PHYSICAL REVIEW B VOLUME 16, NUMBER 2 15 JULY 1977

Calculations of the thermal boundary resistance*

J. A. Katerberg, C. L. Reynolds, Jr., and A. C. Anderson
Department of Physics and Materials Research Laboratory, University of Illinois, Urbana, Illinois 61801

(Received 22 January 1977)

A comparison is made of two versions of the acoustic-mismatch theory of the thermal boundary resistance R which occurs at the interface between a pair of solids. The most recent version predicts $R \rightarrow 0$ as the two solids become identical, while the earlier version indicates a finite R as the interface essentially vanishes. It is shown that the two versions make different assumptions as to what a thermometer measures and that, if appropriate thermometers are used, both versions give the same result. This conclusion is supported by experimental evidence.

I. INTRODUCTION

Calculations of the thermal boundary (Kapitza) resistance between two materials[1,2] generally make use of the acoustic-mismatch model. This model assumes that the resistance arises from the reflection and refraction of acoustic thermal phonons at the interface. Little[3] was the first to explicitly apply the acoustic-mismatch model to an interface between two solids. The Little calculation has often been considered to be in error[4–9] since it predicts the presence of a boundary resistance even when the two materials are identical, i.e., when no interface exists. More recent calculations using the acoustic-mismatch model, such as that of Simons,[5] avoid this apparent paradox. The purpose of the present paper is to compare the calculations of Little and Simons, since the predictions of the two appear at first to be very different, both qualitatively and quantitatively. We will show that in fact, for a given physical interface, the two calculations predict the *same* result. The calculated results are then compared with experimental data.

For convenience we refer to the "Little calculation" and the "Simons calculation" even though others have contributed to these theories.[6,10] The discussion assumes an ideally perfect interface between two ordinary materials, one of which is not a normal metal. A comment is made at the end of the paper concerning the Kapitza resistance to quantum solids and liquids.

II. COMPARISON OF THE LITTLE AND THE SIMONS CALCULATIONS

The net heat flux $\dot{Q}$ between two materials is given by

$$\dot{Q} = \sum_j \int d\omega \int_0^{\pi/2} d\theta_{1j} \int_0^{2\pi} d\phi_{1j} N_{1j}(\omega, \theta_{1j}, \phi_{1j}) w_{1j}(\omega, \theta_{1j})\, \hbar \omega c_{1j} \cos\theta_{1j} \sin\theta_{1j}$$
$$- \sum_j \int d\omega \int_0^{\pi/2} d\theta_{2j} \int_0^{2\pi} d\phi_{2j} N_{2j}(\omega, \theta_{2j}, \phi_{2j}) w_{2j}(\omega, \theta_{2j})\, \hbar \omega c_{2j} \cos\theta_{2j} \sin\theta_{2j}\,, \qquad (1)$$

where $w_{1j}(\omega, \theta_{1j})$ is the fraction of energy transmitted to material 2 due to acoustic phonons of mode j, velocity c_{1j}, and angular frequency ω incident on the interface from material 1 at an angle θ_{1j} measured from the normal. $N_1(\omega, \theta, \phi)$ represents the directional and spectral distribution of phonons *incident* on the interface from material 1. Both materials are here taken to be isotropic.

In the *acoustic-mismatch model* the $w_{ij}(\omega, \theta_{ij})$ are obtained from the boundary conditions of continuum mechanics.[3] This assumption is considered valid in the limit of low temperatures (frequencies), where continuum mechanics provides a reasonable approximation for the atomic lattice[3] ($T \lesssim 1$ K). If one material is a metal, $w(\omega, \theta)$ can be complex; the imaginary portion arises from the electron-phonon interaction near the interface and is again appropriate in the low-frequency limit. A complex $w(\omega, \theta)$ has been developed by Peterson[10,11] for the Little calculation and is discussed in the Appendix for the Simons calculation.

Equation (1), to be useful to an experimentalist, must be rewritten in terms of measurable parameters such as temperatures.[12] To this end Little approximated $N(\omega, \theta, \phi)$ by the equilibrium phonon distribution N_0,

$$N(\omega,\theta,\phi)=N_0(\omega,T)(4\pi)^{-1}$$
$$=\omega^2[8\pi^3c^3(e^{\hbar\omega/kT}-1)]^{-1}. \tag{2}$$

With considerable integration, which for a given pair of materials can only be done on a computer, substitution of Eq. (2) into Eq. (1) gives[3,10]

$$\dot{Q}=\epsilon_1\alpha_{12}(T_H^4-T_C^4)\,, \tag{3}$$

where ϵ_1 represents

$$\frac{\pi^2k^4}{120\hbar^3}\sum_j c_{1j}^{-2}\,,\quad \alpha_{12}=\sum_j \langle w_{1j}\rangle c_{1j}^{-2}\Big/\sum_j c_{1j}^{-2}\,,$$

with $\langle w\rangle$ being an "angle averaged" value of $w(\theta)$, and $T_{H,C}$ are the temperatures of the two materials. (Since there can be no net heat flux across the interface when $T_H=T_C$, it is required that $\epsilon_1\alpha_{12}=\epsilon_2\alpha_{21}$.) When $2(T_H-T_C)(T_H+T_C)^{-1}=\Delta T/T\ll 1$, Eq. (3) reduces to

$$\dot{Q}=4\epsilon_1\alpha_{12}T^3\Delta T\,. \tag{4}$$

A thermal boundary resistance R may be defined as

$$R=\Delta T/\dot{Q}=(4\epsilon_1\alpha_{12}T^3)^{-1}\,, \tag{5}$$

or a conductance h by

$$h=\dot{Q}/\Delta T=4\epsilon_1\alpha_{12}T^3\,. \tag{6}$$

(There has been some opposition to these definitions since resistance and conductance generally refer to bulk properties.) The value of α varies from 0 to 1, the latter value occurring when the two materials are identical (i.e., $w(\omega,\theta)=1$). But when the materials are identical, $R=(4\epsilon\alpha T^3)^{-1}$; i.e., a nonexistent interface seems to produce a finite boundary resistance. This problem was recognized by Little,[3] but a correction was generally ignored[4] since in practice α is often considerably less than unity. Only recently has the problem been looked at in detail by Simons[5] and others.[6,8]

Simons notes that the phonon distribution of Eq. (2) does not provide a heat flux *within* either material. To rectify this problem, $N(\omega,\theta,\phi)$ of Eq. (1) is approximated by

$$N(\omega,\theta,\phi)=N_0(\omega,T)+\delta N(\omega,T)\cos\theta\,, \tag{7}$$

where $N_0(\omega,T)$ is again the equilibrium distribution *within* the material, but close to the interface. δN of necessity then includes phonons transmitted across or reflected from the interface. Equation (7) is valid only if $\delta N<N_0$. Substitution of Eq. (7) into Eq. (1) gives (see Ref. 5 for details)

$$\dot{Q}=\epsilon_1\alpha_{12}(1-\tfrac{1}{2}\beta_{12}-\tfrac{1}{2}\beta_{21})^{-1}(T_H^4-T_C^4)\,, \tag{8}$$

where β_{12} is similar to α_{12} except for an additional factor of $\cos\theta$ within the integral. Again, $0\le\beta\le 1$, but $\epsilon_1\beta_{12}\ne\epsilon_2\beta_{21}$. If $\Delta T/T\ll 1$,

$$\dot{Q}=(4\epsilon_1\alpha_{12}T^3\Delta T)(1-\tfrac{1}{2}\beta_{12}-\tfrac{1}{2}\beta_{21})^{-1} \tag{9}$$

and

$$R=(1-\tfrac{1}{2}\beta_{12}-\tfrac{1}{2}\beta_{21})(4\epsilon_1\alpha_{12}T^3)^{-1}\,, \tag{10}$$

or

$$h=(4\epsilon_1\alpha_{12}T^3)(1-\tfrac{1}{2}\beta_{12}-\tfrac{1}{2}\beta_{21})^{-1}\,. \tag{11}$$

A comparison of Eq. (10) with the Little result, Eq. (5), is shown in Fig. 1 for a range of acoustic parameters.

For acoustically very different materials (i.e., $\alpha\to 0$, $\beta\to 0$), the Simons result agrees with the Little calculation as had been expected.[4] However, if the two materials are identical ($\alpha_{12}=\beta_{12}=\beta_{21}=1$), the Simons calculation yields $R=0$ or $h=\infty$. Hence the calculation avoids the problem found in the Little calculation, namely the occurrence of a finite R at a nonexistent interface.

There is, however, an apparent problem in Eq. (9). For two *slightly* different materials $\dot{Q}$ can become larger than $\approx\epsilon T^4$, which is the maximum heat flux possible in either material. The cause of this is the magnitude of $\delta N/N_0$, which must remain at least less than unity. Thus, Eq. (9) (or 8) is valid only if $\Delta T/T\to 0$ as $\alpha\to 1$.

Since the heat flux across a boundary must be $\lesssim\epsilon T^4$, perhaps it should be worrisome that $h\to\infty$ in Eq. (11) as $\alpha\to 1$. That is, it might seem to be a matter of personal choice whether one preferred, as $\alpha\to 1$, a finite conductance (finite resistance) or zero resistance (infinite conductance) as the most appropriate representation of an imaginary interface within a material. This matter of per-

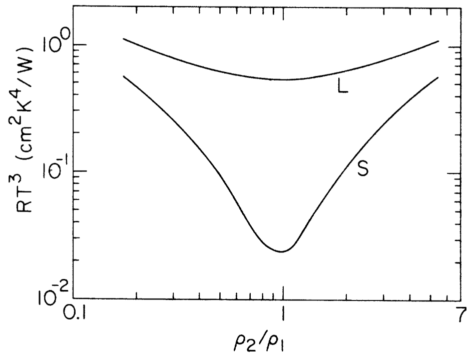

FIG. 1. Thermal boundary resistance obtained from the Little (L) or Simons (S) calculations, as functions of the ratio of densities (ρ_2/ρ_1) of the two materials in contact. The ratio of longitudinal/transverse phonon velocities has been taken as 2 in each material, the ratio of the transverse velocities c_2/c_1 for the two materials as 1.1, and the phonon attenuation as zero.

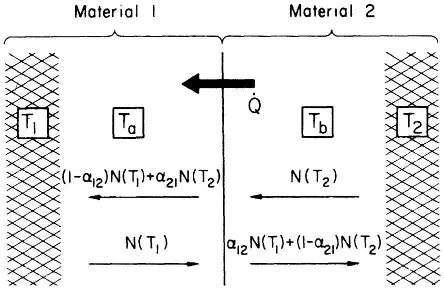

FIG. 2. Representation of an interface between materials 1 and 2, with a heat flux directed to the left. The $T_{1,a,b,2}$ represent both the thermometers and the apparent temperatures recorded by those thermometers. No phonon scattering occurs between the interface and the cross-hatched regions. The $N(T)$ are subsets of phonons having an equilibrium frequency distribution characteristic of temperature T, but moving with a horizontal component of velocity indicated by the arrows.

sonal choice becomes even less important once it is recognized that, for a given pair of materials and ΔT, the Little and the Simons calculations give the same $\dot{Q}$.

The equivalence of the two calculations will be demonstrated using a rather simple and pictorial, but inelegant, approach. The purpose is to provide a clear description of what the experimentalist measures and how this is related to the two calculations. Certain assumptions may at first appear ludicrous, but will be shown in Sec. III to correspond to real physical situations.

Consider the configuration depicted in Fig. 2. Two plates, acoustically different, are in contact. For convenience we take $\alpha_{12} = \alpha_{21} = \beta_{21} = \beta_{12}$. This is valid as $\alpha \to 1$, and it is in this limit that the Little and the Simons calculations appear to be most different. The plates are large in directions parallel to the interface so the presence of edges may be neglected. The cross-hatched region in either plate represents a region which is black, in an optical sense, to phonons. Thus phonons moving within material 1 from left to right have a frequency distribution N_0 characteristic of temperature T_1. Those moving to the left within material 1 may be represented by a frequency distribution $(1-\alpha_{12})N_0(T_1)+\alpha_{21}N_0(T_2)$. Corresponding statements apply to material 2. Thermometers immersed in the four regions, as shown in Fig. 2, monitor the apparent temperature in their respective neighborhoods.

We now compute the heat flux $\dot{Q}$ across this interface. To use the Little result, Eq. (3), we need to know the frequency distribution of phonons *incident* on the interface. This is given by $N_0(T_1)$ and $N_0(T_2)$. If we define $\frac{1}{2}(T_1+T_2) = T$ and $T_2 - T_1 = \Delta T$,

$$\dot{Q}(\text{Little}) = \epsilon_1 \alpha_{12}(T_2^4 - T_1^4) = 4\epsilon_1 \alpha_{12} T_3 \Delta T . \qquad (12)$$

The Simons calculation on the other hand requires the actual frequency and directional distribution of all phonons *within* either material but near the interface. The apparent temperatures in Fig. 2 are T_a and T_b. Assuming the temperature difference is sufficiently small that Eq. (8) is valid and experimentally reasonable (say $10^2 \Delta T/T \lesssim 5\%$), Eq. (8) gives

$$\dot{Q}(\text{Simons}) = \epsilon_1 \alpha_{12}[1-\alpha_{12}]^{-1}(T_b^4 - T_a^4) . \qquad (13)$$

We wish now to compare Eq. (12) with Eq. (13), and so T_b and T_a must be written in terms of T_1 and T_2. In steady state no net energy is exchanged with the thermometers which are internally at temperature T_a or T_b. Equating the energy exchange on the left-hand face of a thermometer with that on the right gives

$$T_a^4 - T_1^4 = \alpha_{21} T_2^4 + (1-\alpha_{12})T_1^4 - T_a^4 \qquad (14)$$

or[13]

$$T_a^4 = T_1^4 + 0.5\, \alpha_{12}(T_2^4 - T_1^4) . \qquad (15)$$

Likewise

$$T_b^4 = T_2^4 + 0.5\alpha_{12}(T_1^4 - T_2^4) . \qquad (16)$$

Use of Eqs. (15) and (16) reduces Eq. (13) to

$$\dot{Q}(\text{Simons}) = \epsilon_1 \alpha_{12}(T_2^4 - T_1^4) = 4\epsilon_1 \alpha_{12} T^3 \Delta T . \qquad (17)$$

The computed heat flux is the same from the two calculations. (More accurately, instead of finding a factor of $\gtrsim 100$ difference between the two results, as reflected in Fig. 1, we find the two identical within the $\approx 10\%$ accuracy of our computation.)

The real problem obviously does not lie in the theory. A finite R as $\alpha \to 1$ (or infinite h as $\alpha \to 1$) has served only as the proverbial red herring. The difficulty lies rather in determining if the experimental thermometers detect T_1 or T_a in Fig. 2, or neither. If the thermometer probes the phonons *incident* on the interface (T_1), the Little calculation is appropriate.[14,15] If the thermometer probes the total phonon distribution within the material but near the interface (T_a), the Simons calculation is to be used. It is even possible that the two calculations must be combined. For example, if T_1 and T_b in Fig. 2 were the only temperatures available experimentally, then $\dot{Q} = \epsilon_1 \alpha_{12}(1 - \frac{1}{2}\beta_{21})^{-1}(T_b^4 - T_1^4)$.

One additional comparison will be made, namely a comparison of the two calculations applied to the "sandwich" arrangement shown in Fig. 3. This configuration is often encountered in experiments. Using the same arguments as previously (in particular, $\Delta T/T$ small),

$$\dot{Q}(\text{Little}) = \epsilon_1 \alpha_{11}(T_2^4 - T_1^4) = 4\epsilon_1 \alpha_{11} T^3 \Delta T . \qquad (18)$$

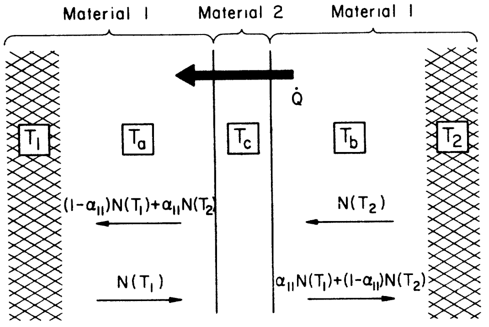

FIG. 3. Representation of two interfaces in a sandwich configuration between materials 1 and 2. Notation is the same as Fig. 2.

Here α_{11} is obtained from the net probability that a phonon incident on material 2 from the left-hand side will exit from material 2 to the right.[16,17] That is,

$$w_{11}(\theta_1)=w_{12}(\theta_1)[2-w_{21}(\theta_2)]^{-1}\,, \tag{19}$$

where it is assumed that no phonon scattering occurs within material 2. For the Simons calculation

$$\dot{Q}(\text{Simons})=\epsilon_1\alpha_{11}[1-\beta_{11}]^{-1}(T_b^4-T_a^4) \tag{20}$$

for the entire sandwich or

$$\begin{aligned}\dot{Q}(\text{Simons})&=\epsilon_1\alpha_{12}(1-\tfrac{1}{2}\beta_{12}-\tfrac{1}{2}\beta_{21})^{-1}(T_b^4-T_c^4)\\&=\epsilon_1\alpha_{12}(1-\tfrac{1}{2}\beta_{12}-\tfrac{1}{2}\beta_{21})^{-1}(T_c^4-T_a^4)\end{aligned} \tag{21}$$

for either half of the sandwich. Substitution of $T_a=T_1+0.5\alpha_{11}\Delta T$, $T_b=T_2+[1-0.5\alpha_{11}]\Delta T$, and $T_c=T_1+0.5\Delta T$ into Eq. (20) or Eq. (21) yields $\dot{Q}(\text{Simons})=4\epsilon_1\alpha_{11}T^3\Delta T$. This is identical to Eq. (18). Using the temperatures T_a, T_b, and/or T_c with the Simons calculation predicts the same $\dot{Q}$ for a given ΔT as using T_1 and T_2 with the Little calculation.

If the phonon mean free path in material 2 (Fig. 3) is not considerably greater than the thickness of material 2, then multiple reflections do not occur, Eq. (18) does not apply, and the phonons have the appearance of being thermalized. If this occurs, the calculated net thermal resistance across the entire sandwich becomes just twice the boundary resistance of a single interface determined using the *same*, Little or Simons, type of calculation plus the bulk thermal resistance[10] of material 2.

In summary, it has been shown that the Little and the Simons calculations are equally valid, but refer to different classes of phonons. In each situation it must be determined to which class of phonons the experimental thermometers respond.

III. COMPARISON OF THE CALCULATIONS WITH EXPERIMENT

A few data with which to test the Little and the Simons calculations are available in the literature. In one series of measurements[10,11] the experimental arrangement was that of Fig. 3, namely a very thin layer of epoxy between two plates of (normal) metal. The thermal conductivity of the metal was large so that each plate was effectively isothermal even in the presence of a heat flux across the sandwich. The mean free path of a thermal phonon in metal is very short, relative to the dimensions of the plate, and the electron-phonon scattering is primarily inelastic.[18] Thus the thermometer attached to each plate could not be influenced by phonons transmitted across or reflected by the dielectric layer; the thermometers recorded the temperature of the conduction electrons. Essentially the only phonons incident on the dielectric are those produced by electrons, thus the thermometers are measuring T_1 and T_2 of Fig. 3 and the Little calculation should be appropriate.

A comparison of experimental results and the Little calculation is presented in Fig. 4 for metals which range by a factor of 10 in acoustic impedance relative to that of the epoxy. The experimental data are reproducible and independent of the epoxy thickness.[10,11] The calculation includes *no* adjustable parameters.[19] As $(\rho c)_M/(\rho c)_D$ becomes large, the theoretical value of R becomes more sensitive[20] to the ratio c_2/c_1. For example, a combined error of only 7% in the measured phonon velocities of tungsten and epoxy would account for the difference between theory and experiment for W in Fig. 4.

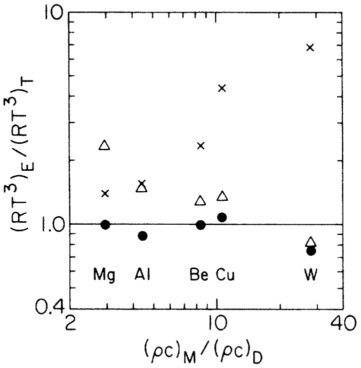

FIG. 4. A comparison of experimental data (from Ref. 10) with the Little (●) or Simons (Δ) calculation. (X) is discussed in the text. $(RT^3)_E$ is the experimental result; $(RT^3)_T$ is the result of the calculation. $(\rho c)_M/(\rho c)_D$ is the ratio of the acoustic impedance of the metal to that of the dielectric (epoxy). $(RT^3)_E/(RT^3)_T$ is also the ratio, for a given T and ΔT, of the calculated heat flux $\dot{Q}$ divided by the experimentally measured value.

Results from the Simons calculation are also included in Fig. 4. Again, there are no adjustable parameters.

Also shown in Fig. 4 (by X) is the maximum ratio of $(RT^3)_E/(RT^3)_T$ which is physically possible due to the finite thermal conductance of the materials, see Sec. II. For magnesium the Simons calculation actually exceeds this value. In other words, using measured temperatures, the Simons calculation not only predicts a $\dot{Q}$ a factor of 2.4 larger than measured, but also a factor of ≈ 2 larger than would be physically possible even if none of the phonons incident on the interfaces underwent reflection.

Summarizing the results contained in Fig. 4, it may be seen that the two calculations give nearly the same result for very different materials [large $(\rho c)_M/(\rho c)_D$] as expected, but diverge as the materials become more similar [small $(\rho c)_M/(\rho c)_D$]. The Little calculation agrees reasonably well with experimental data for all these materials, whereas the Simons calculation is in complete disagreement at small $(\rho c)_M/(\rho c)_D$ where it should nominally be most suitable. The fault does not lie with the Simons calculation, but merely reflects the fact that the experimental thermometers did not monitor the phonon distribution appropriate to that calculation.

A similar comparison can be made using data obtained from a more complicated experimental arrangement.[16] The Little calculation is again found to be in excellent agreement with data on interfaces between a dielectric and (normal state) In, Al, Pb, Sn, or Cu. As determined from the experimental configuration, it is the Little calculation which should be applied.

Experimental data obtained under conditions where the measured temperatures are clearly appropriate to the Simons calculation are more difficult to find in the literature. Measurements involving (normal) metals are almost always ruled out.[21] The rather typical configuration of two long rods of dielectrics (or superconductors) butted end to end would appear logical. The most frequently measured pair of materials has been In and sapphire.[22–25] The data, however, do not reproduce, perhaps due to surface damage[16,25,26] which scatters phonons near the interface. In any case the data fail to support either calculation.

Unfortunately, even a perfect interface between two rods leaves a problem for the experimentalist, namely as to what the apparent temperature measured on the side of the rod actually represents. A similar problem arises in measuring the "thermal conductivity"[27] of a long rod; both "end effects" and the fate of phonons incident on the side of the rod must be considered. The best arrangement to satisfy the Simons calculation is probably two (defect free) dielectric rods having highly polished sides to provide specular reflection. The rod would be sufficiently long so that phonons arriving at the thermometer (near the interface) directly from the heated or refrigerated end subtend only a small solid angle. Using two thermometers to "extrapolate" a temperature to the interface of a dielectric having a long phonon mean free path is at best a dubious practice. If the mean free path is short, extrapolation is possible. But then the temperature drop along the dielectric rod is generally large relative to the drop which is to be measured across the interface.[28]

There are situations in which $w(\omega, \theta)$ apparently cannot be calculated from continuum mechanics,[29] for example, at the interface between an ordinary solid and a quantum solid[30] such as ^{4}He. It is conceivable that $w(\theta, \omega)$ could be measured[31,32] as a function of ω and θ and the empirical result inserted into Eq. (1). Unfortunately such detail is not available as yet.[33]

In conclusion, it has been shown that the Little and the Simons calculations of the thermal boundary resistance predict the same heat flux for a given interface, provided the thermometers used in the measurement are capable of monitoring the phonon spectrum appropriate to the respective calculation. This conclusion is consistent with available experimental data.

ACKNOWLEDGMENT

We thank E. A. Jackson for helpful comments regarding the manuscript.

APPENDIX

As mentioned in Sec. II, the presence of a phonon scattering mechanism necessitates the use of a complex (actually the amplitude of a complex) $w_{ij}(\omega, \theta_{ij})$. In applying this to the Simons calculation, it will be assumed that the reader is familiar with both the derivation of Eq. (8) (see Ref. 5) and the application of a complex $w_{ij}(\omega, \theta_{ij})$ to the Little calculation (see Ref. 10). It will also be assumed that conduction electrons scatter the phonons and that material 2 of Fig. 2 is the normal metal.

In Eq. (8), α_{12} is evaluated as in Ref. 10 and contains the integral

$$\int w_{1j}(\omega, \theta_{1j}) \cos\theta_{1j} \sin\theta_{1j}\, d\theta_{1j}\,. \tag{A1}$$

By calculating w_{1j} rather than w_{2j} one indirectly avoids calculating the heat flux carried within material 2 by the electrons. In addition, the electrons make a contributionto w_{1j} only if the phonon velocity in material 1 is less than that of the metal, materal 2. The quantity β_{12} of Eq. (8) contains the integral

$$\int w_{1j}(\omega, \theta_{1j}) \cos^2\theta_{1j} \sin\theta_{1j}\, d\theta_{1j} \,, \tag{A2}$$

which can be evaluated in the same manner as Eq. (A1). However β_{21} contains

$$\int w_{2j}(\omega, \theta_{2j}) \cos^2\theta_{2j} \sin\theta_{2j}\, d\theta_{2j} \,, \tag{A3}$$

where $w_{2j}(\omega, \theta_{2j})$ generally cannot be written out explicitly if phonon attentuation or scattering is present in material 2.

The term β_{21} arose from the assumption of a $\delta N \cos\theta$ contribution to the phonon spectrum, which provided the necessary heat flux within material 2. Since the electrons also contribute to the heat flux, two corrections should be applied to Eq. (A3). First, only that fraction f of the heat carried by the phonons in material 2 should be included in β_{21}, that is the denominator in Eq. (8) becomes $1 - \frac{1}{2}\beta_{12} - f\frac{1}{2}\beta_{21}$. If $f \to 0$ as for a good metallic conductor, one obtains the Little result for that side of the interface.

Second, $w_{2j}(\omega, \theta_{2j})$ in Eq. (A3) needs to be known only for those phonons which exist within material 2. Hence the integral over θ_{2j} is taken over the real values only and Eq. (A3) can be evaluated. This step is equivalent to evaluating β_{21} only for phonons which strike or emerge within the critical cone. Larger angles are associated with the electron-phonon interaction at the interface. This is an approximation, but since the correction is generally small, serious errors are avoided. In addition, real experimental thermometers usually cannot monitor the phonon spectrum within the metal close to the interface. Hence, as discussed in detail in the text, the above procedure for the Simons calculation is generally *not* needed.

---

*Work supported in part by the U. S. ERDA Contract No. EY-76-C-02-1198.

[1] A brief review and bibliography is given by A. C. Anderson, in *Phonon Scattering in Solids*, edited by L. J. Challis, V. W. Rampton, and A. F. G. Wyatt (Plenum, New York, 1976), p. 1.

[2] A review of solid/He interfaces is given by L. J. Challis, J. Phys. C 7, 481 (1974).

[3] W. A. Little, Can. J. Phys. 37, 334 (1959). At very low frequencies ($\approx 10^9$ Hz, or ≈ 0.1 K) experimental values of $w(\omega, 0)$ agree with calculations based on continuum mechanics. See Th. zur Nieden and O. Weis, Z. Phys. B 21, 11 (1975); see also J. Wilks, *Liquid and Solid Helium* (Clarendon, Oxford, 1967), p. 519, and papers cited therein.

[4] A. F. Andreev, Sov. Phys.-JETP 16, 1084 (1963).

[5] S. Simons, J. Phys. C 7, 4048 (1974).

[6] W. M. Visscher, Los Alamos Scientific Laboratory, LA-UR-74-1753 (unpublished).

[7] W. M. Saslow, Phys. Rev. B 11, 2544 (1975).

[8] M. C. Phillips and F. W. Sheard, in *Phonon Scattering in Solids*, edited by L. J. Challis, V. W. Rampton, and A. F. G. Wyatt (Plenum, New York, 1976), p. 24. (The result $R_{\text{net}} = 2R$ obtained for a sandwich arrangement in a Simons-type calculation is not valid in general for an interface between three-dimensional solids.)

[9] See also H. Nanlofer, W. Tlirring, and R. Sexl, Ann. Phys. (N.Y.) 57, 350 (1970).

[10] R. E. Peterson and A. C. Anderson, Solid State Commun. 10, 891 (1972); J. Low Temp. Phys. 11, 639 (1973).

[11] R. E. Peterson, Ph.D. thesis (University of Illinois, 1973) (unpublished). A computer program in this thesis contains an error when dense integration is used.

[12] Temperature may not always be an appropriate parameter as, for example, in work using nearly monochromatic thermal phonons. See C. H. Anderson and E. S. Sabisky, in *Physical Acoustics*, edited by W. P. Mason and R. N. Thurston (Academic, New York, 1971), Vol. 8, p. 1.

[13] For the conditions stated, Eqs. (15) and (16) reduce to $T_a = T_1 + 0.5\alpha_{12}\Delta T$ and $T_b = T_2 - 0.5\alpha_{12}\Delta T$.

[14] This fact has been emphasized previously, see Refs. 1 and 30.

[15] Tables of transmission coefficients (from which one may calculate boundary resistance) as a function of acoustic parameters are given by J. D. N. Cheeke, H. Ettinger, and B. Hebral, Can. J. Phys. 54, 1749 (1976), from the Little calculation for isotropic materials containing no (normal) electrons. The tables agree with our calculations except for the case of very different ratios $\rho_2/\rho_1(=0.1)$, $C_2^l/C_1^l(=0.1)$, in which the ratios of longitudinal to transverse phonon velocities in medium 1 and 2 are 1.50 and 1.75, respectively. For this particular case, our calculated value is 9% larger than that deduced from the tables. This difference may be due to the density of integration points used in the two calculations (we used 2000 points, while Cheeke *et al*. used 40). One would expect the most significant deviations to occur for those interfaces in which a small critical angle is involved.

[16] S. G. O'Hara and A. C. Anderson, J. Phys. Chem. Solids 35, 1677 (1974).

[17] Equation (18) assumes no mode conversion of phonons which are reflected back into material 2. This results in a small error as discussed in Ref. 16. The expression for $w_{11}(\theta)$ is substituted into the appropriate integrals to obtain α_{11} and β_{11}. The $(2-\bar{w})$ approximation used in Ref. 16 and other papers is accurate only to order 10%.

[18] R. E. Peterson and A. C. Anderson, Phys. Lett. A 38, 519 (1972).

[19] The values for attenuation of the phonons due to electrons and the densities and phonon velocities have been taken from Refs. 10, 11, and from A. C. Anderson and S. G. O'Hara, J. Low Temp. Phys. 15, 323 (1974).

[20] This is true for either the Little or the Simons calculation and results from the narrowing of the critical cone in the dielectric.

[21] Unpublished data (taken primarily for diagnostic pur-

poses) associated with Ref. 18 do provide a rough test of the Simons calculation. Copper block C_1 shown in Fig. 1 of Ref. 18 was also monitored with a resistance thermometer. Hence, in the notation of the present paper, that thermometer measured T_2 whereas, the thermometer on block C_2 measured the local phonon temperature T_a. The measured $\dot{Q}/\Delta T$ between C_1 and the copper foil for six samples was a factor of 1.3 ± 0.2 larger than for the type of measurement of Fig. 4 of the present paper. The calculated factor for these two different arrangements of thermometers is 1.14.

[22] D. A. Neeper and J. R. Dillinger, Phys. Rev. 135, A1028 (1964).

[23] M. W. Wolfmeyer, G. T. Fox, and J. R. Dillinger, Phys. Lett. A 31, 401 (1970).

[24] B. S. Park and T. Narahara, J. Phys. Soc. Jpn. 30, 760 (1970).

[25] C. Schmidt and E. Umlauf, J. Low Temp. Phys. 22, 597 (1976).

[26] J. D. N. Cheeke and C. Martinon, Solid State Commun. 11, 1771 (1972); J. D. N. Cheeke, Cryogenics 10, 463 (1970).

[27] Thermal conductivity, an intrinsic property, cannot be determined if the sample boundary is the primary phonon scatterer. Nevertheless it is the practice to deduce a "thermal conductivity" from such thermal transport measurements. A review of boundary scattering is presented by M. P. Zaitlin, L. M. Scherr, and A. C. Anderson, Phys. Rev. B 12, 4487 (1975).

[28] We attempted to avoid these problems by constructing a sandwich as in Fig. 3 for use at temperatures $\lesssim 1$ K. Material 1 was to be (normal state) Ga or In, while material 2 was a polished single crystal of sapphire of 4×10^{-2} cm thickness. In Fig. 3 temperatures T_1, T_2, and T_c would be monitored. The two calculations for measurements involving T_1 and T_c, or T_2 and T_c differ by a factor of ≈ 2.5 for Ga and by ≈ 50% for In. Unfortunately the Ga fractured on cooling to low temperatures, with some regions pulling loose from the sapphire. With In, we could not obtain a satisfactory interface with the sapphire using any technique (including those described in Refs. 22–25) unless the crystalline surface of the sapphire was physically damaged. A typical interface could be examined from the opposite side of the sapphire using an optical microscope, or with an electron microscope after the In was removed. Sputtering techniques were not used because of the resulting surface damage to the sapphire. Our failure to achieve a satisfactory interface may be related to the lack of reproducibility in data for indium/sapphire boundaries mentioned in the text.

[29] See, however, D. Cheeke and H. Ettinger, Phys. Rev. Lett. 37, 1625 (1976).

[30] C. L. Reynolds, Jr. and A. C. Anderson, Phys. Rev. B 14, 4114 (1976).

[31] E. S. Sabisky and C. H. Anderson, Solid State Commun. 17, 1095 (1975).

[32] R. A. Sherlock, A. F. G. Wyatt, N. G. Mills, and N. A. Lockerbie, Phys. Rev. Lett. 29, 1299 (1972).

[33] It is known that, from Pb to He II, $\alpha_{12} \approx 0.2$ for either the normal or superconducting state (Ref. 2, 26). Thus for a given temperature difference the Simons calculation predicts a ≈ 10% greater heat flux than the Little calculation. A. Ridner, E. N. Martinez, and F. de la Cruz, Phys. Rev. Lett. 35, 855 (1975), used surface superconductivity on otherwise normal Pb to serve as a highly localized surface thermometer. An ordinary thermometer was also placed in thermal contact with the conduction electrons and another with the He II. The difference in apparent boundary resistance as calculated by the two sets of thermometers differed by a factor of ≈ 2, which is much greater than the ≈ 10% change expected if the surface thermometer probed the local phonon distribution near the interface but within the Pb. However it is not known with what thermal excitations the superconducting surface sheath is in most intimate contact; it may be with the surface ("Rayleigh") waves excited by phonons in the He. In fact the temperature indicated by the surface thermometer is strongly dependent on surface conditions. In brief, the data may be telling more about surface superconductivity than about boundary resistance.