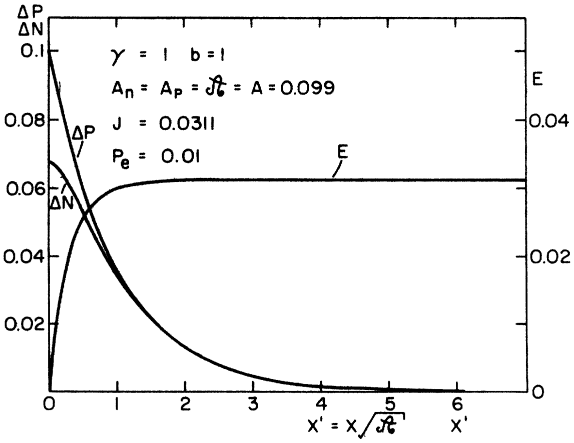

FIG. 1. Concentration and field contours for a case characterized by unit injection ratio ($\gamma=1$), unit mobility ratio ($b=1$), and $A_n=\mathcal{A}<1$ (lifetime regime).

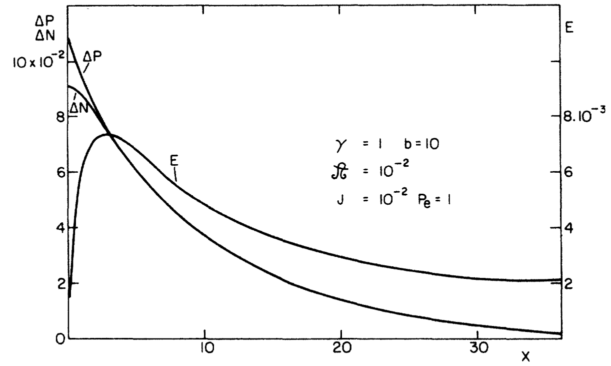

FIG. 2. Concentration and field contour for a case characterized by $\gamma = 1$, $b > 1$.

maximum field is

$$E(X_m) \approx 0.36 \text{ V/cm}, \; X_m = 6.9 \; L_D ,$$

and the field at $X = 700 \; L_D$ is

$$E(700 L_D) \approx 0.25 \text{ V/cm},$$

which means that the resistance increase should be easily measurable. Intrinsic Ge is particularly suitable for this purpose because the relatively high value of P_e makes it possible to pursue the linear approximation to high current densities. Moreover, the injection effects are more important here than for highly extrinsic material (see Appendix).

Case of $\gamma = 1$ for extrinsic germanium ($\approx 1.7 \; \Omega \cdot \text{cm}$)

Figure 5 refers to a typical extrinsic Ge (n type $\rho \simeq 1.7 \; \Omega \, \text{cm}$) and shows the situation near $X = 0$ on an expanded scale. The crossover of the ΔN and ΔP contours occurs at $X_m \approx 7.95$, but $\Delta P - \Delta N$ is even here too small to be visible at greater distances.

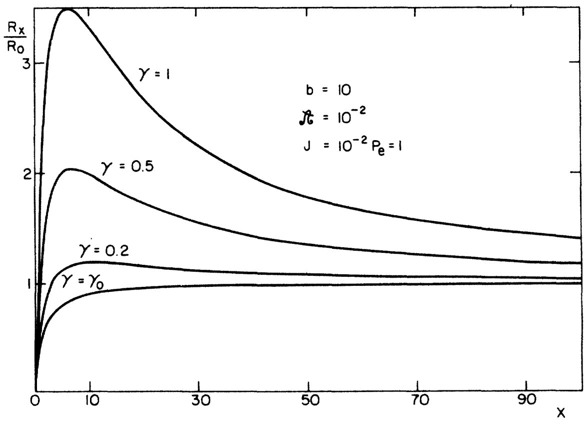

FIG. 3. Resistance increase between the injecting boundary $X = 0$ and bulk $X = X$ [Eq. (36)] for a case characterized by γ variable, and $b > 1$; R_0 is the resistance of same interval in the absence of injection.

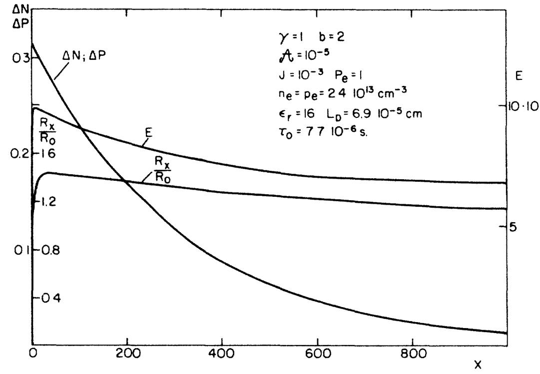

FIG. 4. Resistance increase between the injecting boundary $X = 0$ and bulk $X = X$ [Eq. (36)] for hole injection into intrinsic germanium; R_0 is the resistance of the same interval in the absence of injection.

BOUNDARY BETWEEN RELAXATION AND LIFETIME REGIMES

For the trap-free case, $A_n = 1$ has hitherto been regarded as the boundary between the lifetime regime (conventionally defined by $\Delta N > 0$) and the relaxation regime (conventionally defined by $\Delta N < 0$), in each case irrespective of ΔP. However, this boundary is valid *only* as long as the injection ratio is unity ($\gamma = 1$), as assumed in all the Popescu and Henisch calculations.[1,2] Equations (33) and (34) then yield solutions which make $\Delta N > 0$ or $\Delta N < 0$ irrespective of X, and for $A_n = 1$, $\Delta N = 0$ everywhere. However, when $\gamma < 1$, a set of conditions also covered by the present calculations, $A_n = 1$ ceases to be a boundary in this sense: there is no longer any value of A_n which makes $\Delta N = 0$, irrespective of X. This means that there is no longer any simple boundary between the two operating regimes, though it remains true that $\Delta N < 0$ when $A_n \gg 1$ and $\Delta N > 0$ when $A_n \ll 1$ in the trap-free case. The complications relating to boundary conditions have also been recognized by Popescu and Stoica.[13]

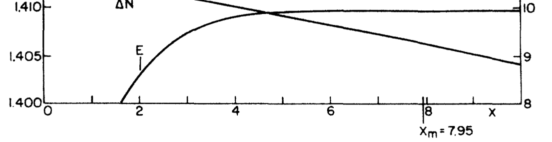

FIG. 5. Concentration and field contours for minority-carrier injection into typical extrinsic germanium, situation close to the injecting boundary.

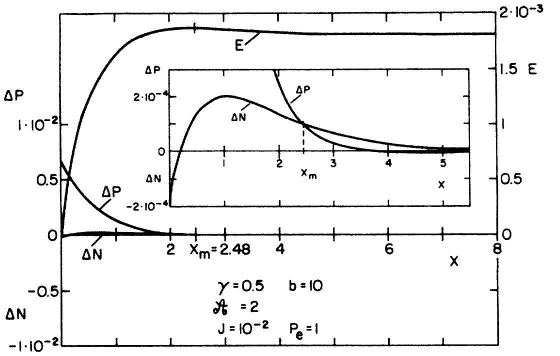

FIG. 6. Concentration and field contours for minority-carrier injection under conditions which make $\Delta N \approx 0$.

Figure 6 shows ΔN and ΔP profiles for the same material as that considered in Figs. 2 and 3, except that the lifetime is different. The value $A_n = 0.364$ was chosen to correspond to $\mathfrak{A} = 2$ by Eq. (22). This value of a A_n is close to that given by the boundary inequality (37b), namely, $A_n = 0.55$. On the normal scale, we see that ΔN *is* approximately zero everywhere, but contours drawn on the expanded scale make it clear that there is in fact a crossover for ΔN from $\Delta N < 0$ to $\Delta N > 0$, and again a maximum field E_m. For $A_n > 0.55$, however, we have $\Delta P > 0$ and $\Delta N < 0$ for any value of X.

DISCUSSION

Equations (20) and (21) are general and can be used in a variety of other contexts, e.g., minority-carrier extraction, accumulation, exclusion,[14] etc., depending only on the boundary conditions. Their form calls for further discussion.

The conventional procedure is to make the assumption of neutrality ($\Delta N = \Delta P$) in Eqs. (15) and (16) but not in Poisson's Eq. (17).[9,11] By reference to Eqs. (15) and (16) the consequences of this assumption can be readily assessed. After elimination of dE/dX, the equations yield

$$\frac{d^2 \Delta N}{dX^2} = \frac{A_p + P_e A_n}{1 + P_e}\, \Delta N = \mathfrak{A}\, \Delta N. \tag{38}$$

The solutions for $\Delta N = \Delta P$ as functions of X involve only terms containing $\exp(\pm\sqrt{\mathfrak{A}}X)$, or in unnormalized form $\exp\{\pm [x/(\tau_0 D_a)^{1/2}]\}$. Comparison with Eqs. (20) and (21) shows that this leads to the arbitrary elimination of the first two terms in $e^{\pm X}$. It will be clear that these two terms can have a critical influence on the question whether ΔP and ΔN vary monotonously with X or not. In the same way, they influence field profile. The numerical results on Figs. 2–4 demonstrate that this is a matter of importance; it is directly responsible for the appearance of a maximum in the field contour, and thus for the resistance increase. It is, in all likelihood, because $\Delta N = \Delta P$ has been a superficially plausible and, in any event, a very popular assumption, that these new effects were missed by analysis in the past.

The physical interpretation of the resistance increase as such is here very similar to that formulated by Popescu and Henisch for the lifetime case with traps.[2] In that case, trapped minority carries attract free majority carriers to the region in the neighborhood of the injection boundary. They do this under all conditions, but the degree of charge compensation (the approach to neutrality) will be greatest in lifetime semiconductors. In such materials, charge compensation results in a concentration gradient $dN/dx < 0$. Corresponding to this gradient there is diffusion current, and that flows in opposition to the injection current. However, the calculations are made for a given *total* current, which means that the presence of the concentration gradient must be compensated by an extra field. Only then can the current remain unchanged. In the present (trap-free) lifetime cases, majority carriers are likewise attracted to the injection region by a positive space charge. However, that charge owes its existance to a different mechanism, namely, the high density of free minority carriers that is needed to carry the (total) current in the vicintiy of $x = 0$ when $\mu_p < \mu_n$. Under such conditions, therefore, the resistance increase due to minority-carrier injection can occur without traps. Under all other conditions; traps are needed.

The resistance increases exemplified by the extent of the field "overshoot" data on Figs. 2 and 4 are substantial, and the question arises as to why they have not been found and noted in the course of experimentation. There are two likely explanations:

(i) As shown above, the resistance increase is expected only in the range of very low currents, whereas practical measurements are usually carried out at higher current densities for which the conventional expectation of a resistance decrease is correct. This can be demonstrated in a very simple way: we are dealing with the lifetime case ($A_n \ll 1$), far away from the injecting contact in the region near the unperturbed bulk where $\Delta p \simeq \Delta n$ and $(d\Delta n/dx \simeq (d\Delta p/dx)$. With $\Delta n \ll n_e$, $\Delta p \ll p_e$ the total conduction current J_C can be written

$$J_C \approx q \Delta n(x)(\mu_n + \mu_p)E + q(n_e \mu_n + p_e \mu_p)E; \tag{39}$$

the total diffusion current J_D can be written

$$J_D \approx kT\mu_p(b-1)\,\frac{d\Delta n}{dx}\,. \tag{40}$$

As is well known, in this region the excess concentration $\Delta n(x)$ varies as: $\Delta n \propto \exp\{-[x/(\tau_0 D_a)^{1/2}]\}$, where D_a is the ambipolar diffusion length, Eq. (23), leading to $(d\Delta n/dx) = -[\Delta n(x)/(\tau_0 D_a)^{1/2}]$, and

$$J_D \approx -kT\mu_p(b-1)\frac{\Delta n(x)}{(\tau_0 D_a)^{1/2}} . \tag{41}$$

It is easy to see that this diffusion current is opposite to the conduction current when $b > 1$. The "equivalent conductivity" of the sample will decrease due to the injected carriers (compared to the conductivity of the unperturbed bulk) as long as the conduction current due to Δn is less than the opposing diffusion current, i.e., by Eqs. (39) and (41):

$$\begin{gathered} q\Delta n(x)(\mu_n + \mu_p)E_c < \frac{kT\mu_p(b-1)\Delta n(x)}{(\tau_0 D_a)^{1/2}} , \\ E_c < \frac{b-1}{b+1} \frac{kT}{q\tau_0}\left(\frac{(\mu_n n_e + \mu_p p_e)}{(n_e + p_e)\mu_n \mu_p}\right)^{1/2} , \end{gathered} \tag{42}$$

which define a critical field E_c. In a practical case, for intrinsic germanium ($T = 300$ K; $\mu_n = 3600$ cm^{2}V^{-1} sec^{-1}; $\mu_p = 1800$ cm^{2}V^{-1} sec^{-1}; $n_e = P_e = 2.4 \times 10^{13}$ cm^{-3}; $\tau_0 = 7 \times 10^{-6}$ sec), the critical field $E_c = 0.41$ V cm^{-1}. For $E_\infty < E_c$ one should observe the resistance increase as predicted by probing along the sample; for $E_\infty > E_c$, the conventional resistance decrease. The quantitative results suggest that the changeover of conduction patterns should be easily measurable. The overall situation is thus as summarized by Fig. 7.

(ii) When contact characteristics are measured, the contact barrier is an integral part of the system, whereas the present considerations apply only to the bulk material beyond the barrier. The effects here predicted are therefore likely to be masked by the inherent superlinearity of the contact barrier itself. As a result, this superlinearity (measured between the injecting contact and a potential probe at some point X) would be diminished, and it is a well-known fact that the forward characteristics of rectifying junctions are always more resistive than expectations based on contact theories alone lead us to suppose. From the point of view of rectifier design this is, of course, a disadvantage, but it is already known that a high dark resistance in the forward direction (high empirical ideality factor) is actually helpful to solar cell operation.[15] This follows from the fact that the open circuit condition involves a situation in which the (essentially) reverse current produced by light is balanced by an equal forward current produced by forward (self-) bias. Experiments on systems which are specifically designed to demonstrate the present effects remain to be performed (see Appendix).

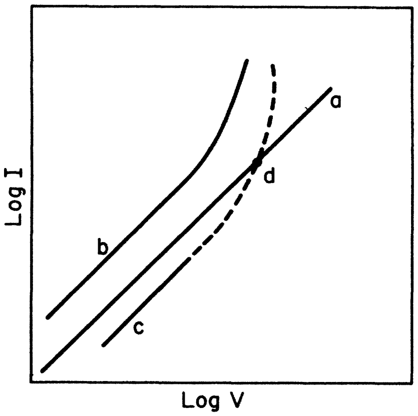

FIG. 7. Departures from linearity as a result of minority-carrier injection (schematic). (a) Linear characteristic for $\gamma = \gamma_0$. (b) Typical behavior calculated by Popescu and Henisch (Ref. 1) for the trap-free relaxation case and $b = 1$. (c) Range of the present calculations ($b > 1$); low-current regime. (d) Crossover point to the high current regime.

ACKNOWLEDGMENTS

The present work was carried out under NSF Grant No. INT 77-04433 issued by the Office of International Program, which is hereby acknowledged. Grateful thanks are also due to C. Popescu for much helpful correspondence, and to Professor J. P. Fillard and Professor M. Savelli for generous support.

APPENDIX: CONDITIONS FOR THE VERIFICATION OF THE RESISTANCE INCREASE

Intrinsic Ge offers the best opportunities, because its room-temperature conductivity is still relatively high, and because highly injecting contacts can be achieved. There are two ways of calculating the hole injection ratio γ: one on the basis of diffusion theory,[16,17] and one on the basis of thermionic emission (diode theory).[18] Of these, the diffusion theory gives lower values, and thus leads to conservation estimates of injection effect. From the work of Yu and Snow,[17] modified for the semi-infinite geometry here considered, one derives

$$\gamma \simeq \frac{qD_p P_e}{L_p} \Big/ \left(\frac{qD_p P_e}{L_p} + J_s\right)$$

for the injection ratio of a Schottky barrier, where J_s is the saturation current density. The barrier height controls J_s, and for high barriers J_s can be very small compared with the diffusion term $qD_p P_e/L_p$. For the parameters of near-intrinsic

Ge, the equation yields $\gamma \approx 0.3$ to 0.7 for typical barrier heights, which should be quite high enough for the resistance increase to be comfortably observed. Even more favorable estimates of γ can be derived from the work of Sze,[18] and Green and Shewchun.[19] The fact that a great deal of hole injection can be detected even in slightly extrinsic ($\rho \sim 10\ \Omega\,\mathrm{cm}$) Ge has already been demonstrated.[20]

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