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Critical fields of weakly coupled superconductors*

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The parallel critical field of a stack of superconducting and insulating layers is calculated as a function of the superconducting layer's coherence length ξ and thickness d, interlayer coherence length $\xi_\perp$, and insulating thickness s. It is shown that the critical field shows an upturn when the stack crosses over from the strongly coupled or three-dimensional regime ($\xi_\perp > s + d$) to the weakly coupled or two-dimensional regime ($\xi_\perp < s + d$). This upturn is a general property of such stacks, though it is more pronounced when $s > d$.

I. INTRODUCTION

The properties of arrays of weakly coupled superconductors, such as stacks of thin layers or bundles of fibers weakly coupled through thin insulating junctions, have been recently the subject of several investigations.[1-3] These geometries usually result in strongly anisotropic superconducting properties (critical fields, critical currents), as found, for instance, in intercalated compounds[1,2] and the polymeric superconductor $(SN)_x$.[3]

The critical field of layered superconductors where the thickness of the superconductor layer can be neglected (intercalated compounds) was calculated by Klemm *et al.*[4]

This calculation predicts a remarkable behavior for the parallel critical field: It should diverge (within limitations imposed by the paramagnetic limit) at some temperature below T_c where the interlayer coherence length $\xi_\perp(T)$ is of the order of the interlayer spacing.

We show here that this upturn in $H_\parallel(T)$ is in fact a general property of layered systems, i.e., it is not limited to the case where the superconducting layer thickness d can be neglected compared to the interlayer spacing s. We find that the upturn always occurs at the temperature where $\xi_\perp(T)$ is of the order of the periodicity $d + s$. It is already weakly present when $d/s >> 1$ and is increasingly pronounced for smaller values of d/s.

Anisotropic superconductors can be considered as bulk as long as the coherence length is larger than the period of the system in all directions. In that case, one finds for a layered superconductor[5]

$$H_\perp = \Phi_0/2\pi\xi_\parallel^2, \quad H_\parallel = \Phi_0/2\pi\xi_\parallel\xi_\perp \ , \tag{1}$$

where Φ_0 is the flux quantum $hc/2e$.

In the more general case where the anisotropic tensor is described by three different coherence lengths ξ_1, ξ_2, and ξ_3, one obtains from similar considerations

$$H = (\Phi_0/2\pi)\xi_1^2\xi_2^2\xi_3^2[(\xi_1^{-2}\sin^2\phi + \xi_2^{-2}\cos^2\phi) \times(\sin^2\theta + \xi_3^{-2}\cos^2\theta)]^{-1/2} \ , \tag{2}$$

where θ, ϕ are the polar and azimuthal angles, respectively. Note that the anisotropy ratios are temperature independent.

In the opposite limit where the coherence length is smaller than the period in at least one direction, one deals, in fact, with isolated (or quasi-isolated) layers or fibers for which quite different results are to be expected; in particular, the anisotropy ratio should then be strongly temperature dependent.[6] We now apply the method of Klemm *et al.* to calculate the crossover behavior of a layered superconductor.

II. PARALLEL UPPER CRITICAL FIELD OF A LAYERED SUPERCONDUCTOR

We consider a stack of superconducting slabs of thickness d coupled through Josephson junctions in an array of period D. The layers are perpendicular to the z direction, and the external field H is in the y direction, with the gauge

$$\vec{A} = (0, 0, Hx) \ . \tag{3}$$

For thin slabs ($d << \xi$, where ξ is the temperature dependent intrinsic coherence length), the order parameter is

$$\psi(x,z) = f(x)e^{i\Phi(x,z)} \ . \tag{4}$$

Assuming that the amplitude f is the same in neighboring layers,[7] the phase in each layer depends linearly on z, x, and the phase difference $\Delta\Phi$ between two succesive layers is[4]

$$\Delta\Phi = (2eHD/\hbar c)x, \quad D = d + s \ . \tag{5}$$

The Ginzburg-Landau equation for one layer taking into account the coupling energy for one period is

$$\int_{-d/2}^{d/2} dz \left[\alpha f - \frac{\hbar^2}{2m} \frac{d^2}{dx^2} f + \frac{1}{2m} \hbar^2 \left(\frac{\partial \Phi}{\partial x} \right)^2 f \right] + 2\eta d f \left(1 - \cos \frac{2eHDx}{\hbar c} \right) = 0 , \quad (6)$$

where η is the coupling energy, and $\partial\Phi/\partial x = 2eHz/\hbar c$ inside one slab. Note that the phase in each layer is $2eHx(z-\bar{z})/\hbar c$, where $\bar{z}$ is the center of the layer. Carrying out the integration, (6) becomes

$$-\frac{d^2}{dx^2} f + \frac{2}{t^2}\left(1 - \cos \frac{2eHDx}{\hbar c} \right) f = \frac{1-(H/H_{\parallel})^2}{\xi^2} f . \quad (7)$$

Here we have introduced

$$t^2 = \hbar^2/2m\eta , \quad (8)$$

and $H_{\parallel}$ is the parallel critical field for a single layer,[6]

$$H_{\parallel} = (\hbar c/2e)\sqrt{12}/d\xi . \quad (9)$$

Equation (7) is the Mathieu equation, whose general solution is quite complicated. However, a simple solution can be found in two extreme cases.

a. Strong coupling, weak field. Here, the cosine term can be expanded, so that (7) becomes a harmonic oscillator equation, with a quadratic perturbative potential. The lowest energy level yields

$$h = (d/\sqrt{12}D)(t/\xi)[1 - h^2 + \tfrac{3}{4}(D/d)^2 h^2] + O(h^3) , \quad (10)$$

$$h = H/H_{\parallel} . \quad (11)$$

This approximation is valid as long as the range of f is much smaller than $2eHD/\hbar c$, i.e., when

$$h\sqrt{12}D/d < \xi/t . \quad (12)$$

We see that in that case, to lowest order, the upper critical field is that of an anisotropic superconductor as given in Eq. (1), with $\xi_{\perp} = \xi D/t$, $\xi_{\parallel} = \xi$.

b. Weak coupling, strong field. Here we transform (7) into the usual form of Mathieu equation[8]

$$\frac{d^2 f}{dx^2} + (a - 2q\cos 2x) f = 0 , \quad (13)$$

with

$$q = -\left(\frac{\xi}{t} \right)^2 \frac{1}{3} \left(\frac{d}{D} \right)^2 \frac{1}{h^2} , \quad (14)$$

$$a = \left(\frac{d}{D} \right)^2 \frac{1}{3h^2} \left[1 - h^2 - 2\left(\frac{\xi}{t} \right)^2 \right] .$$

The smallest eigenvalue for $q^2 < 1$ yields

$$\frac{1}{3}\left(\frac{d}{D} \right)^2 \frac{1}{h^2} \left[1 - h^2 - 2\left(\frac{\xi}{t} \right)^2 \right] = -\frac{1}{2}\left(\frac{\xi}{t} \right)^4 \left(\frac{d^2}{3D^2} \right)^2 \frac{1}{h^4} + O(h^{-8}) . \quad (15)$$

This approximation is valid for $q^2 < 1$, i.e.,

$$h\sqrt{3}D/d > \xi/t . \quad (16)$$

We see that as ξ/t tends to zero, $h \to 1$. The upper critical field approaches that of a single layer.

The crossover point between the two cases occurs at $\xi/t \sim 1$, independent of d/D. In the strong coupling case, putting $h = (d/\sqrt{12}D)(t/\xi)$ from Eq. (10) into Eq. (12) gives

$$1 \ll (\xi/t)^2 .$$

In the weak coupling case, Eq. (15) yields, to lowest order, $h^2 = 1 - 2(\xi/t)^2$, which implies

$$(\xi/t)^2 < \tfrac{1}{2}$$

for this result to be physical. Hence, the crossover point occurs for $\xi/t \sim 1$. Noting that $\xi_{\perp} = \xi D/t$, we see that this condition gives, for the coherence length perpendicular to the layers, $\xi_{\perp} \sim D$.

In Fig. 1, we have plotted h as a function of t/ξ for $D = d$, $D = 2d$, and $D = 4d$. In each case we plotted the solutions of (10) and (15), and indicated by dotted lines the regions of validity of the two approximations [using (12) and (16)]. The crossover between the two curves (the crosses) was calculated from numerical solutions of the Mathieu equation.[8]

For temperatures not too far from T_c, we can take $\xi = \xi(T=0)[T_c/(T_c - T)]^{1/2}$, where $\xi(T=0) = 0.74(\xi_0 l)^{1/2}$ in the dirty limit and l is the intralayer mean-free path. Hence, Fig. 1 has essentially the shape of the $H(T_c - T)$ plot, which can be obtained from Fig. 1 by multiplying both coordinates by $H_{\parallel}$ [as given by Eq. (9)] and then the $(t/\xi)H_{\parallel}$ axis by the appropriate constant, i.e., $T_c[\xi^2 + (T=0)d/t]2\pi/\sqrt{12}\Phi_0$.

If we choose to define the crossover point from weak to strong coupling as that where $h(t/\xi)$ shows an inflexion point, we observe from Fig. 1 that this occurs for $t/\xi = D/\xi_{\perp} \sim 1.4$, independent of D/d. This choice for crossover is consistent with the behavior for $d/s \ll 1$, where H was shown[4] to diverge for $\xi_{\perp}(T) = s/\sqrt{2}$.

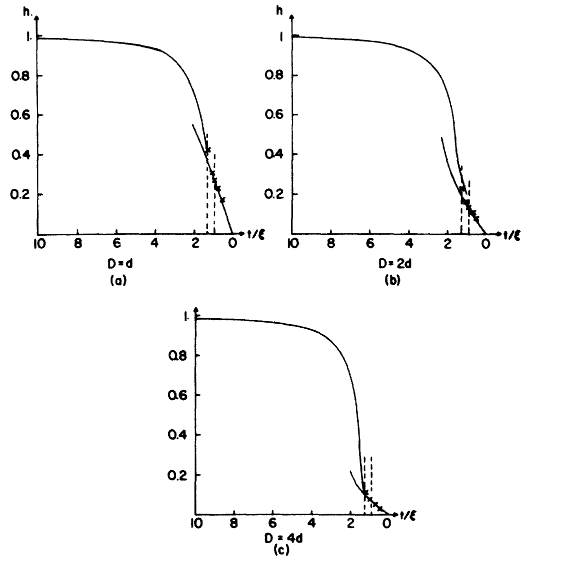

FIG. 1. Critical fields of layered superconductors. D is the period of the stack and d the thickness of the superconducting layers. The field is measured in units of the critical field of the isolated layer. The variable t/ξ characterizes the interlayer coupling strength ($t/\xi >> 1$ weak coupling; $t/\xi << 1$ strong coupling. The interrupted lines mark the transition region.) In practice, this $h(t/\xi)$ representation is very similar to $H(T_c - T)$ (see text). (a) Upturn of the critical field is already present for the case of negligible interlayer spacing. (b) and (c) Increasing interlayer spacings, approaching the divergent behavior predicted for intercalated compounds (Ref. 4).

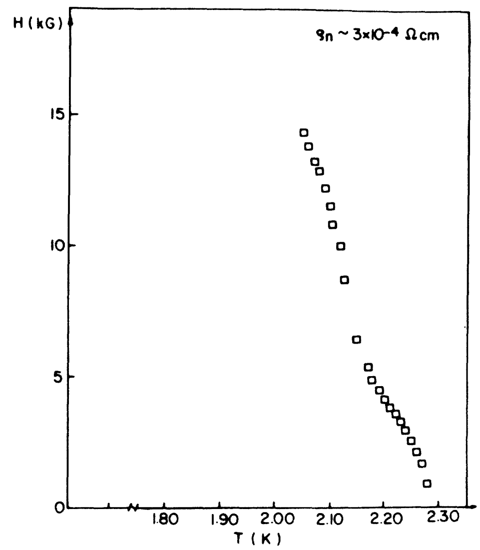

FIG. 2. Upturn in the parallel critical field observed in a granular aluminium film (after Ref. 9). The normal-state resistivity is ρ_n.

III. DISCUSSION

Although an upward curvature of $H(T)$ was obtained in some intercalated compounds,[2] the evidence in that case is not very clear cut and the quantitative analysis is complicated by the Pauli paramagnetism and the strength of the spin-orbit coupling, generally not known *a priori* with sufficient accuracy.

A more convincing evidence for crossover behavior may be found in granular Al-Al_2O_3 films produced by evaporation in an atmosphere of oxygen. These films were recently shown to have an anisotropic structure. The coupling, being weaker between superimposed grains than between collateral grains, produces a sort of "layered" structure on the scale of ~100 Å.[9] The oxide content can be adjusted so that superimposed "layers" are strongly coupled close to T_c and weakly coupled far from T_c.[9] The resulting behavior of $H(T)$ (Fig. 2) is seen to be strikingly similar to that predicted by theory (Fig. 1).

The same approach may be useful for the understanding of the critical fields of other systems. $(SN)_x$ crystals have critical fields that may be interpreted as resulting from a fibrous structure, the fibers being apparently always in the weak coupling limit in a parallel field.[3] We note also that the perpendicular critical field of granular films shows a definite tendency to an upward curvature,[9,10] which may be interpreted as a progressive decoupling of the grains in the lateral directions.

Finally, it has been noted[11] that the low-temperature critical fields of granular NbN and similar films are practically independent of the normal-state resistivity of the films, i.e., they seem to be characteristic of the grains themselves, which is the result that one would obtain in the weak coupling limit. If this argument is correct, a region of upward curvature of $H(T)$ should exist near T_c. We suggest that this would be worth checking experimentally.

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