Gap enhancement in superconducting thin films due to quasiparticle tunnel injection

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The enhancement of superconductivity due to tunnel injection was studied by a symmetric superconductor-insulator-superconductor junction. Nonequilibrium phonons have been taken into account in this investigation.

There has been tremendous interest in nonequilibrium superconductivity in recent years. Of particular interest is the observation that superconductivity in thin films can be enhanced by driving the films away from equilibrium. About a decade ago, Dayem and Wiegand[1] and Wyatt *et al.*[2] discovered that the critical current of a superconducting microbridge can be enhanced by irradiating the bridge with microwaves. Recently, Klapwijk *et al.*[3] reported that microwave irradiation can induce superconductivity 16 mK above the equilibrium transition temperature in an Al thin strip. Kommers and Clarke[4] have observed the enhanced energy gap in superconducting thin film irradiated with microwaves by tunneling measurements.

Enhancement of the critical superconducting current has also been observed using high-frequency phonons. Tredwell and Jacobsen[5] reported that in a microbridge or point contact junction, the critical superconducting current can be enhanced by phonon injection with phonon frequency $\Omega < 2\Delta/\hbar$. Here Δ is the energy gap of the superconductor.

Recently, Gray[6] has observed gap enhancement in superconducting thin films due to quasiparticle tunnel injection. The idea was first proposed by Parmenter[7] and was further studied by Peskovatskii and Seminozhenko[8] using a linearized quasiparticle kinetic equation. In their discussions, phonons were assumed to be in thermal equilibrium with the ambient temperature. In this short paper we discuss the gap enhancement in thin superconducting film due to quasiparticle tunnel injection with nonequilibrium phonons taken into account.

The reason that superconductivity can be enhanced by tunnel injection of quasiparticles is easy to understand. When a junction is biased at a voltage eV smaller than the sum of the gaps of the junction films, there will be no net pair breaking due to the primary tunneling processes. The dissipated power is due to the shoveling of low-energy quasiparticles into higher-energy states. Therefore, as in the case of microwave irradiation, the superconductivity can be enhanced.[9-11]

We calculate the energy gap in the steady state for a thin superconducting film which is part of a symmetric superconductor-insulator-superconductor (SIS) tunnel junction. Our study is based upon the coupled kinetic equations for quasiparticle and phonon distributions and a modified BCS gap equation[12] with the equilibrium quasiparticle distribution function replaced by its steady-state value. This set of nonlinear integral equations has been discussed before[13] and will not be discussed here again.

In the case of tunnel injection, a simple Golden-Rule calculation gives the quasiparticle injection rate per unit volume for the film with lower electronic potential:

$$\begin{aligned} I_{qp}^{\gtrless}(E) = \frac{\sigma_n}{2N(0)e^2 d} \Bigg\{ & \Bigg[\begin{Bmatrix} u^2(E) \\ v^2(E) \end{Bmatrix} u^2(E-eV)[f_<(E-eV) - f_{\gtrless}(E)] + \begin{Bmatrix} u^2(E) \\ v^2(E) \end{Bmatrix} v^2(E-eV)[f_>(E-eV) - f_{\gtrless}(E)] \Bigg] \\ & \times \theta(E-eV-\Delta)\rho(E-eV) + \Bigg[\begin{Bmatrix} v^2(E) \\ u^2(E) \end{Bmatrix} v^2(E+eV)[f_<(E+eV) - f_{\gtrless}(E)] + \begin{Bmatrix} v^2(E) \\ u^2(E) \end{Bmatrix} \\ & \times u^2(E+eV)[f_>(E+eV) - f_{\gtrless}(E)] \Bigg] \theta(E+eV-\Delta)\rho(E+eV) \\ & + \Bigg[\begin{Bmatrix} u^2(E) \\ v^2(E) \end{Bmatrix} u^2(eV-E)[1 - f_{\gtrless}(E) - f_>(eV-E)] + \begin{Bmatrix} u^2(E) \\ v^2(E) \end{Bmatrix} v^2(eV-E) \\ & \times [1 - f_{\gtrless}(E) - f_<(eV-E)] \Bigg] \theta(eV-E-\Delta)\rho(eV-E) \Bigg\} , \end{aligned} \quad (1)$$

where $f_{\gtrless}(E)$ is the steady-state quasiparticle distribution of energy E on the branch with wave vector $|k| \gtrless |k_f|$, σ_n is the normal-state conductivity per unit area of the tunnel junction, d is the film thickness, and $N(0)$ is the single-spin normal-state density of states at the Fermi surface. For the film with higher electronic potential, $I_{qp}^{>}$ and $I_{qp}^{<}$ change roles.

To obtain the above expression for $i_{qp}^{\gtrless}(E)$ we have used the definitions

$$\begin{Bmatrix} u^2(E) \\ v^2(E) \end{Bmatrix} \equiv \frac{1}{2}\left[1 \pm \frac{(E^2-\Delta^2)}{E}\right]^{1/2} \tag{2}$$

and

$$\rho(E) \equiv [E/(E^2-\Delta^2)^{1/2}]\theta(E-\Delta) \quad . \tag{3}$$

We have also assumed electron-hole symmetry in the symmetric SIS junction to take advantage of the fact that $f_{\gtrless}^{L}(E)$, the distribution function on the left-hand side of the junction is equal to $f_{\lessgtr}^{R}(E)$ on the right-hand side.

If the branch imbalance effects are neglected, Eq. (1) can be simplified to give

$$I_{qp}(E) \simeq [\sigma_N/2N(0)e^2d]\left[\left\{\rho(E+eV)[f(E+eV)-f(E)]\theta(E+eV\Delta)\right\}+\{eV\to -eV\}-\{E\to -E\}\right] , \tag{4a}$$

with

$$I_{qp}(E) = I_{qp}^{>}(E) + I_{qp}^{<}(E) \quad , \tag{4b}$$

where

$$f(E) \simeq f_{>}(E) \simeq f_{<}(E) \quad .$$

This is a good approximation if (i) the branch-mixing time due to elastic process is shorter than the inelastic lifetime of the quasiparticles or (ii) $eV + k_BT \lesssim 2\Delta$, so that only a few high-energy quasiparticle states are involved. Since the size of the energy gap depends upon the energy distribution of the quasiparticles and does not depend upon the branch imbalance, we believe that the branch imbalance effect will not require significant modification of our results obtained by using Eq. (4).

Equation (4) has the same form as that for the case of microwave irradiation on a thin film except that in the present case there are no coherence factors involved. In this case, the convenient parameter to measure the coupling strength is

$$A = [\sigma_N \tau_0/2N(0)e^2d](T_c/\Delta_0)^3 \quad , \tag{5}$$

where τ_0 is the normal-state electron scattering time of an electron with energy k_BT_c above the Fermi surface at zero temperature (Eq. 6 of Ref. 13).

The calculated results for the steady-state quasiparticle distribution $f(E)$ and phonon spectrum [phonon distribution $n(\Omega)$ multiplied by Ω^2 which is taken to be proportional to the phonon density of states] using Eq. (4) for the injection term, are shown as the dashed curves in Fig. 1.[14] In the figure we have used

$$\tau_{es} = 8\tau_B(2\Delta_0) \quad .$$

Here τ_{es} is the phonon escape time and $\tau_B(2\Delta_0)$ is the phonon pair-breaking lifetime for phonons with energy

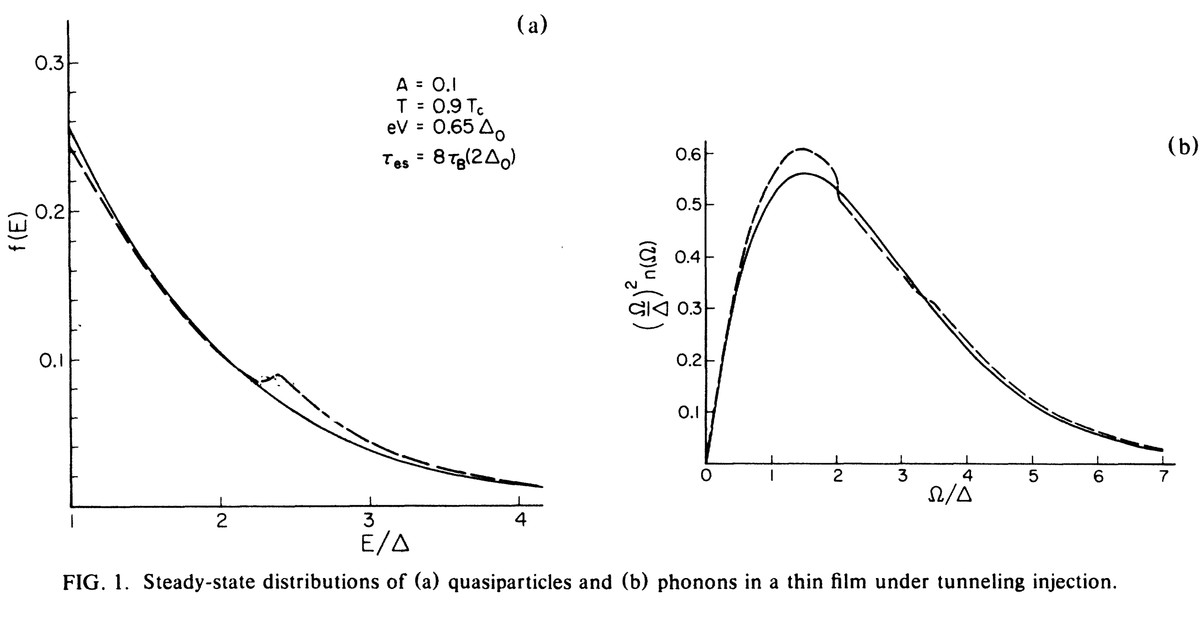

FIG. 1. Steady-state distributions of (a) quasiparticles and (b) phonons in a thin film under tunneling injection.

$\hbar\Omega = 2\Delta_0$ in an equilibrium superconductor at zero temperature. The corresponding thermal-equilibrium values are plotted in solid curves. The general structures in the steady-state curves are similar to those in the case of microwave irradiation. From Fig. 1(a), it is clear that there is shoveling of quasiparticles from states of $E \lesssim \Delta + eV$ into states of $E \gtrsim \Delta + eV$ as discussed before. Figure 1(b) shows that there are fewer phonons with energy $\hbar\Omega$ between 2Δ and $2\Delta + eV$ due to the deficiency of quasiparticles of energy $E \lesssim \Delta + eV$. The excess phonons of energy $\hbar\Omega < 2\Delta$ and $\hbar\Omega > 2\Delta + eV$ are mainly due to, respectively, the scattering and recombination relaxations of the quasiparticles. All these features can be understood in the same manner as discussed in Ref. 11 for the case of microwave irradiation.

The dependence of the energy gap upon the bias voltage is shown in Fig. 2 for $A = 0.05$ and $A = 0.1$.[15] At ambient temperature $T = 0.9T_c$ and phonon escape time $\tau_{es} = 8\tau_B(2\Delta_0)$, the maximum gap enhancement is slightly less than 4%. We note that the energy gap begins to drop before the bias voltage eV reaches 2Δ when the pair-breaking process sets in. This is because of the fact that, for eV slightly less than 2Δ, there will be a significant number of quasiparticles with energy $E > 3\Delta$ being excited. When these quasiparticles relax, phonons of energy $\hbar\Omega > 2\Delta$ can be emitted to break pairs and, therefore, to reduce the energy gap. This result is different from that given in Ref. 8, where the phonons were assumed to be unperturbed and it was found that the gap enhancement became more pronounced as $eV \rightarrow 2\Delta$ (but still $eV < 2\Delta$).

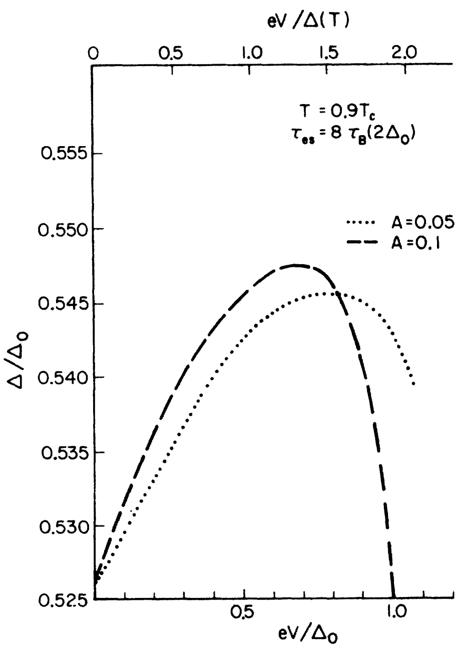

FIG. 2. Bias-voltage dependence of the gap enhancement in thin superconducting film.

One of the advantages of using tunnel injection of quasiparticles to study gap enhancement is that the parameters involved can be determined with reasonable accuracy and, hence, comparison of theoretical calculations and experimental results can be easily made. Qualitatively, our calculated curve for gap enhancement as a function of junction voltage agrees with the experimental curve.[6] But, the maximum gap enhancement measured at $T = 0.92T_c$ is about 1% of the equilibrium value while the calculated one is about 4% at $T = 0.9T_c$. Among other reasons this could be due to the fact that our calculation uses for the relaxation time τ_0, the value calculated by Kaplan *et al.*[16] This value may be too large considering that the actual film is slightly doped with oxygen and has a transition temperature of 1.3 °K.[6] Also, it could be due to the fact that in this calculation the magnetic field generated by the tunneling current is ignored. Both the above mentioned effects tend to reduce the energy gap. It is not unreasonable to expect that these factors will lead to a correction of a factor of 5.

In conclusion, we have studied the gap enhancement in superconducting thin films due to quasiparticle tunnel injection. Our study is based upon the coupled kinetic equations for the quasiparticle and phonon distributions, and a modified BCS gap equation[12] with the equilibrium quasiparticle distribution function replaced by the steady-state distribution function. Using reasonable parameters we find that the energy gap can be enhanced by about 4% of its equilibrium value at $T = 0.9T_c$. The bias voltage at which the maximum gap enhancement occurs depends upon both the coupling strength and the phonon escape times τ_{es} and is smaller than 2Δ. Our calculation also shows that it is possible to have gap enhancement even when the bias voltage is greater than twice the *equilibrium* energy gap $2\Delta(T)$. This can happen when either the coupling strength is not too big [as shown in Fig. 2, for $A = 0.05$ the gap is enhanced when $eV = 2\Delta(T)$, while for $A = 0.1$ the gap is depressed at the same voltage] or the phonon escape time is short enough. It will be interesting to study junctions made of clean Al films which have long quasiparticle relaxation times and to see how large a gap enhancement one can get. Also, it is desirable to include branch imbalance in the future study of this problem.

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[9]As Eliashberg (Ref. 10) pointed out, it is the low-energy quasiparticles which are most effective to inhibit the pairing process. Therefore, by scooping quasiparticles from low-energy states into high-energy states, superconductivity can be enhanced. Recently, Chang and Scalapino (Ref. 11) suggested that since the recombination rate of quasiparticle depends upon the energy of the emitted phonon, the recombination rate increases when quasiparticles are shoveled into higher-energy states. This leads to a steady-state quasiparticle density smaller than that in the equilibrium state and hence an enhancement of energy gap.

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[14]Essentially, we have solved three coupled singular integral equations to obtain the steady-state values for the energy gap and the distribution function $f(E)$ and $n(\Omega)$. We use iteration method. The criterion for convergence is the iterated results being within 1% of the trial values.

[15]Consider an $Al/Al_2O_3/Al$ tunnel junction of $\sigma_N = 10^4(\Omega\mathrm{cm})^{-1}$, $N(0) = 12 \times 10^{21}$ states/eV cm^3 and film thickness $d = 300$ Å. Assuming $\tau_0 = 438 \times 10^{-7}$ sec, we have $A = 0.07$.

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