PHYSICAL REVIEW B VOLUME 17, NUMBER 3 1 FEBRUARY 1978

Motion of flux lines in nearly pure superconductors

John Bardeen
Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
(Received 7 October 1977)

By use of a simple semiphenomenological argument, Sherman and the author showed that in nearly pure superconductors (mean free path large compared with the core radius) at low temperatures, the transport current flows directly through the core of a vortex line without backflow. Expressions they derived for the flux-flow resistivity and Hall angle are nearly identical with those derived later by Larkin and Ovchinnikov (resistivity) and by Kopnin and Kravtsov (Hall angle) using Green's-function methods except for a numerical factor of 4/3. It is shown that this factor comes from a difference in relaxation time between the vortex core and the normal metal.

Sherman and the author[1] gave a nonlocal generalization of an early local model of Bardeen and Stephen[2] to show that in the presence of a transport current J_T the vortex line moves so as to generate an electric field which drives the current directly through the core without backflow. The total current density, the sum of super and normal current components, is for $T \ll T_c$,

$$\vec{J} = \vec{J}_s + \vec{J}_n = \vec{J}_T + \vec{J}_0(\vec{r} - \vec{v}_L t) , \tag{1}$$

where $\vec{J}_T$ is the uniform transport current, $\vec{J}_0(\vec{r})$ is the circulating current around a stationary vortex and $\vec{v}_L$ is the velocity of the vortex line. The Fermi sea is everywhere displaced by the transport velocity $\vec{v}_T$ ($\vec{J}_T = ne\vec{v}_T$). In the vicinity of the core there is a balance between decrease of the current caused by scattering from bound states in the core and an increase from acceleration by an electric field. Taking v_T in the x direction, as a result of the Hall effect, $v_{Lx} = v_T$, or the vortex moves in the direction of the transport current with the transport velocity.

At low temperatures, only the bound states with energies less than the bulk pair potential, Δ_∞, can be excited and contribute to the scattering. The effective area of the core A_c, is that of a normal region with the same number of states in energy as that of the core. Equivalently, one may define a function $g(r)$ which represents the fraction of the density of states at the Fermi surface from the bound states and varies from unity on the axis $r = 0$ to zero as $r \to \infty$. Then

$$A_c = 2\pi \int g(r) r \, dr . \tag{2}$$

We found that the resistive loss from motion of the vortex line is that of a uniform current J_T flowing through a normal region of area A_c. We also showed that the line moves at a Hall angle equal to that of the normal metal in an effective field Φ_0 / A_c, where Φ_0 is the flux quantum.

Our results have been confirmed by a microscopic Green's-function calculation by Larkin and Ovchinnikov[3] for the resistive loss and to a close approximation by Kopnin and Kravtsov[4] for the Hall angle, except for a factor of $\frac{3}{4}$. The former authors suggested that the effective relaxation time for electrons in the core τ_c differs from that of a normal metal τ_n by this factor, that is $\tau_c = \frac{3}{4}\tau_n$. We agree and will show that this factor arises from the way averages are taken over the Fermi surface.

The expression we used for the density in energy of bound states in the core was taken from one derived by Kramer and Pesch.[5] Their expression for the core area A_c differed from the one we used by an equivalent factor of $\frac{3}{4}$. Both the energy loss and Hall angle involve the ratio τ / A_c. If we had taken $\tau = \tau_n$ and used the Kramer and Pesch value for A_c, we would have obtained results practically identical with those of the Soviet workers.

The bound states in the core, with Bogoliubov amplitudes $u_{n\uparrow}(r)$ and $v_{n\downarrow}(r)$, are designated by the quantum numbers (μ, k_z), where the magnetic quantum number μ is that of u_n. States of positive energy may be characterized by the pairs (u_n, v_n^*) as follows:

$$\begin{aligned} &(1,0), \quad (2,-1), \quad (3,-2), \ldots \\ &(0,1), \quad (-1,2), \quad (-3,4), \ldots , \end{aligned} \tag{3}$$

where the second row are the spin reversed states of the first-row.

According to Kramer and Pesch,[3] the energies of the states corresponding to $(\mu, -\mu+1)$, or its reversal, are for small μ such that $\epsilon_\mu \ll \Delta_\infty$

$$\epsilon_\mu = \frac{2\mu\Delta_\infty}{k_F v_F \cos^2\theta} \ln \frac{\pi \xi_0 \cos\theta}{2\xi_1} , \tag{4}$$

where $k_z = k_F \cos\theta$. From the behavior of the wave functions near the core axis as $T \to 0$, they felt that ξ_1 should be proportional to T at low T (corre-

 © 1978 The American Physical Society

sponding to "shrinkage" of the core as $T\to 0$). However, Ovchinnikov[6] in a recent calculation has pointed out that ξ_1 does not vanish but approaches a limiting value as $T\to 0$. This is irrelevant for the present discussion which depends only on the angular dependence of the density of states.

The density of states of one spin at the Fermi surface is

$$N_\uparrow(\theta)\,d\theta = \frac{d\mu}{d\epsilon_\mu}\,\frac{dk_z}{2\pi}\,\frac{\pi m \xi_0^2 k_F \cos^3\theta\, d\theta}{4\hbar^2 \ln(\pi\xi_0\cos\theta/\xi_1)}\,. \qquad (5)$$

Disregarding the slowly varying logarithmic term, the main difference in angular dependence from a normal metal is a factor of $\cos^2\theta$, as illustrated in Fig. 2 of Ref. 1. The angular dependence would be the same as that of a normal metal if one included only excitations of k_r, not those in k_z. In Ref. 1 we defined A_c such that the radial excitations are the same as for the normal metal. Since the relaxation rate is proportional to the density of states, we should have defined τ_c such that

$$\left\langle \frac{1}{\tau_c}\right\rangle = \left\langle \frac{2\cos^2\theta}{\tau_n}\right\rangle = \frac{4}{3\tau_n} \qquad (6)$$

to be consistent with our definition of A_c. The factor of 2 corresponds to the possibility of scattering between both signs of μ, as shown in (3). Thus in Ref. 1, τ should be interpreted as τ_c or replaced by $\frac{3}{4}\tau_n$, which would bring the result for flux-flow resistivity into agreement with Ref. 3.

The same considerations apply to the Hall angle α which is given in Eq. (33) of Ref. 1 as

$$\tan\alpha = v_{Lx}/v_{Ly} = \omega_c\tau = \pi\hbar\tau/mA_c\,. \qquad (7)$$

If A_c is as defined by Eq. (29) of Ref. 1,

$$A_c = \tfrac{1}{2}\pi\xi_0^2 \ln(\xi_0/\xi_1)\,, \qquad (8)$$

then τ should be $\tau_c = \frac{3}{4}\tau_n$. Although the way the angular averages are calculated is different, this result is very close to that of Kopnin and Kravtsov.[4]

These authors express their results in the following form:

$$\vec{v}_T = f_1(x)\,\vec{v}_L + b^{-1} f_2(x)(\vec{1}_H\times\vec{v}_L)\,, \qquad (9)$$

where $\vec{1}_H$ is a unit vector in the direction of the axis of the vortex line, $b=16/3\pi$, $x=4\hbar\tau_n/mA_c$, and f_1 and f_2 are given by rather complicated averages over the Fermi surface. To show how close their theory is to the simple picture outlined above, we shall compare their calculations of $f_1(x)$ and $f_2(x)$ with those that follow from (7) and the requirement that with no backflow, $v_{Lx}=v_T$.

From (9) it follows that

$$\tan\alpha = bf_1/f_2 \qquad (10)$$

and from $v_{Lx}=v_T$,

$$v_T = v_L\sin\alpha = v_L(f_1\sin\alpha + b^{-1}f_2\cos\alpha)\,, \qquad (11)$$

which gives

$$\begin{aligned} f_1 &= \sin^2\alpha = x^2/(x^2+b^2)\,,\\ f_2 &= b\sin\alpha\cos\alpha = xb^2/(x^2+b^2)\,. \end{aligned} \qquad (12)$$

Limiting forms of the exact expressions derived by Kopnin and Kravtsov are for x large

$$f_1 = 1-3.015/x^2\,,\quad f_2 = 1.735/x\,;$$

and for x small

$$f_1 = \tfrac{3}{8}x^2 = x^2/2.67\,,\quad f_2 = x\,.$$

Since $b=16/3\pi=1.695$ and $b^2=2.88$, these are very close to expansions of (12). Values of f_1 and f_2 for intermediate values of x are also very close.

Thus at low temperatures in relatively pure superconductors, the approximation that a vortex line moves without backflow is a good one. This is not true at temperatures such that quasiparticles are excited above the gap in the bulk superconductor when scattering of quasiparticles by the vortex lines must be taken into account. The simplifying assumptions that scattering is proportional to the density of states at the Fermi surface and that in steady state there is an electric field to balance this scattering may be useful in other problems as well.

ACKNOWLEDGMENTS

The author is indebted to Larkin and Ovchinnikov and to Kopnin and Kravtsov for helpful discussions and for sending preliminary versions of their publications.

[1]John Bardeen and Richard D. Sherman, Phys. Rev. B 12, 2634 (1975).
[2]John Bardeen and M. J. Stephen, Phys. Rev. 140, A1197 (1965).
[3]A. I. Larkin and Yu N. Ovchinnikov, Pis'ma Zh. Exsp. Teor. Fiz. 23, 210 (1976) [JETP Lett. 23, 187 (1976)].
[4]N. B. Kopnin and V. E. Kravtsov, Pis'ma Zh. Exsp. Teor. Fiz. 23, 631 (1976) [JETP Lett. 23, 578 (1976)].
[5]L. Kramer and W. Pesch, Z. Phys. 269, 59 (1974).
[6]Yu. N. Ovchinnikov (private communication).