Temperature fluctuations in freely suspended tin films at the superconducting transition*

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We have used a superconducting-quantum-interference-device voltmeter to measure the spectral density $S_v(f)$ of the voltage noise across current-biased tin films at the superconducting transition. Each film was freely suspended between two thermal clamps a distance L apart in a vacuum can. The transverse dimensions of the films were small compared with $(D/f)^{1/2}$ at the frequencies of interest (D is the thermal diffusivity), so that the heat flow was one dimensional. A thin layer of lead was evaporated on some of the tin films to leave an uncoated middle region of length l. $S_v(f)$ was proportional to the square of the bias current and to the square of the temperature coefficient of resistance, indicating that the noise was generated by temperature fluctuations. $S_v(f)$ was flat at frequencies below $f_L \approx D/L^2$. For samples without the lead overlay, $S_v(f)$ typically varied as $f^{-1.3}$ at frequencies above f_L. For samples with the lead overlay, a second knee was usually observed at $f_l \approx (L/l)^2 f_L$. At frequencies between f_L and f_l the slope was typically -0.8, while at frequencies above f_l the slope was somewhat less steep than -1.5. This behavior is in reasonable agreement with the predictions of an equilibrium temperature-fluctuation model in which the equal-time temperature fluctuations are spatially uncorrelated. The magnitude of $S_v(f)$ is within a factor of 2 or 3 of the model predictions. The autocorrelation function of the voltage noise was obtained from the time derivative of the response of the film to a step function in power. The cosine transform of the autocorrelation function was in excellent agreement with the measured spectral density. These results are in marked contrast with those obtained for normal and superconducting films supported by substrates, for which a model is required with spatially correlated fluctuations. We conclude that the $1/f$ noise for films on substrates is mediated by an interaction between substrate and film.

I. INTRODUCTION

Voss and Clarke[1] used a model that involved equilibrium temperature fluctuations to quantitatively predict the $1/f$ voltage noise observed at room temperature in metal films deposited on glass substrates. Fluctuations in the temperature T generate fluctuations in the film resistance R_F provided that the temperature coefficient of resistance $\beta = (1/R_F)dR_F/dT$ is nonzero. In the presence of a steady current, the resistance fluctuations produce voltage fluctuations. Subsequently, the measurements of Clarke and Hsiang[2] of the $1/f$ noise in current-biased films on substrates at the superconducting transition further supported the thermal-fluctuation model. A simple argument shows that the mean-square voltage fluctuation $\langle(\Delta V)^2\rangle$ is given by

$$I_1^2\left(\frac{dR_F}{dT}\right)^2\langle(\Delta T)^2\rangle = I_1^2\left(\frac{dR_F}{dT}\right)^2\frac{k_B T^2}{C_v};$$

C_v is the heat capacity of the film, and I_1 is the bias current. In both sets of experiments, the spectral density of the voltage fluctuations $S_v(f)$ was proportional to $I_1^2(dR_F/dT)^2/\Omega$, where Ω was the volume of the film. Furthermore, at a given frequency the $1/f$ noise was found to be spatially correlated over a distance $\lambda(f) \approx (D/f)^{1/2}$, where D was the thermal diffusivity of the film. This result suggests that the fluctuations were governed by a thermal-diffusion equation. In a further set of experiments, Clarke and Hawkins[3] studied the $1/f$ noise in resistively shunted Josephson tunnel junctions biased with a current I at a nonzero voltage. In this case, the temperature fluctuations generate fluctuations in the critical current I_c. It was found that $S_v(f)$ was proportional to $(dI_c/dT)^2 \times(\partial V/\partial I_c)_I^2$, as predicted by the thermal-fluctuation model.

However, there is a serious problem with the thermal-fluctuation model, namely, the difficulty in obtaining a $1/f$ power spectrum. The usual Langevin treatment of temperature fluctuations[4] leads to the equation

$$\frac{\partial T(\vec{r},t)}{\partial t} - D\nabla^2 T(\vec{r},t) = \frac{1}{c_v}\vec{\nabla}\cdot\vec{F}(\vec{r},t), \qquad (1.1)$$

where $T(\vec{r},t)$ is the local temperature of a volume element, $\vec{F}(\vec{r},t)$ represents a fluctuating energy flux of the form

$$\langle\vec{F}(\vec{r}+\vec{s},t+\tau)\cdot\vec{F}(\vec{r},t)\rangle = (2\pi)^3 F_0^2\delta(\vec{s})\delta(\tau),$$

and c_v is the specific heat. The resulting equal-time temperature fluctuations are spatially uncorrelated, that is $\langle\Delta T(\vec{r}+\vec{s},t)\Delta T(\vec{r},t)\rangle \propto \delta(\vec{s})$. Let $\overline{T}(t)$ be the spatially averaged temperature of a rectangular region of volume $l_1\cdots l_m$ in an infinite homogeneous m-dimensional medium. From Eq. (1.1), one finds that the spectral density of the fluctuations of $\overline{T}(t)$ is given by

$$S_T(f)=\frac{F_0^2}{(2\pi)^m c_v^2}\int_{-\infty}^{\infty}\frac{d^m k k^2}{D^2k^4+(2\pi f)^2}\times\prod_{i=1}^{m}\frac{\sin^2(\frac{1}{2}k_i l_i)}{(\frac{1}{2}kl_i)^2}. \quad (1.2)$$

For $m=3$ and $l_1 \gg l_2 \gg l_3$, $S_T(f)$ has the following limiting forms: $S_T(f)\propto f^0$ $(f \ll f_1)$, $S_T(f)\propto ln[(\mathrm{const})/f]$ $(f_1 \ll f \ll f_2)$, $S_T(f) \propto f^{-1/2}$ $(f_2 \ll f \ll f_3)$, and $S_T(f)\propto f^{-3/2}$ $(f_3 \ll f)$, where $f_i \approx D/l_i^2$. There is no region in which $S_T(f)\propto f^{-1}$.

Voss and Clarke[1] proposed an alternative treatment leading to an equation of the form

$$\frac{\partial T(\vec{r},t)}{\partial t}-D\nabla^2 T(\vec{r},t)=\frac{1}{c_v}P(\vec{r},t), \quad (1.3)$$

where $P(\vec{r},t)$ represents fluctuating sources and sinks of energy, and is of the form

$$\langle P(\vec{r}+\vec{s},t+\tau)P(\vec{r},t)\rangle=(2\pi)^3P_0^2\delta(s)\delta(\tau).$$

The resulting equal-time temperature fluctuations are correlated, $\langle \Delta T(\vec{r}+\vec{s},t)\Delta T(\vec{r},t)\rangle \propto |\vec{s}|^{-1}$. For the rectangular region described previously, the power spectrum of the fluctations of $\overline{T}(t)$ is given by

$$S_T(f)=\frac{P_0^2}{(2\pi)^m c_v^2}\int_{-\infty}^{\infty}\frac{d^m k}{D^2k^4+(2\pi f)^2}\times\prod_{i=1}^{m}\frac{\sin^2(\frac{1}{2}k_i l_i)}{(\frac{1}{2}k_i l_i)^2}. \quad (1.4)$$

For $m=3$ and $l_1 \gg l_2 \gg l_3$, $S_T(f)$ has the limiting forms: $S_T(f)\propto f^{-1/2}$ $(f\ll f_1)$, $S_T(f)\propto f^{-1}$ $(f_1 \ll f \ll f_2)$, $S_T(f)\propto f^{-3/2}$ $(f_2 \ll f \ll f_3)$, and $S_T(f)\propto f^{-2}$ $(f_3 \ll f)$. Thus, this spectral density does contain an explicit $1/f$ region. Furthermore, if one normalizes $S_T(f)$ by setting $k_B T^2/C_v=\int_0^\infty S_T(f)\,df$, one obtains a spectral density in the $1/f$ region that is in good agreement with the noise measured in metal films on substrates at room temperature,[1,5] metal films on substrates at the superconducting transition,[2] and Josephson junctions.[3]

Problems remain with the P model: First, the physical origin of the driving term is unclear; second, if the fluctuations are assumed to be intrinsic to the metal, one finds that the magnitude of P_0 depends on the volume of the sample; and third, there is no measured change in slope of the power spectrum at f_1 for films on substrates. Nevertheless, one final set of experiments further supported the P model. An alternative way to determine the spectral density is to perturb the system from thermal equilibrium and to measure as a function of time the decay of the spatially averaged temperature $\overline{T}(t)$ back to equilibrium. For the proper perturbation, the decay represents the autocorrelation function $c_T(\tau)$, which is related to $S_T(f)$ via

$$S_T(f)=4\int_0^\infty c_T(\tau)\cos(2\pi f\tau)\,d\tau. \quad (1.5)$$

The autocorrelation function is normalized by setting $c_T(0)=k_B T^2/C_v$. Voss and Clarke[1] showed that if a δ function of power is applied to the rectangular region described previously, the cosine transform of $[\overline{T}(t)-T_0]$ has the form of the integral given in Eq. (1.2). On the other hand, if one applies a step function in power, the cosine transform of $[\overline{T}(t)-T_0]$ yields the integral given in Eq. (1.4). The response of metal films on substrates at room temperature to δ and step functions was measured. It was found that the cosine transform of the response to the step function was in excellent agreement with the measured power spectrum. This result indicates that the P model describes the temperature fluctuations.

An unresolved problem in these experiments was the role of the substrate in the fluctuation spectra. The model calculations were always made for a thermally homogeneous medium, whereas the substrate obviously has different thermal properties from the metal film, and there is a thermal boundary resistance between the film and the substrate. In addition, Clarke and Hsiang[2] found that a thin (~5-nm) layer of aluminum deposited on the substrate prior to the deposition of a tin film considerably modified the spectral density at the superconducting transition. The noise was no longer $1/f$-like, and flattened at low frequencies. This result suggests that the substrate or the substrate–film interface plays a major role in the generation of the $1/f$ noise.

In the present paper, we describe experiments on freely suspended tin films at the superconducting transition that were performed to investigate the nature of the low-frequency noise in a well-defined 1-dimensional system with no substrate. Each film was freely suspended between two pairs of clamps that thermally grounded the ends of the film, and also allowed us to make two- or four-terminal current-voltage measurements. The voltage fluctuations in the presence of a current were measured with a dc superconducting-quantum-interference-device (SQUID) voltmeter,[6] and their spectral density computed. The response of the film to δ- and step-function inputs was also determined. In contrast to the films on substrates, the results are in close agreement with the predictions of the $\vec{\nabla}\cdot\vec{F}$ model.

II. TEMPERATURE FLUCTUATIONS IN A FINITE ONE-DIMENSIONAL SYSTEM

Figure 1 shows a film of length L suspended between two thermal clamps at temperature T_0. The transverse dimensions of the film are small com-

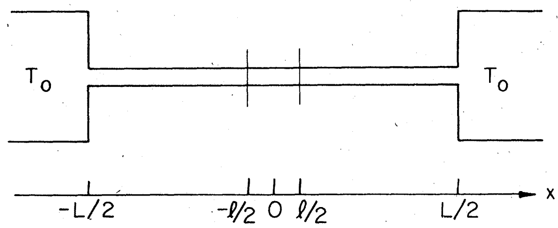

FIG. 1. One-dimensional system with ends clamped at constant temperature T_0.

pared with $(D/f)^{1/2}$ at any frequency of interest, so that at any instant of time a given cross section of the film has a uniform temperature. Consider now the region $|x| \leq \frac{1}{2}l \leq \frac{1}{2}L$ in Fig. 1. We wish to find the power spectrum of the fluctuations[7] in the average temperature of the length l,

$$\overline{T}(t) = l^{-1} \int_{-l/2}^{l/2} T(x,t)\, dx,$$

subject to the constraint $T(-\frac{1}{2}L, t) = T(\frac{1}{2}L, t) = T_0$.

According to the $\vec{\nabla}\cdot\vec{F}$ model, the temperature fluctuations obey the equation

$$\frac{\partial T(x,t)}{\partial t} - D\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{c}\,\frac{\partial F(x,t)}{\partial x}, \qquad (2.1)$$

where c is the heat capacity per unit length. The techniques used by Voss and Clarke[1] to calculate the spectral density of temperature fluctuations in a rectangular section of an infinite medium may be applied to the present problem with $m = 1$ provided that one replaces the continuous variable k with discrete k values: $k_n = (n\pi/L)$, $n = 1, 2, 3, \ldots$. The allowed modes associated with even values of n are of the form $\sin(n\pi x/L)$, and do not contribute to $\overline{T}(t)$. The allowed modes associated with odd values of n are of the form $\cos(n\pi x/L)$, and do contribute to $\overline{T}(t)$. In the analysis integrals over k space weighted by e^{ikx} become sums over odd values of n weighted by $\cos(n\pi x/L)$. Setting $\int_0^\infty S_T(f)\,df = k_B T^2/lc$, we find that Eq. (1.2) becomes

$$S_T(f) = \frac{32 k_B T^2 D}{l^2 L c} \sum_{n_{\mathrm{odd}}} \frac{\sin^2(n\pi l/2L)}{(n^2\pi^2 D/L^2)^2 + (2\pi f)^2}. \qquad (2.2)$$

At frequencies $f \ll D/L^2$ the spectral density is white, with magnitude

$$S_T(0) = (k_B T^2 L/3cD)[3 - 2(l/L)]. \qquad (2.3)$$

In Fig. 2, we plot $S_T(f)$ for several values of L/l. In general there are two knees in the power spectrum, one at $f_L \approx D/L^2$ and the other at $f_l \approx D/l^2$. At frequencies much greater than f_l the slope is -1.5. For $L/l = 100$, the slope for frequencies in the range $f_L \ll f \ll f_l$ is about -0.5, as predicted by the theory for $L/l \to \infty$. For $L/l \lesssim 10$, the slope in this range is considerably steeper than -0.5.

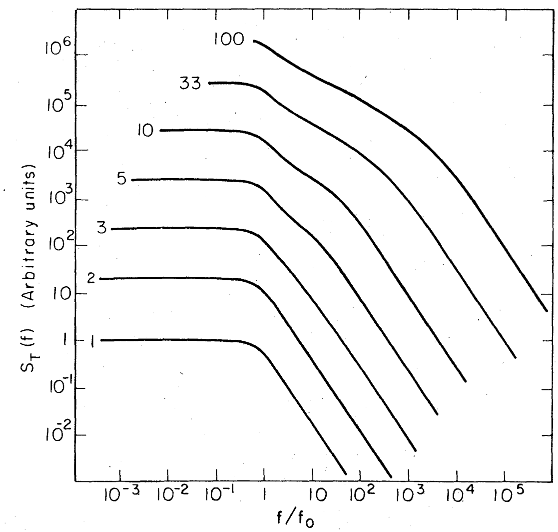

FIG. 2. Computed power spectra for the configuration of Fig. 1 for several values of L/l, with frequency normalized to $f_0 = \pi D/2L^2$. For clarity, as L/l increases each spectrum has been displaced upwards by one decade relative to the spectrum below.

Similarly, one can obtain from Eq. (1.4) the spectral density for the 1-dimensional system using the P model

$$S_T(f) = \frac{384 k_B T^2 D}{l^3 c\pi^2 [3 - 2(l/L)]} \times \sum_{n_{\mathrm{odd}}} \frac{\sin^2(n\pi l/2L)}{n^2[(n^2\pi^2 D/L^2)^2 + (2\pi f)^2]}. \qquad (2.4)$$

In view of the good agreement between our experimental results and Eq. (2.2) we have not calculated the shape of Eq. (2.4) in detail. However, in the limit $L/l \gg 1$, this power spectrum varies as f^0 $(f \ll f_L)$, $f^{-3/2}$ $(f_L \ll f \ll f_l)$, and f^{-2} $(f_l \ll f)$. For smaller values of L/l the slope between f_L and f_l is somewhat steeper. At frequencies $f \ll D/L^2$, the spectral density is white

$$S_T(0) = \frac{k_B T^2 L^2}{5cDl}\,\frac{5 - 5(l/L)^2 + 2(l/L)^3}{3 - 2(l/L)}. \qquad (2.5)$$

For $L \gg l$, this expression exceeds that given by Eq. (2.3) by a factor $L/3l$.

The calculations of Voss and Clarke[1] with regard to the cosine transform of the response to step and δ functions can be similarly modified for a 1-dimensional system with the allowed discrete values of k. The cosine transforms of the responses to a step function and to a δ function yield spectral densities of the form of Eq. (2.4) and Eq (2.2), respectively.

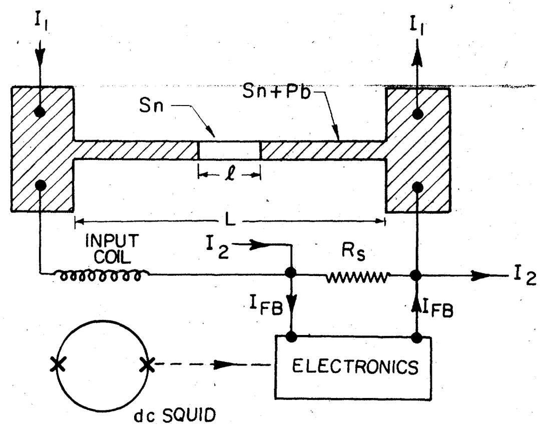

FIG. 3. Configuration of tin film and four-terminal measurement circuit.

III. EXPERIMENTAL PROCEDURES

Strips of tin (or tin + 3-wt.% In), 1.5 μm thick, were evaporated onto glass microscope slides that had been previously coated with a thin layer of Duco cement. Two parallel 6-mm-wide 0.4-μm-thick lead films were then deposited across the ends of the tin strips, with an inside separation L of about 30 mm. The remainder of the tin strips were sometimes coated with a 0.1-μm-thick lead film except for a central region of length l. The films were then cut in the configuration shown in Fig. 3 with a diamond knife. The film width was 50 μm, and the mounting tabs were 2×3 mm. The substrate was then immersed in acetone, and the films floated off.

The two tabs at the ends of the film were placed, Pb side uppermost, on two copper mounts. Small copper clamps, coated with Pb/Sn solder, were fastened down on each tab. The tin side of each tab was then pressed firmly against a copper block that provided a thermal ground, while the lead side was in contact with superconducting electrical leads.[8] The two copper blocks were attached to but electrically isolated from a copper plate on the reverse side of which were mounted a heater and a thermometer. This plate was suspended below a second copper plate on which were mounted a shielded dc SQUID and a standard resistor. Using superconducting wire, we connected the tin sample in series with the standard resistor ($R_S = 0.214\ \Omega$) and a superconducting coil wound on the SQUID (see Fig. 3). The whole assembly was mounted in a vacuum can. At liquid-helium temperatures, the plate supporting the SQUID was in good thermal contact with the helium bath, while the thermal time constant of the plate supporting the sample was about 70 sec. This low-pass filter reduced the effects of temperature fluctuations in the helium bath, which was regulated to $\pm 10\ \mu$K at about 1.7 K. Two μ-metal shields around the cryostat reduced the ambient field to about 2 μT. The cryostat was operated in a shielded room to eliminate rf interference.

The temperature of the copper T_0 was adjusted to be within the width of the superconducting transition of the tin film (a few mK). In the four-terminal configuration[8] shown in Fig. 3 the output current of the SQUID electronics I_{FB} was fed back into R_S. The currents I_1 and I_2 were adjusted so that $I_1\bar{R}_F = I_2 R_S$, where $\bar{R}_F$ was the average value of the resistance of the tin film. $I_{FB}(t)$ was then proportional to the resistance fluctuations of the sample. Thus, the SQUID system was a voltmeter with a nearly infinte input resistance. The frequency response of the system was flat from 0 to 1 kHz. The resolution was limited by Johnson noise in the sample and standard resistor. The output from the SQUID electronics was digitized, and stored in a PDP-11 computer that subsequently computed the spectral density of the fluctuations.

IV. RESULTS

A. Resistive transition

The shape of the resistive transition was measured for each sample. Figure 4 shows a representative plot of the film resistance R_F versus the temperature of the copper clamps T_0 at several values of the bias current I_1 for an In-doped sample with $L = 30$ mm and $l = 2.2$ mm. The absolute temperature was known to ± 0.02 K. The shift of the curves to lower temperatures for increasing I_1 arises from the self-field of the current. At the currents shown self-heating effects were negligible. Values of dR_F/dT as a function of R_F and I_1 were obtained from these curves.

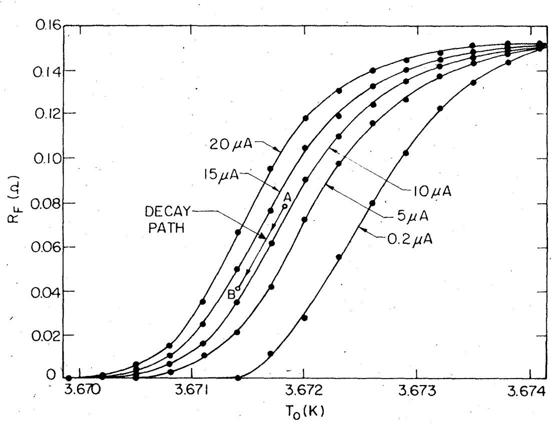

FIG. 4. Film resistance R_F vs temperature of clamps T_0 for the sample on which decay measurements were made.

B. Measured noise power spectra

Data were obtained from a total of 15 films. The noise spectral densities in Figs. 5(a) and 5(b) and the information in Table I for six samples summarize the essential results of these experiments. For each sample, Table I lists the values of L and l, the indium content of the film, the value of I_1 at which the spectrum was taken, the resistance of the film just above the transition (R_N), the values of R_F and of dR_F/dT at which the spectrum was taken, the measured value of f_L, the predicted value of f_l, and the thermal diffusivity calculated from f_L. The value of f_L for each sample was defined as the intersection of the horizontal line representing the low-frequency limit with the line drawn through the points in the region $f_L < f < f_l$. The value of D in each case was related to f_L using the same construction on the theoretical power spectra of Fig. 2 with a comparable value of (L/l). Using this value of D, the measured values of I_1 and dR_F/dT, and the handbook value[9] of C_v we calculated the power spectrum from Eq. (2.5) with $S_v(f) = I_1^2(dR_F/dT)^2 S_T(f)$. Straight-line approximations of the predicted spectra are shown in Fig. 5, with f_L (measured) and $f_l = \alpha(L/l)^2 f_L$ denoted by heavy dots. The value of α, which is close to unity, was determined for each value of L/l from the theoretical curves in Fig. 2. All the spectral densities shown were measured with insignificant self-heating.

Figure 5(a) shows three spectral densities for undoped samples. Over the frequency range investigated, the magnitudes of the spatial densities are within a factor of 2 or 3 of the predicted values. The spectral density of sample 1 with $L = l = 30$ mm is shown in Fig. 5(a). The value of f_L is 17 Hz, and the slope above f_L is about -1.3. In several similar samples, we found that the slope above f_L was between -1.1 and -1.4. The reason for the variation in slope is not known. Also shown for sample 1, as open circles, is the prediction of the semiempirical model of Voss and Clarke[1] for the amplitude of $1/f$ noise in metal films on glass substrates,

$$S_v(f) = \overline{V}^2\beta^2 k_B T^2/C_v[3 + 2\ln(L/w)]\,f\,,$$

where w is the film width. Above f_L, the observed slope is steeper than $1/f$, but the magnitude is in

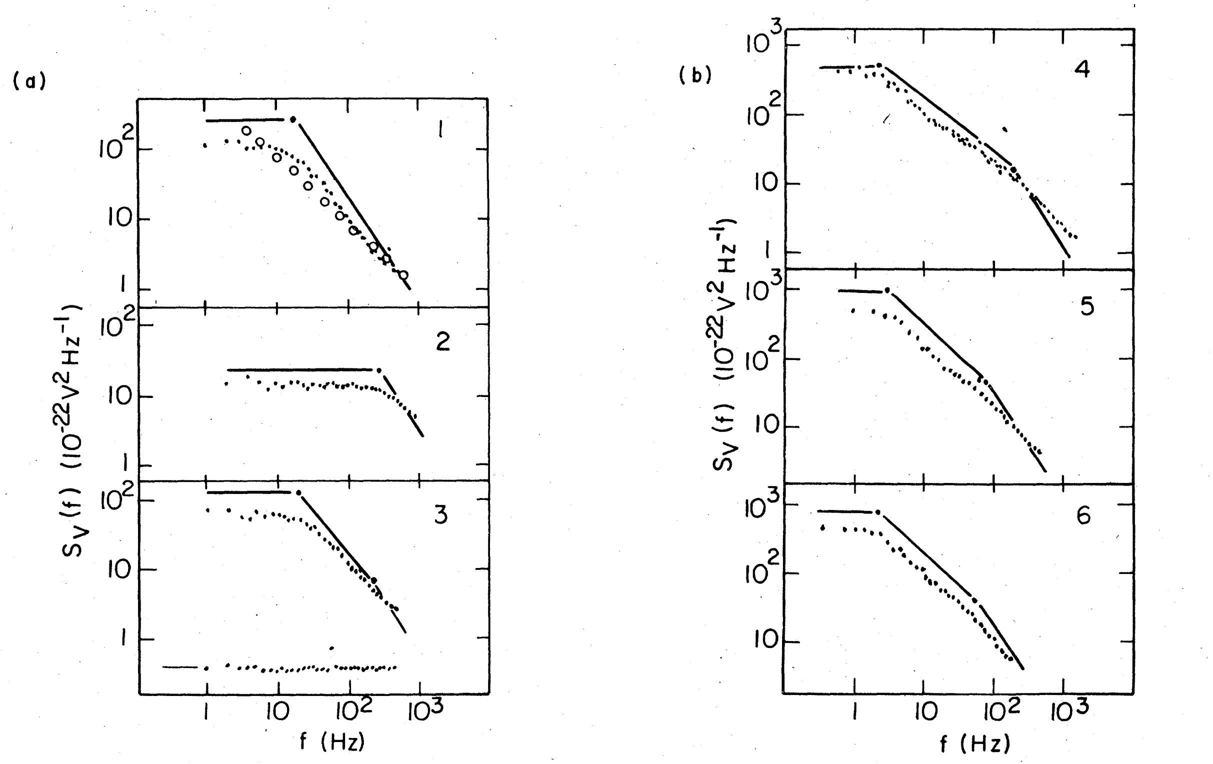

FIG. 5. Spectral densities for (a) three undoped samples, and (b) three doped samples, the parameters of which are shown in Table I. The heavy solid lines represent the predictions of the $\vec{\nabla}\cdot\vec{F}$ model; the heavy dots indicate the values of f_L and f_l. For sample 3, the dotted line in the lower part of the figure is the measured Johnson noise, and its solid continuation is the calculated Johnson noise. For sample 1, the open circles represent the prediction of the semiempirical model of Voss and Clarke (Ref. 1) for the $1/f$ noise in metal films on substrates.

TABLE I. Parameters of films.

Sample No.	L (mm)	l (mm)	wt.% In	I_1 (μA)	R_N (Ω)	R_F (Ω)	dR_F/dT (Ω K^{-1})	f_L (Hz) observed	f_l (Hz) predicted	D (cm^2 sec^{-1}) from f_L
1	30	30	0	3.6	1.12	0.27	1350	17	···	150
2	10	10	0	27.5	0.17	0.07	110	250	···	245
3	30	10	0	14.6	0.20	0.11	160	20	220	205
4	30	3.2	3	7.8	0.71	0.21	200	2	175	25
5	30	6.4	3	7.8	1.09	0.21	360	3	75	40
6	30	6.4	3	6.0	1.25	0.40	335	2	50	30

remarkably good agreement. In sample 2, $L=l$ $=10$ mm. The knee frequency is higher than that of sample 1 by a factor 15, implying that D was about 60% greater for sample 2 than for sample 1. We found that D varied by as much as a factor of $2\frac{1}{2}$ for identically prepared films. In sample 3, $L=30$ mm, and $l=10$ mm. There is no discernible knee at $f_l \approx 220$ Hz for reasons that we do not understand. The resistance of the film just above the lead transition was about a factor of 3 greater than that just below the lead transition. This result indicates that the diffusion of the lead into the tin was not sufficient to significantly reduce the electrical resistivity of the tin, or, consequently, its electronic thermal conductivity. Also shown in Fig. 5(a) for sample 3 is the power spectrum measured with zero current in the film. The power spectrum is white, and in excellent agreement with the calculated Johnson noise of the film and the standard resistor (indicated as a solid line to the left of the spectrum). The equivalent voltage spectral density of the SQUID noise was smaller by about a factor of 10.

Figure 5 (b) shows the power spectra for samples 4–6 that were doped with 3 wt.% of indium. This doping reduced the thermal diffusivity to the order of 30 cm sec^{-1}, so that the values of f_L were appropriately lower than for samples 1–3. In each case a knee is discernible at $f_l \approx (L/l)^2 f_L$. In the frequency range investigated, the measured spectral densities are everywhere within a factor of 2 or 3 of the spectral densities predicted by the $\vec{\nabla}\cdot\vec{F}$ model. The slopes between f_L and f_l are in excellent agreement with the theory. The slopes above f_l are less steep than predicted, and consequently, the knees at f_l are less pronounced than expected. The small anomalies near f_L in the theoretical spectral densities of Fig. 2 are evident in Fig. 5(b).

For all spectral densities measured, the slopes are never steeper than the prediction of the $\vec{\nabla}\cdot\vec{F}$ model. The slopes are therefore in considerably greater disagreement with the predictions of the P model, for which the slopes above f_L are steeper than those of the $\vec{\nabla}\cdot\vec{F}$ model. Additionally, the values of $S_v(0)$ for the P model are a factor of 2 to 4 greater than those of the $\vec{\nabla}\cdot\vec{F}$ model for samples 4–6. Thus, the results favor the $\vec{\nabla}\cdot\vec{F}$ model over the P model.

The values of D obtained from f_L are always less than the values estimated independently, but never by more than about a factor of 2. The electronic contribution to D was estimated from the Wiedemann-Franz law using the measured electrical resistivity and the handbook value of the heat capacity.[10] Assuming the mean free path of the phonons to be boundary limited,[10] we estimated a phonon contribution to D of about 10 cm^2 sec^{-1}. For example, the diffusivity of sample 1 was estimated to be 240 cm^2 sec^{-1}, a value about 60% higher than the value of 150 cm^2 sec^{-1} obtained from f_L. The estimated and measured values of D for sample 4 are 50 and 25 cm^2 sec^{-1} respectively. In addition, we obtained an estimate of the thermal conductivity of sample 4 from the self-heating of the film at large currents, and deduced a value of D of about 50 cm^2 sec^{-1}. Given the uncertainties involved, we consider the agreement among the various determinations of D for each film to be satisfactory.

The measured dependence of $S_v(0)$ on I_1 is shown in Fig. 6 for an undoped sample for which thermal runaway occurred at $I_1 \approx 150$ μA. The data were taken at a constant film resistance (0.04 Ω), near the midpoint of the transition. A line of slope 2 has been fitted to the data. There is an excellent fit over four decades of power. Figure 7 shows $S_v(0)$ vs dR_F/dT for sample 6. Each value of dR_F/dT was obtained by maintaining the film at a different point on the 6-μA curve of R_F vs T_0. The data are a reasonably good fit to a line of slope 2. T^2/C_v does not change significantly over the range of temperature represented in Fig. 7.

C. Determination of the autocorrelation function

The results presented in Sec. IV B provide very strong evidence that the observed voltage fluctuations are generated by thermal fluctuations. The measured power spectra agree more closely with the $\vec{\nabla}\cdot\vec{F}$ model than with the P model. To further

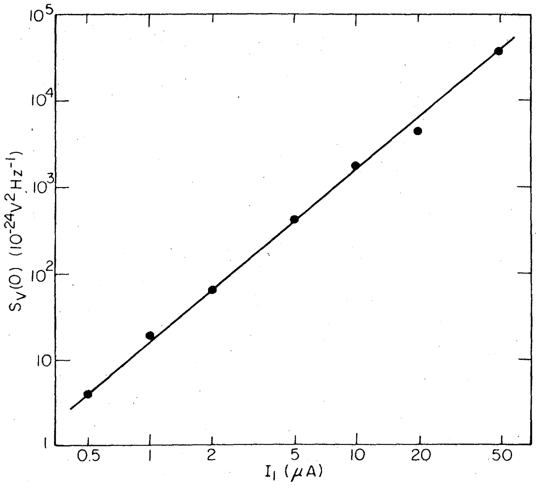

FIG. 6. $S_v(0)$ vs I_1 for an undoped sample.

distinguish between the two models we applied a step-function of power to one film, and measured the subsequent change in average temperature as a function of time. We computed a noise spectral density by taking the cosine transform of this transient [Eq. (1.5)]. We calculated a second noise spectral density by taking the cosine transform of the time derivative of the step response. The two spectral densities correspond to the P and $\vec{\nabla}\cdot\vec{F}$ models, respectively.

We used the sample for which the R_F-vs-T_0 curves are shown in Fig. 4. The sample was In-doped, with $L = 30$ mm, $l = 2.2$ mm, $f_L \approx 5$ Hz, and $R_N \approx 0.2\ \Omega$. A step function in power was applied by suddenly decreasing I_1 by an amount $\Delta I \lesssim 20\ \mu$A. The SQUID feedback loop was unable to track the sudden change in voltage that occurred ($\lesssim 2\ \mu$V), and instantaneously became unlocked, so that ΔI_1 was distributed between R_F and R_S (see Fig. 3). The SQUID feedback loop then recovered lock, and the subsequent change of voltage across R_F was

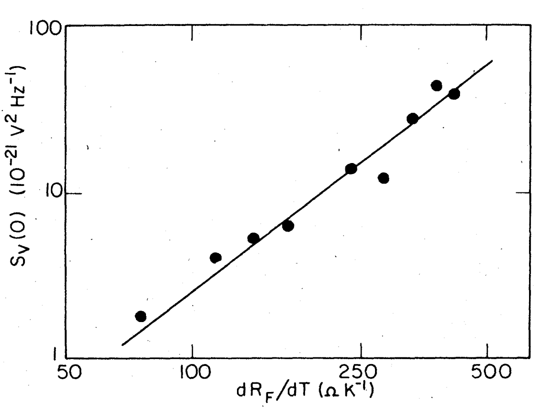

FIG. 7. $S_v(0)$ vs dR_F/dT for sample 6.

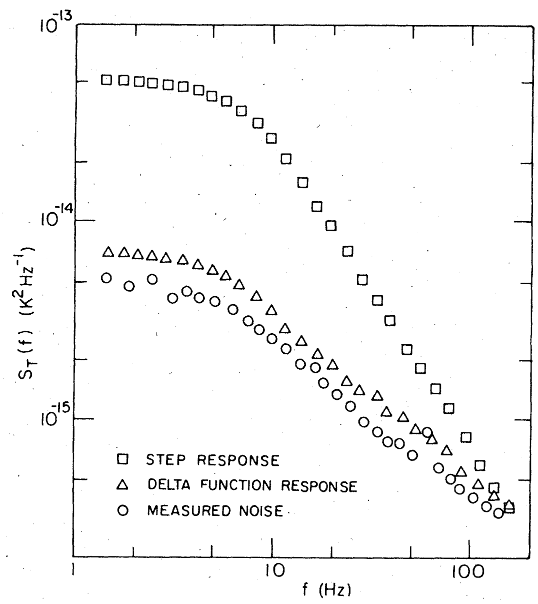

FIG. 8. $S_T(f)$ obtained from direct measurement (○), and cosine transforms of response to step function (□) and δ function (△).

measured in the usual way. Immediately after the reduction of I_1, the film was biased (for example) at A in Fig. 4. Because the dissipation in the film had been lowered, the temperature of the film decayed to a new value at B. The resulting voltage decay was recorded. R_F was very nearly a linear function of temperature along AB. When the decay was complete, the voltage noise spectral density was measured at B. The spectral density $S_T(f) = S_v(f)/I_{1B}^2(dR_F/dT)^2$ is plotted in Fig. 8. The measured decay curve was normalized by setting $c_T(0) = k_B T^2/C_v$. The cosine transforms of this curve and of its properly normalized derivative with respect to time are plotted in Fig. 8. There is a factor of 2 uncertainty in the initial value of the derivative that leads to a factor of 2 uncertainty in the vertical scaling of the corresponding $S_T(f)$.

The shape and magnitude of the cosine transform of the δ-function response are in excellent agreement with the measured spectral density. On the other hand, the shape of the cosine transform of the step-function response is quite different from the measured noise power spectrum, and its low-frequency value is an order of magnitude higher. We conclude that the δ-function response has the shape of the true autocorrelation function, and that $\vec{\nabla}\cdot\vec{F}$ is the appropriate driving term in the diffusion equation.

V. DISCUSSION

The measured power spectra of the freely suspended films correspond more nearly to the predictions of the $\vec{\nabla}\cdot\vec{F}$ model than to the predictions of the P model. However, the slopes of the spectra between f_L and f_t, and above f_t are not always exactly correct. The reasons for these discrepancies are unknown. Despite these discrepancies, the magnitudes of the measured spectral densities are generally in good agreement with the predictions of the $\vec{\nabla}\cdot\vec{F}$ model over the range of frequencies studied. The spectral density obtained from the cosine transform of the response to a δ function of power is in excellent agreement with the measured spectral density. This result is in contrast with the work of Voss and Clarke[1] on room-temperature films on substrates, where the cosine transform of the response to a step function of power was in excellent agreement with the measured spectral density. We conclude that the $1/f$ noise measured in films on substrates must be strongly influenced by an interaction between the metal film and the substrate. The mechanism for this interaction is, at present, unknown, and further work should be directed towards understanding it.

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[8]In some of the earlier experiments, a two-terminal configuration was used. The results obtained were indistinguishable from those obtained using the four-terminal arrangement.

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