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Theory of cyclotron resonance of interacting electrons in a semiconducting surface inversion layer*

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The frequency-dependent conductivity of the quasi-two-dimensional electron gas of a semiconducting surface inversion layer in the presence of a dc magnetic field is calculated. Both electron-electron and electron-impurity scattering are included in the model. The conductivity is evaluated using the memory-function technique. The line shape of the fundamental cyclotron resonance is studied for a variety of values of frequency, transport relaxation time, and quasiparticle lifetime. The frequency-dependent shift in the cyclotron mass obtained in the calculations is found to be quite small.

I. INTRODUCTION

The amplitude of the Shubnikov–de Haas oscillations in the conductivity of a surface inversion layer is known to depend upon the quasiparticle mass m^*. The experimentally observed[1] dependence of m^* on the electron concentration has been interpreted[2] as resulting from electron-electron interactions. The agreement of the experimental data with the mass enhancement calculated within the framework of the dynamic random-phase approximation (RPA) is surprisingly good. As a check on the validity of the theoretical interpretation several groups[3] decided independently to study cyclotron resonance of the inversion-layer carriers using far-infrared radiation. An elementary argument[4] shows that electron-electron interactions cannot affect the position of the resonance for a translationally invariant electron gas. Therefore, only the bare band mass should appear in the expression for the electron cyclotron frequency. Any concentration-dependent enhancement of the mass determined in these experiments, where electron-electron interaction effects are not expected to occur, would certainly cast doubt upon the interpretation of the mass enhancement obtained in the dc experiments (i.e., from the temperature dependence of the amplitude of the Shubnikov–de Haas oscillations).

Far-infrared cyclotron-resonance experiments have been performed and repeated by a large number of investigators.[3,5-7] The results have led to a number of lively discussions since data from different laboratories, or at least the interpretation of their data by different experimental groups, do not always agree. A number of unexpected results have been obtained. Among them are: (i) the occurrence of relatively strong "harmonics" of the fundamental cyclotron resonance at not quite integral multiples of the fundamental resonance[5]; (ii) a concentration dependence of the "harmonic" mass (obtained by equating the frequency of the lth harmonic to leB/m^*c) quite similar to that observed in the dc experiments[5]; (iii) an apparent dependence of the mass associated with the fundamental resonance on concentration and on frequency[6]; and (iv) a strong dependence of the fundamental mass on temperature.[7]

We have recently proposed a model,[8] that is capable of explaining at least some of these observations. In this model both electron-electron and electron-impurity interactions are included. A similar model in which electron-electron interactions are treated in an approximate way has been studied by Ando.[9] In the absence of a magnetic field the zero-temperature ac conductivity has also been calculated by Tzoar *et al.*[10] The suggestion that electron-electron interaction effects in cyclotron resonance of inversion-layer carriers could become apparent in the presence of impurities was first suggested by Kennedy *et al.*,[6] on the basis of an apparent dependence of the cyclotron mass on the value of $\omega\tau_{tr}$. Here ω is the frequency of the far-infrared radiation and τ_{tr} is the transport lifetime. For very large $\omega\tau_{tr}$ the observed cyclotron mass is close to the bare band mass, but as $\omega\tau_{tr}$ is decreased the apparent cyclotron mass appears to increase. In this paper we present some numerical calculations based on the model[8] which we proposed previously. We find the line shape of the resonance is sensitive to the single-particle lifetime τ. Because τ is affected not only by the electron-impurity interaction but also by electron-electron scattering, we have not

attempted to evaluate τ from first principles. What we have done is to choose τ empirically by fitting one of the cyclotron-resonance line shapes at each concentration. Although the calculated line shapes agree reasonably well with experiment, our results for the cyclotron-mass shift are too small to account for the measurements of Ref. 6.

In Sec. II we briefly describe the model. The memory-function approach to the ac conductivity in the presence of a magnetic field is given. The expression for the cyclotron mass in terms of the electron-impurity potential is derived. The total Hamiltonian which describes the system is also given there. In Sec. III the definition of the single-particle Green's function in the single-level approximation is given. The calculations of the memory function and the approximations involved are discussed in detail, and numerical results are presented. Section IV contains a summary and discussion of our results.

II. FORMULATION

In the presence of a dc magnetic field $\vec{H}=(0,0,H)$ oriented normal to the surface, the power transmitted through the surface inversion layer can be expressed in terms of the frequency-dependent conductivity $\sigma_{\pm}(\omega)=\sigma_{xx}\mp i\sigma_{xy}$. The conductivity itself can be written[11]

$$\sigma_{\alpha\beta}(\omega)=\frac{i}{\omega}\frac{Ne^2}{\omega}\delta_{\alpha\beta}+\frac{i}{\omega}\int_{-\infty}^{\infty}dt\,e^{i\omega t}Q_{\alpha\beta}(t)\,, \tag{1}$$

where $Q_{\alpha\beta}(t)$ is the current-current correlation function, N is the total number of carriers per unit area with charge $-e$ and the bare band mass m, and ω is the frequency of the far-infrared electric field. $Q_{\alpha\beta}(t)$ is of the form

$$Q_{\alpha\beta}(t)=-i\theta(t)\langle[J_{\alpha}(t),J_{\beta}(0)]\rangle\,, \tag{2}$$

where $\theta(t)$ stands for the unit-step function; $\theta(t)=0$ if $t<0$, and $\theta(t)=1$ if $t>0$. The current $\vec{J}$ is equal to $(-e/m)[\vec{P}+(Ne/c)\vec{A}(\vec{R})]$, where $\vec{P}$ is the momentum of the center of mass of the electrons and $\vec{A}(\vec{R})$ is the dc vector potential at the position of the center of mass. The Hamiltonian H of the system can be written

$$H=\sum_i\frac{1}{2m}\left(\vec{p}_i+\frac{e}{c}\vec{A}(r_i)\right)^2+\sum_{i>j}V(\vec{r}_i-\vec{r}_j)+\sum_{q,j,l}u(q)\exp[i\vec{q}\cdot(\vec{r}_j-\vec{R}_l)]\,. \tag{3}$$

In this equation $\vec{p}_i$ is the momentum of the ith electron, and $V(\vec{r}_i-\vec{r}_j)$ describes the electron-electron interaction. $u(q)$ is the Fourier transform of the electron-impurity potential, and $\vec{R}_l$ and $\vec{r}_j$ are the positions of the lth impurity and the jth electron, respectively. By using the fact that for a uniform magnetic field the vector potential $\vec{A}(\vec{r}_i)$ can be written

$$\vec{A}(\vec{r}_i)=\vec{A}(\vec{r}_i-\vec{R})+\vec{A}(\vec{R})\,, \tag{4}$$

where $\vec{R}$ is the center-of-mass coordinate of the electrons, we can rewrite the Hamiltonian in Eq. (3) in the form

$$H=\frac{[\vec{P}+(Ne/c)\vec{A}(\vec{R})]^2}{2M}+H_R+\sum_{q,j,l}e^{i\vec{q}\cdot\vec{R}}[u(\vec{q})\,e^{i\vec{q}\cdot(\vec{r}_j'-\vec{R}_l)}]\,. \tag{5}$$

The first term on the right-hand side of Eq. (5) is the kinetic energy of the center of mass. The center of mass acts exactly like a particle with charge $-Ne$ and mass $M=Nm$ in a uniform magnetic field. H_R is the Hamiltonian describing the relative motion of electrons and includes electron-electron interactions. The last term in Eq. (5) is the electron-impurity interaction which couples the internal coordinates $\vec{r}_j'=\vec{r}_j-\vec{R}$ to the center-of-mass coordinate $\vec{R}$. In the absence of electron-impurity scattering $[u(q)=0]$, the center of mass and the relative degrees of freedom are completely uncoupled. A spatially uniform ac electric field couples only to the center-of-mass coordinate, so that Kohn's argument[4] about the absence of electron-electron interaction effects in cyclotron resonance is clearly valid.

We study the equation of motion $Q_{\alpha\beta}(t)$,

$$i\frac{d}{dt}Q_{\alpha\beta}(t)=\delta(t)\langle[J_{\alpha}(t),J_{\beta}(0)]\rangle+\theta(t)\langle[\dot{J}_{\alpha}(t),J_{\beta}(0)]\rangle\,. \tag{6}$$

For a two-dimensional electron gas (i.e., neglecting all of the higher subbands of the surface potential well), the α and β appearing in Eq. (6) can be either x or y. It is apparent $Q_{xx}=Q_{yy}$ and that $Q_{xy}=-Q_{yx}$. Taking the Fourier transform with respect to time gives

$$\omega Q_{xx}(\omega)=\langle[J_x(0),J_x(0)]\rangle+i\int_{-\infty}^{\infty}dt\,e^{i\omega t}(-i)\theta(t)\langle[\dot{J}_x(t),J_x(0)]\rangle \tag{7}$$

and

$$\omega Q_{xy}(\omega)=\langle[J_x(0),J_y(0)]\rangle+i\int_{-\infty}^{\infty}dt\,e^{i\omega t}(-i)\theta(t)\langle[\dot{J}_x(t),J_y(0)]\rangle\,. \tag{8}$$

Recall that

$$\vec{J}(0)=-\frac{e}{m}\vec{\pi}(0)=-\frac{e}{m}\left(\vec{P}+\frac{Ne}{c}\vec{A}\right).$$

Thus,

$$J_x(0) = -\frac{e}{m}\pi_x(0) = \frac{ie}{m}\frac{\partial}{\partial X}, \tag{9}$$

$$J_y(0) = -\frac{e}{m}\pi_y(0) = \frac{ie}{m}\frac{\partial}{\partial Y} - \frac{eM\omega_c}{m}X. \tag{10}$$

Here we have used the relations $\vec{A}(\vec{R}) = (0, HX, 0)$ and $\omega_c = eH/mc$. Throughout this paper we take $\hbar$ equal to unity. It is clear that

$$[J_x(0), J_y(0)] = -i(e/m)^2 M\omega_c. \tag{11}$$

The time derivative of J_x is given by

$$\dot{J}_x = -i[J_x, H] = -\omega_c J_y + (e/m)U_x, \tag{12}$$

where $U_\alpha = \partial U/\partial R_\alpha$, and U is the last term in Eq. (5), namely, the electron-impurity interaction

$$U = \sum_{\vec{q}, j, l} e^{i\vec{q}\cdot\vec{R}}[u(q)\, e^{i\vec{q}\cdot(\vec{r}_j' - \vec{R}_l)}]. \tag{13}$$

Similarly one can show that $\dot{J}_y = \omega_c J_x + (e/m)U_y$. Substituting these results into the equations of motion [Eqs. (7) and (8)] gives

$$\omega Q_{xx}(\omega) = -i\omega_c Q_{yx}(\omega) - iI_{xx}(\omega) \tag{14}$$

and

$$\omega Q_{xy}(\omega) = -i(e/m)^2 M\omega_c - i\omega_c Q_{yy}(\omega) - iI_{xy}(\omega). \tag{15}$$

In these equations

$$I_{\alpha\beta}(\omega) = \frac{e}{m}\int_{-\infty}^{\infty} dt\, e^{i\omega t}(-i)\theta(t)\,\langle[U_\alpha(t), J_\beta(0)]\rangle. \tag{16}$$

We can go a step further by writing the equations of motion for $I_{\alpha\beta}$; they can be shown to have the form

$$\omega I_{xx}(\omega) = A_{xx}(t=0) + i\omega_c I_{xy}(\omega) + i\phi_{xx}(\omega) \tag{17}$$

and

$$\omega I_{xy}(\omega) = A_{xy}(t=0) - i\omega_c I_{xx}(\omega) + i\phi_{xy}(\omega), \tag{18}$$

where

$$A_{\alpha\beta}(t=0) = (e/m)^2\langle[U_\alpha, \pi_\beta]\rangle$$

and

$$\phi_{\alpha\beta}(\omega) = \left(\frac{e}{m}\right)^2\int_{-\infty}^{\infty} dt\, e^{i\omega t}[-i\theta(t)] \times\langle[U_\alpha(t), U_\beta(0)]\rangle. \tag{19}$$

By symmetry arguments, if we average Eqs. (19) and (18) over impurity coordinates, the contribution from $\phi_{xy}(\omega)$ vanishes. We assume that all the retarded Green's functions $I_{\alpha\beta}(\omega)$ and $\phi_{\alpha\beta}(\omega)$ remain finite as $\omega \to 0$. Then from Eqs. (17) and (18) we find

$$A_{xx}(t=0) = -i\omega_c I_{xy}(0) - i\phi_{xx}(0)$$

and

$$A_{xy}(t=0) = i\omega_c I_{xx}(0). \tag{20}$$

In Appendix A, we show that $I_{xx}(0) = I_{xy}(0) = 0$; therefore Eq. (20) implies

$$A_{xy}(t=0) = 0, \quad A_{xx}(t=0) = -i\phi_{xx}(0).$$

Solving the equations of motion [Eqs. (14), (15), (17), and (18)] for $Q_\pm = Q_{xx} \mp iQ_{xy}$ to lowest order in the concentration of impurities gives

$$Q_\pm = \left(\frac{e}{m}\right)^2\left(\pm\frac{M\omega_c}{\omega\mp\omega_c} + \frac{\tilde{\phi}_{xx}(\omega)}{\omega\mp\omega_c^2}\right), \tag{21}$$

where $\tilde{\phi}_{xx}(\omega) = (m/e)^2[\phi_{xx}(\omega) - \phi_{xy}(0)]$. After averaging over the impurity coordinates, $\tilde{\phi}_{xx}(\omega)$ becomes

$$\tilde{\phi}_{xx}(\omega) = n_i\sum_q |u(q)|^2 q_x^2[\chi(q,\omega) - \chi(q,0)], \tag{21'}$$

where

$$\chi(q,\omega) = -i\int dt\, e^{i\omega t}\theta(t)\,\langle[\hat{\rho}_q(t), \hat{\rho}_{-q}(0)]\rangle$$

is the density-density correlation function, and $\hat{\rho}_q$ is the density fluctuation operator. n_i is the concentration of impurities. By using the holomorphic memory function approach of Götze and Wölfle,[12] we can express the conductivity as

$$\sigma_\pm(\omega) = \frac{iNe^2/m}{\omega\mp\omega_c + M(\omega)}, \tag{22}$$

where

$$M(\omega) = M_1(\omega) + iM_2(\omega) = (1/Nm\omega)\tilde{\phi}_{xx}(\omega).$$

The conductivity can be expressed in a more familiar form by introducing a frequency-dependent mass and relaxation time defined by

$$m^*(\omega) = m[1 + M_1(\omega)/\omega], \tag{23}$$

$$\tau(\omega) = M_2^{-1}(\omega)[1 + M_1(\omega)/\omega]. \tag{24}$$

Then the conductivity takes the Drude form

$$\sigma_\pm(\omega) = \frac{iNe^2/m^*(\omega)}{[\omega \mp eH/m^*(\omega)c] + i\tau^{-1}(\omega)}. \tag{25}$$

The fundamental resonance occurs at the frequency satisfying the equation $\omega = eH/m^*(\omega)c$. This is renormalized from the bare resonance by a factor $[1 + M_1(\omega)/\omega]$. It should be emphasized that $M(\omega)$ depends both on frequency and on magnetic field. The function $\chi(q,\omega)$ appearing in Eq. (21) is the density-density correlation function of the interacting electron gas. Electron-electron interactings enter the expression for $\sigma_\pm(\omega)$ only through the presence of the function $[\chi(q,\omega) - \chi(q,0)]$. In random-phase approximation $\chi(q,\omega)$ can be written

$$\chi(q,\omega) = \chi_0(q,\omega)/[1 - V(q)\chi_0(q,\omega)], \tag{26}$$

where $V(q)$ is the Fourier transform of the effec-

tive electron-electron interaction in the inversion layer, and $\chi_0(q,\omega)$ is the density-density correlation function in the absence of $V(q)$. Chiu and Quinn[13] have evaluated $\chi_0(q,\omega)$ for a two-dimensional electron gas in the presence of a dc magnetic field. We have used this $\chi_0(q,\omega)$ in a previous paper[8] to discuss the behavior of $\sigma_\pm(\omega)$. Some of the unexpected results outlined in Sec. I can be explained qualitatively. In Sec III we shall calculate the dynamical conductivity of the electrons in the surface inversion layer in the presence of a magnetic field. Before we do that, let us rewrite the Hamiltonian given in Eq. (3) in the form of second quantization

$$H=\sum_{n,k_y} E_n C^\dagger_{n,k_y} C_{n,k_y} + V_{ee} + U\,, \tag{27}$$

where

$$V_{ee}=\tfrac{1}{2}\sum_q \sum_{\substack{n,m\\ n',m'}} V(q) J_{nm}(q_x, k_y+q_y, k_y) \times J_{n'm'}(-q_x, k'_y-q_y, k'_y) \times C^\dagger_{n,k_y+q_y} C^\dagger_{n',k'_y-q_y} C_{m',k'_y} C_{m,k_y}$$

and

$$U=\sum_{q,l}\sum_{n,n',k_y} u(q) e^{-i\vec{q}\cdot\vec{R}_l} J_{nn'}(q_x,k_y,k_y-q_y) C^\dagger_{n,k_y} C_{n',k_y+q_y}\,.$$

In these equations the symbol J_{nm} is given by

$$J_{nm}(q_x,k_y,k'_y)=\int_{-\infty}^{\infty} e^{iq_x x}\phi_n\left(\frac{x}{\alpha}+\alpha k_y\right) \times \phi_m\left(\frac{x}{\alpha}+\alpha k_y\right)dx\,. \tag{28}$$

$E_n=(n+\frac{1}{2})\omega_c$ is the Landau-level energy, and $\alpha=1/m\omega_c$. $C^\dagger_{n,k_y}$ and C_{n,k_y} are, respectively, the creation and the annihilation operators for electrons in the Landau level (n,k_y). The wave function associated with the Landau level is

$$\psi_{n,k_y}(r)=e^{-ik_y y}\phi_n(x/\alpha+\alpha k_y)\,. \tag{29}$$

$\phi_n(x/\alpha+\alpha k_y)$ is the nth eigenfunction of a simple harmonic oscillator. It can be expressed in terms of Hermite polynomials by the equation

$$\phi_n(x/\alpha+\alpha k_y)=(\sqrt{\pi}2^n n!\,\alpha)^{-1/2} H_n(x/\alpha+\alpha k_y) \times \exp[-\tfrac{1}{2}(x/\alpha+\alpha k_y)^2]\,. \tag{30}$$

III. CALCULATION

In order to calculate the dynamical conductivity $\sigma_\pm(\omega)$, we need to determine the memory function $M(\omega)$ which is given by Eq. (22). $M(\omega)$ depends on the density-density correlation function $\chi(q,\omega)$. In random phase approximation $\chi(q,\omega)$ is of the form given by Eq. (26). In Eq. (26), $\chi_0(q,\omega)$ is the density-density correlation function in the absence of electron-electron interactions. In time and configuration spaces $\chi_0(r,t)$ can be written

$$\chi_0(r,t)=-i\theta(t)\langle[\hat{\rho}(r,t),\hat{\rho}(0,0)]\rangle\,, \tag{31}$$

where $\hat{\rho}(r)$ is the density operator

$$\hat{\rho}(r)=\psi^\dagger(r)\psi(r)\,. \tag{32}$$

$\psi(r)$ is the annihilation operator for a particle at position $\vec{r}$. $\psi(r)$ can be written

$$\psi(r)=\sum_{n,k_y} e^{-ik_y y}\phi_n(x+\alpha k_y) C_{n,k_y}\,. \tag{33}$$

From Eqs. (32) and (33), the Fourier transform of $\hat{\rho}(r)$ in momentum space can be written

$$\hat{\rho}(q)=\sum_{k_y}\sum_{nn'} J_{nn'}(-q_x,k_y,k_y+q_y)\, C^\dagger_{n,k_y} C_{n',k_y+q_y}\,, \tag{34}$$

where $J_{nn'}$ has been given by Eq. (28). From Eqs. (31) and (34), we can show that $\chi_0(q,\omega)$ is given by the expression

$$\chi_0(q,\omega)=\sum_{k_y}\sum_{nn'} J_{nn'}(q_x,k_y,k_y-q_y) J_{nn'}(-q_x,k_y-q_y,k_y) \times \pi_{n,n'}(k_y,q_y,\omega)\,, \tag{35}$$

where $\pi_{nn'}(k_y,q_y,\omega)$ is the Fourier transform with respect to time of the function

$$\pi_{nn'}(k_y,q,t) = -i\theta(t)\langle[C^\dagger_{n,k_y}(t)C_{n',k_y-q_y}(t), C^\dagger_{n',k_y-q_y}C_{n,k_y}]\rangle\,. \tag{36}$$

The right-hand side of Eq. (36) can be written in terms of the product of two causal Green's functions[14] which are defined, respectively,

$$G_n(k_y,t)=-i\langle T[C_{n,k_y}(t)C^\dagger_{n,k_y}]\rangle\,,$$
$$G_{n'}(k_y-q_y,t)=-i\langle T[C_{n',k_y-q_y}(t)C^\dagger_{n',k_y-q_y}]\rangle\,. \tag{37}$$

Because we are considering a two-dimensional electron gas in the presence of a magnetic field, the Landau energies of these electrons depend only on the quantum number n but not on the momentum k_y or q_y. It is straightforward to show that the Green's functions in Eq. (37) do not depend on the momentum indices. For simplicity we can drop k_y and q_y in Eq. (37) and write

$$G_n(t)=G_n(k_y,t),\quad G_{n'}(t)=G_{n'}(k_y-q_y,t)\,. \tag{38}$$

In terms of the Green's functions we can rewrite Eq. (35) as

$$\chi_0(q,\omega)=\sum_{nn'} C_{nn'}(q)\pi_{n,n'}(\omega)\,, \tag{39}$$

where

$$C_{nn'}(q)=\sum_{k_y} J_{nn'}(q_x,k_y,k_y-q_y) \times J_{n'n}(-q_x,k_y-q_y,k_y) \tag{40}$$

and

$$\pi_{nn'}(\omega)=T\sum_{l} G_n(\omega_l+\omega)G_{n'}(\omega_l)\,.$$

In the above expression for $\pi_{nn'}(\omega)$, T is the temperature and $G_n(\omega_l)$ is the finite-temperature Green's function.[14]

A. Evaluation of $C_{nn'}(q)$

The function $C_{nn'}(q)$ appearing in Eq. (40) has been discussed for a three-dimensional electron plasma.[15] Since the wave function $\phi_n(x/\alpha)$ which enters Eq. (40) through $J_{nn'}$ is independent of whether the electron gas is two dimensional or three dimensional, the quantity $C_{nn'}(q)$ is the same for both cases. From Ref. 15, $C_{nn'}$ can be written

$$C_{nn'}(q)=p\,|F_{nn'}(q)|^2$$

and (41)

$$F_{nn'}(q)=\int_{-\infty}^{\infty} dx\, e^{iqx}\phi_n\left(\frac{x}{\alpha}\right)\phi_{n'}\left(\frac{x}{\alpha}\right).$$

Here p stands for the degeneracy or the number of states on each Landau level. If we include the spin and the valley degeneracies for the Si(100) surface, then p has the value

$$p=(2A/\pi)eH/hc\,, \tag{42}$$

where A is the area of the inversion layer. The integral appearing in Eq. (41) can be evaluated using standard tables of integrals.[16] If we replace n' by $n'=n+l$ with l an integer, the explicit final form for $C_{n,n+l}(q)$ can be written

$$C_{n,n+l}(q)=p\,\frac{n!}{(n+l)!}\left(\frac{\alpha q}{\sqrt{2}}\right)^{2l}e^{-\alpha^2q^2/2}[L_n^l(\tfrac{1}{2}\alpha^2q^2)]^2\,. \tag{43}$$

The function $L_n^l(\frac{1}{2}\alpha^2q^2)$ is the associated Laguerre polynomial. For $l>0$, $L_n^l(x)$ is given by

$$L_n^l(x)=\sum_{m=0}^{n}(-1)^m\frac{(n+l)!}{(l+m)!\,(n-m)!}\,\frac{x^m}{m!}\,, \tag{44}$$

and for $l<0$, it can be written

$$L_n^l(x)=\sum_{m=-l}^{n}(-1)^m\frac{(n+l)!}{(l+m)!\,(n-m)!}\,\frac{x^m}{m!}\,. \tag{45}$$

B. Evaluation of $\pi_{nn'}(\omega)$

In Sec. II, we have described the method of obtaining the memory function $M(\omega)$. The memory function $M(\omega)$ is correctly evaluated to the lowest order in the impurity concentration. Therefore in evaluating the density-density correlation function, $\chi(q,\omega)$, the effects of electron-impurity scattering should no longer be included. If this is the case, and the Green's functions appearing in $\pi_{nn'}(\omega)$ are not renormalized by the impurity potential, and we should have

$$G_n(\omega)=1/(\omega-\epsilon_n)\,,$$

where

$$\epsilon_n=(n+\tfrac{1}{2})\omega_c-\epsilon_F\,. \tag{46}$$

Here ϵ_F is the Fermi energy of the electron gas. At finite temperature ω becomes $\omega_l=(2l+1)\pi T$. From Eq. (40), $\pi_{nn'}(\omega+i\delta)$ can be written explicitly as

$$\pi_{nn'}(\omega+i\delta)=\frac{f(\epsilon_{n'})-f(\epsilon_n)}{\omega+\epsilon_{n'}-\epsilon_n+i\delta}\,, \tag{47}$$

where $f(x)$ is the Fermi distribution. If $f(\epsilon_{n'})-f(\epsilon_n)\neq 0$, Eq. (47) predicts that whenever ω equals $\epsilon_n-\epsilon_{n'}\equiv(n-n')\omega_c$, $\pi_{nn'}(\omega+i\delta)$ diverges. These divergences will appear in the memory function $M(\omega)$ through the function $\chi_0(q,\omega)$ appearing in Eq. (26). On the other hand these sharp divergences have not been found in experimental measurements,[6,7] and thus they have to be removed by including higher-order effects of electron-impurity scatterings. The most intuitive way to remove these singularities is to renormalize the Green's function in Eq. (38) by electron-impurity interactions. To do this certainly needs justification. The only justification we can give is to note that in the absence of a magnetic field and of electron-electron interaction[17] the inverse of the transport lifetime or the memory function appearing in Eq. (22), can be obtained from a density-density correlation function which is a product of two Green's functions, and these two Green's functions should be renormalized by electron-impurity scatterings. Although the final value of the transport lifetime in Ref. 17 does not depend on the Green's functions renormalization, in the present case $\chi(q,\omega)$ must be renormalized by electron-impurity scatterings in order to remove the singularities.

The renormalized Green's function can be written in the form[18]

$$G_n^{-1}(\omega)=\omega-\epsilon_n-\tfrac{1}{4}\sum_{n=0}^{\infty}\Gamma_{nn'}^2G_{n'}(\omega)\,, \tag{48}$$

where $\Gamma_{nn'}^2$ measures the level width which comes from the electron-impurity scattering. It is given

by

$$\tfrac{1}{4}\Gamma_{nn'}^2 = n_i \sum_q |u_s(q)|^2 |F_{nn'}(q)|^2 , \tag{49}$$

where $u_s(q)$ is the value of the electron-impurity scattering potential properly screened by electrons on the Fermi surface. In general, Eq. (48) can only be solved numerically. For simplicity, we shall adopt the single-level approximation for the Green's function.[18] We have

$$G_n^{-1}(\omega) = \omega - \epsilon_n - \tfrac{1}{4}\Gamma_{nn}^2 G_n(\omega) . \tag{50}$$

For short-range interaction $u_s(q)$ can be regarded as constant; it is then straightforward to show that Γ_{nn} is independent of n. If we replace Γ_{nn} by Γ, then Γ has the value[19]

$$\Gamma^2 = \omega_c(2/\pi\tau) , \tag{51}$$

where τ, the scattering lifetime of an electron in the absence of the magnetic field, is given by

$$\frac{1}{\tau} = \frac{n_i m}{2\pi}\int_0^{2\pi} |u_s(\phi)|^2 d\phi = n_i m |u_s|^2 . \tag{52}$$

This last result for τ is obtained by assuming $u_s(\phi)$ to be a constant u_s. The Green's function $G_n(\omega)$ given by Eq. (50) can be solved for self-consistently. For $(\omega - \epsilon_n)^2 > \Gamma^2$ we find

$$G_n(\omega + i\delta) = (2/\Gamma^2)\{\omega - \epsilon_n - [(\omega - \epsilon_n)^2 - \Gamma^2]^{1/2}\} ; \tag{53}$$

and for $(\omega - \epsilon_n)^2 < \Gamma^2$, we have

$$G_n(\omega + i\delta) = (2/\Gamma^2)\{\omega - \epsilon_n - i[\Gamma^2 - (\omega - \epsilon_n)^2]^{1/2}\} . \tag{54}$$

With the Green's function given by the above equations, the function $\pi_{nn'}(\omega)$ appearing in Eq. (40) can be evaluated rather easily. We obtain the real part and the imaginary part of the retarded $\pi_{nn'}(\omega + i\delta) = \pi_{nn'}^R(\omega)$ as follows:

$$\mathrm{Re}\pi_{nn'}^R(\omega) = -\frac{1}{\pi}\int_{-\infty}^{\infty} dz\, f(z)[\mathrm{Re}G_n^R(z+\omega)\,\mathrm{Im}G_{n'}^R(z) + \mathrm{Re}G_{n'}^R(z-\omega)\,\mathrm{Im}G_n^R(z)] , \tag{55}$$

$$\mathrm{Im}\pi_{nn'}^R(\omega) = -\frac{1}{\pi}\int_{-\infty}^{\infty} dz\,[f(z) - f(z+\omega)] \times \mathrm{Im}G_n^R(z+\omega)\,\mathrm{Im}G_{n'}^R(z) .$$

The function $f(z)$ is defined as $f(z) = (1 + e^{z/T})^{-1}$. Because the Green's function depends on the Fermi energy ϵ_F through ϵ_n, ϵ_F must be determined self-consistently through the condition which conserves the number of electrons N in the inversion layer

$$-\frac{1}{\pi}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty} dz\, f(z)\,\mathrm{Im}G_n^R(z) = \frac{N}{p} . \tag{56}$$

From Eq. (56), ϵ_F not only depends on temperature T, but also depends on the magnetic field H.

C. Numerical calculation of the memory function $M(\omega)$ and the ac conductivity $\sigma_+(\omega)$

The electron-impurity potential $u(q)$ appearing in Eq. (21′) should not be renormalized by electrons on the Fermi surface. Since we have assumed that the screened potential $u_s(q)$ appearing in Eq. (52) is a constant for short-range interaction, $u(q)$ can be approximated in terms of $u_s(q) = u_s$ through the equation

$$u_s = u(q)/\epsilon(q, 0) , \tag{57}$$

where $\epsilon(q,\omega) = 1 - V(q)\chi_0(q,\omega)$. For $\omega = 0$, $\epsilon(q,\omega)$ is just the static dielectric constant of the electron gas. $M(\omega)$ in Eq. (22) can be rewritten

$$M(\omega) = (4\pi m^2 N\omega\tau_{\mathrm{tr}})^{-1}\int_0^{\infty} dq\, q^3 \frac{\epsilon^2(q,0)}{V(q)} \times [\epsilon^{-1}(q,\omega) - \epsilon^{-1}(q,0)] . \tag{58}$$

Here τ_{tr} is the electron transport lifetime in the absence of H and is defined as

$$\frac{1}{\tau_{\mathrm{tr}}} = \frac{n_i m}{2\pi}\int_0^{2\pi} |u_s(\phi)|^2(1 - \cos\phi)\, d\phi . \tag{59}$$

We have assumed here that $u_s(\phi)$ is a constant and thus $\tau_{\mathrm{tr}}^{-1} = n_i m u_s^2$. The effective electron-electron interaction $V(q)$ has been obtained previously[20]; it is given by

$$V(q) = (2\pi e^2/\epsilon_s q) I(q/b) ,$$

where

$$I(x) = (1+x)^{-6}[\tfrac{1}{8}x(33 + 54x + 44x^2 + 18x^3 + 3x^4) + 2\epsilon_s(\epsilon_s + \epsilon_0 \coth qD)^{-1}] . \tag{60}$$

$\epsilon_s = 11.8$ and $\epsilon_0 = 3.8$ are, respectively, the dielectric constants of Si and its oxide. $D \simeq 1000$ Å is the thickness of the oxide. The average value of the thickness of the inversion layer is given by $\langle z\rangle = 3b^{-1}$, and it depends on the number of electrons. The real part $M_1(\omega)$ and the imaginary part $M_2(\omega)$ of $M(\omega)$ can then be calculated numerically. The real part of the conductivity $\sigma_+(\omega)$ which is responsible for the cyclotron resonance is given by

$$\mathrm{Re}\sigma_+(\omega) = \frac{Ne^2}{m^*(\omega)}\frac{\tau^{-1}(\omega)}{[\omega - eH/m^*(\omega)c]^2 + \tau^{-2}(\omega)} \tag{61}$$

where the renormalized mass $m^*(\omega)$ and the relaxation time $\tau(\omega)$ are defined in Eqs. (23) and (24) in terms of $M_1(\omega)$ and $M_2(\omega)$. We have calculated $\mathrm{Re}\sigma_+(\omega)$ for the sample of Abstreiter *et al*,[21] by using the IBM 3600 Computer at Brown University. The carrier concentration for this sample is $N = 2.6\times 10^{12}$ electrons cm^{-2}, and its mobility μ at $T = 4.5$ °K is $\mu = 7000$ cm^2/V sec. The transport lifetime τ_{tr} appearing in the expression for $M(\omega)$

can be fixed by the relationship $\tau_{tr}=m/e\mu$. The bare mass of the electron is $m=0.19m_0$, where m_0 is the free-electron mass. With the carrier concentration of this sample, the thickness $\langle z\rangle$ of the inversion layer is estimated[22] to be $\langle z\rangle \simeq 20$ Å. Using the parameters given above and putting the single-particle lifetime τ of Eq. (52) equal to $\tau=\tau_{tr}$, we have evaluated $\mathrm{Re}\sigma_+(\omega)$ as a function of H (in kG) by fixing the incident $\omega=3.68$ meV. The result is shown in curve (a) of Fig. 1. The solid curve is our calculation and the dashed curve is the classical line shape. The dotted curve appearing in the figure is the result of experimental measurements.[9,21] Figure 1 shows clearly that the calculated line shape, curve (a), does not agree very well with the experimental result. We believe that the reason of this discrepancy is primarily due to the fact that we have assumed that $\tau=\tau_{tr}$ by assuming a zero-range electron-impurity interaction. The relationship $\tau=\tau_{tr}$ is no longer true if we include the effect of electron-electron interaction in the calculation of the electron self-energy. It is well known that the electron-electron interaction can directly contribute to the lifetime τ of a single particle but that it cannot contribute to the transport lifetime τ_{tr} in the absence of impurities or phonons.[17] Strictly speaking, the single-particle lifetime τ defined in Eq. (52) should be replaced by

$$\frac{1}{\tau}=\frac{n_i m}{2\pi}\int_0^{2\pi}|u_s(\phi)|^2 d\phi+\frac{1}{\tau_{ee}}\ , \tag{62}$$

where τ_{ee} is the correction to τ due to electron-electron interaction only. The screened electron-impurity potential in general is not of zero range so that the factor of $(1-\cos\phi)$ appearing in the right-hand side of Eq. (59) makes τ_{tr} different from the time τ appearing in Eq. (52). Although τ_{tr} can be determined by the mobility of the sample, τ is not known at all. The best we can do here is to put $\tau_{tr}=\beta\tau$ and regard β as a parameter of the order of magnitude of 1. For curve (b) of Fig. 1, we have chosen $\beta=1.8$, the obtained line shape agrees reasonably well with the experimental data. At $H\simeq 31$ kG both curves (a) and (b) show a weak "harmonic" structure at $2\omega_c\simeq\omega$. The cyclotron resonance peak is at $H\simeq 62$ kG. The remaining structure of the line shape is due to the oscillations of the Fermi energy as a function of magnetic field. ϵ_F has been calculated self-consistently from Eq. (56). The plot of ϵ_F as a function of H is shown in Fig. 2. The functions $M_1(\omega)$ and $M_2(\omega)$ have also been numerically calculated; the results are shown in Fig. 3. We also did similar calculations for the sample of Kennedy *et al.* The sample has a mobility $\mu=6500$ cm^2/Vsec at $T=4.5$ °K and an electron concentration $N=1.5\times10^{12}$ electrons cm^{-2}. The thickness of the inversion layer $\langle z\rangle$ for this

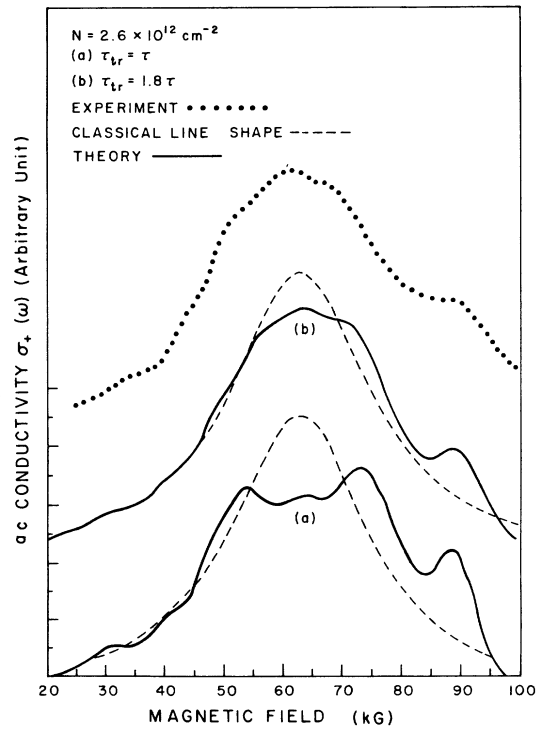

FIG. 1. $\mathrm{Re}\sigma_+(\omega)$ vs magnetic field in kG. The solid curves labeled (a) and (b) are theoretical results for $\tau=\tau_{tr}/1.8$. τ_{tr} is fixed from the value of the dc mobility μ(7000 cm^2/V sec) at $T=4.5$ °K. The external frequency ω is set at $\omega=3.68m$ eV. All the parameters are taken to agree with the experimental situation (Ref. 21).

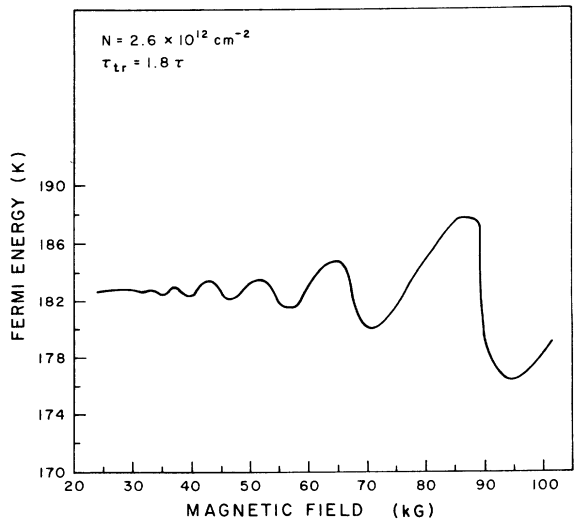

FIG. 2. Fermi energy (in units of temperature) vs magnetic field. All parameters needed are taken to agree with the sample of Ref. 21.

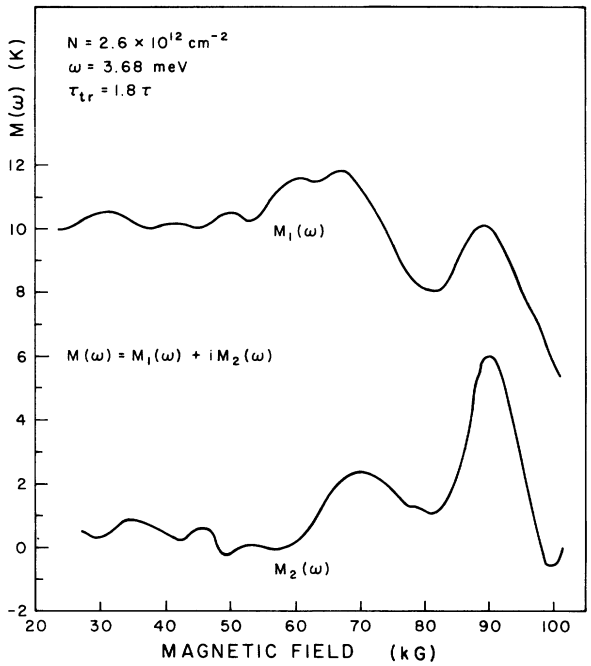

FIG. 3. Memory function $M(\omega)=M_1(\omega)+iM_2(\omega)$ (in units of temperature) vs magnetic field is calculated by using the parameters of Ref. 21.

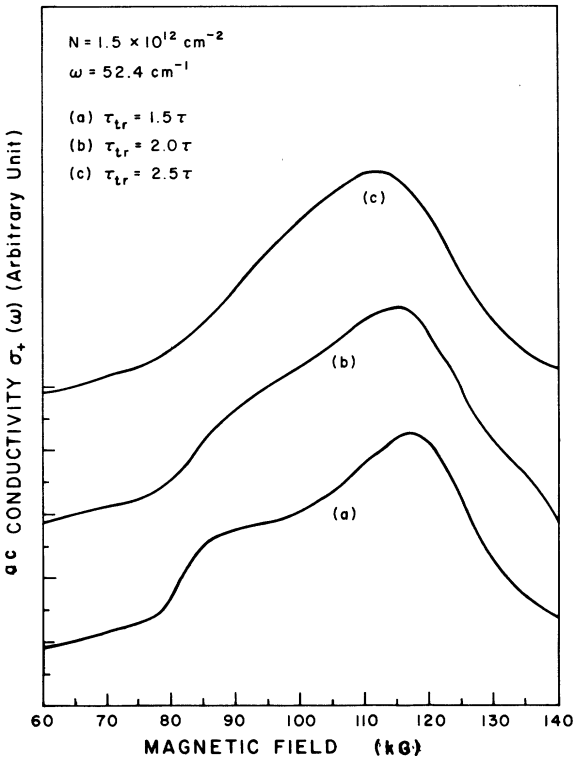

FIG. 4. $\mathrm{Re}\,\sigma_+(\omega)$ vs magnetic field for experimental parameters corresponding to the data of Kennedy *et al.* (Ref. 6). The mobility μ for this sample is $\mu=6500$ cm^2/V sec. The solid curves labeled (a), (b), and (c) are theoretical results for $\tau=\tau_{tr}$, $\tau=\tau_{tr}/2.0$, $\tau=\tau_{tr}/2.5$ at a frequency of $\omega=52.4$ cm^{-1}.

value of N is estimated to be $\langle z\rangle\simeq 30$ Å. The cyclotron-resonance line shape for the external frequency $\omega=52.4$ cm^{-1} has been calculated for $\tau^{-1}=1.5\tau_{tr}^{-1}$, $2.0\tau_{tr}^{-1}$, and $2.5\tau_{tr}^{-1}$. The results are shown in Fig. 4. From Fig. 5, it seems that the line shape with $\tau^{-1}=2.5\tau_{tr}^{-1}$ agrees reasonably well with the experimental measurements.[6] By fixing the value of the single-particle lifetime $\tau^{-1}=2.5\tau_{tr}^{-1}$, the cyclotron-resonance line shapes for $\omega=25.4$ cm^{-1} and $\omega=11.2$ cm^{-1} have also been computed. The results shown in Fig. 6 and 7, respectively. In Fig. 7 the calculation has not been extended below 20 kG, because too much computing time is needed there. We also have calculated the line shape as a function of ω in Fig. 8 by fixing the magnetic field $H=52$ kG. The cyclotron mass shift $\Delta m=m^*-m$, and m^* depends on the resonance frequency ω through $M_1(\omega)$ as given in Eq. (23). The results for $\Delta m/m$ as a function of the cyclotron-resonance frequency ω are shown in Fig. 9 for both samples considered previously. The structure of the curves in Fig. 9 is due to the oscillation of the Fermi level as a function of the magnetic field. Our calculations show that the values of $\Delta m/m$ are always less than 5% and oscillate as the resonance frequency ω or the magnetic field H increases. Our results are within the experimental error bars of Abstreiter *et al.*,[21] but do not agree with the experimental measurements of Kennedy *et al.*[6] who obtained almost a 10% effect on $\Delta m/m$. This latter discrepancy is possibly due to the method of defining $m^*(\omega)$. The authors in Ref. 6 essentially fit their experimental line shape with the best Drude formula for $\sigma_\pm$ allowing m^* to be a parameter for the fit.

IV. DISCUSSION

The main shortcoming of the present calculation seems to be the necessity of introducing a quasiparticle lifetime τ which is not calculated from first principles. When no dc magnetic field is present, the function $\chi(q,\omega)-\chi(q,0)$ appearing in Eq. (21) can be approximated by its value in the absence of electron-impurity scattering, and τ never enters the calculation. The reason for this is that we need to evaluate $\tilde{\phi}_{xx}$ only to first order in the concentration of impurities. Since the right-hand side of Eq. (21) contains a factor of n_i, it is already of first order even when $\chi(q,\omega)-\chi(q,0)$ is independent of impurity scattering. We have evaluated the memory function $M(\omega)$ in the absence of a dc magnetic field by approximating the unscreened electron-impurity potential u_q by

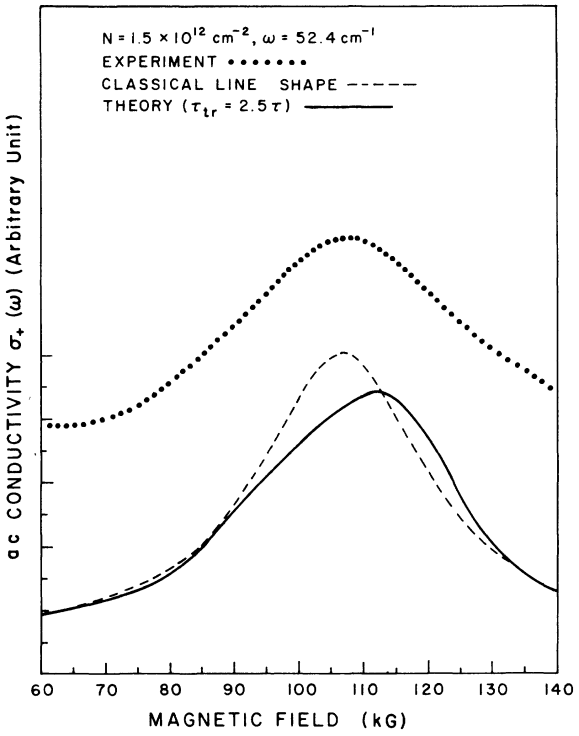

FIG. 5. Curve (c) of Fig. 4 is replotted and is compared with the experimental measurement of Ref. 6.

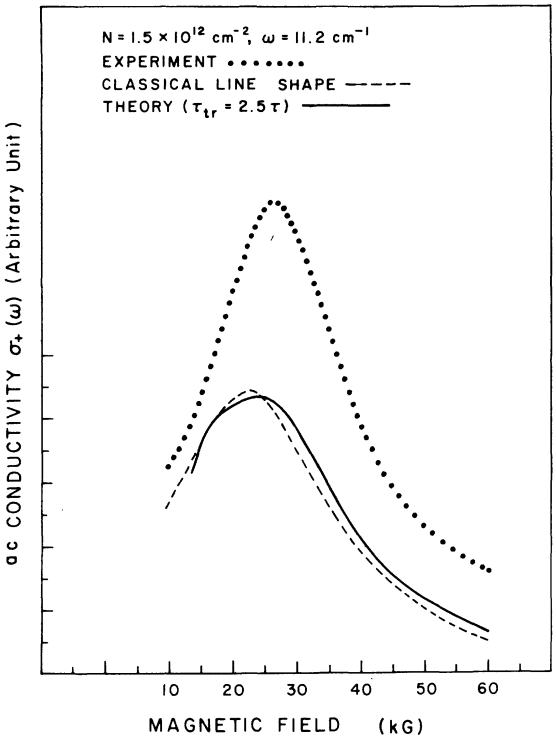

FIG. 7. Same as Fig. 6 but a frequency of $\omega = 11.2$ cm^{-1}.

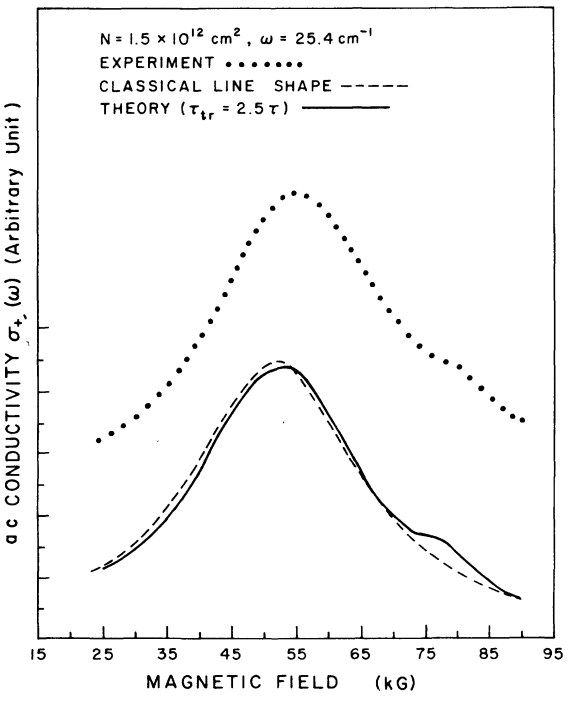

FIG. 6. Same as Fig. 5 but a frequency of $\omega = 25.4$ cm^{-1}.

$\epsilon(q,0)(n_i m\tau_{tr})^{-1/2}$ as discussed in Sec. III. The expression for $M(\omega)$ can be written

$$M(\omega) = (4\pi m^2 N\tau_{tr})^{-1} \int dq\, q^3 V_q^{-1} \epsilon^2(q,0)\omega^{-1} \times [\epsilon^{-1}(q,\omega) - \epsilon^{-1}(q,0)]\,. \quad (63)$$

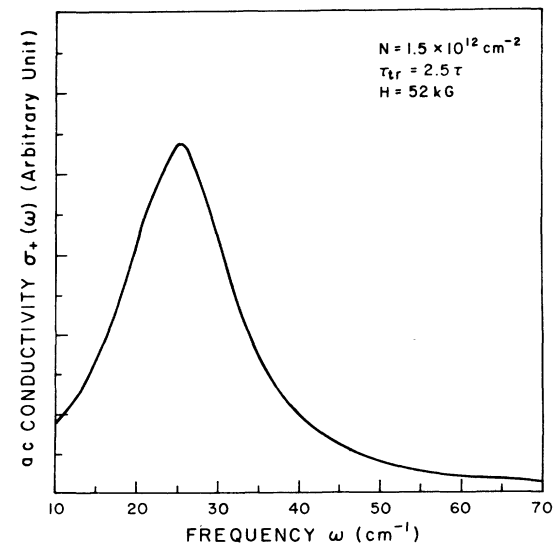

FIG. 8. Re$\sigma_+(\omega)$ vs frequency for the sample of Ref. 6. The magnetic field is fixed at $H = 52$ kG.

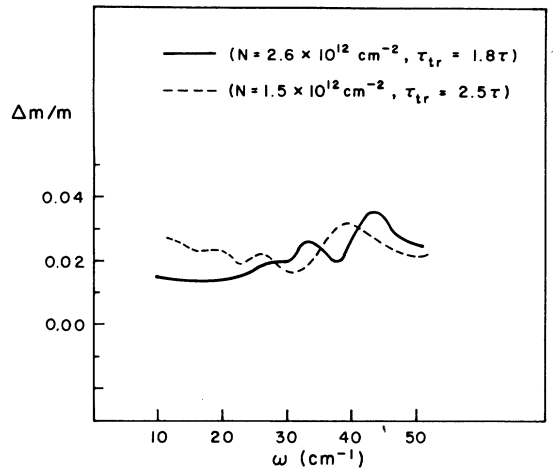

FIG. 9. Mass shift $\Delta m/m$ vs frequency is calculated for the sample of Kennedy *et al.* (Ref. 6) (solid line) and the sample of Abstreiter *et al.* (Ref. 21) (dashed line), respectively.

This expression is essentially identical to that of Tzoar *et al.* except that we have approximated the unscreened electron-impurity potential as described above. By noting that the real part of $\epsilon(q,\omega)$ is an even function of ω, and the imaginary part is an odd function of ω, one can demonstrate that for very small ω,

$$M_2(\omega) \simeq 1/\tau_{tr} + O(\omega^2) ,$$
$$M_1(\omega) = \alpha\omega + O(\omega^3) ,$$

where α is a constant. For larger values of ω we have evaluated $M_1(\omega)$ numerically, using the effective electron-electron interaction which takes account of image charges in the oxide layer. Results for the mass shift $\Delta m/m$ as a function of frequency are similar in shape to those of Tzoar *et al.*,[10] but the amplitude of the shift is smaller.

In the presence of a dc magnetic field $\chi(q,\omega)$ has resonances at $\omega = n\omega_c$, where n is an integer. These resonances correspond to transitions between Landau levels, and they take the form of δ-function singularities in the absence of broadening associated with electron-impurity and electron-electron scattering. In order to broaden these resonances, we have introduced a finite quasiparticle lifetime into the Green's functions which enter the expression for $\chi(q,\omega)$. We must do this in order to obtain a reasonable line shape for the absorption spectrum. The quasiparticle lifetime τ is expected to differ from the transport lifetime τ_{tr} if the impurities have a finite range. We do not attempt to calculate τ from first principles but instead assume the τ is proportional to τ_{tr}, and let the constant of proportionality be an adjustable parameter. Reasonably good agreement with the experimental data can be obtained in this way. However, our numerical calculations indicate that the constant of proportionality depends somewhat on frequency (as well as on concentration); this dependence is unexpected and somewhat disappointing.

In summary, we have shown how electron-electron interactions together with electron-impurity scattering can affect cyclotron resonance in semiconducting surface inversion layers. The fundamental resonance is shifted and its width is affected by these interactions. Both m^* and τ can be thought of as functions of both frequency and magnetic field. The magnitude of the shift in the fundamental cyclotron mass turns out to be less than 5% for reasonable values of the experimental parameters.

In addition to the calculation in Ref. 13, the quantity $\chi_0(q,\omega)$ was also calculated by Morgenstein Horing *et al*[23].

APPENDIX A

In this appendix we shall show that

$$I_{xx}(0) = I_{xy}(0) = 0 . \tag{A1}$$

In Sec. II we have derived Eqs. (14), (15), (17), and (18) by using the vector potential $\vec{A} = (0, HX, 0)$. It is straightforward to show that those equations are still valid even if a different vector potential is chosen. This conclusion means that our results are gauge invariant. It is much easier to obtain Eq. (A1) if we choose the vector potential to be $A = (-HY, 0, 0)$. In this gauge, the current operators appearing in $Q_{\alpha\beta}(\omega)$ and $I_{\alpha\beta}(\omega)$ are given by

$$J_x = -(e/m)\pi_x = -(e/m)(P_x - M\omega_c Y) \tag{A2}$$

and

$$J_y = -(e/m)\pi_y = -(e/m)P_y . \tag{A3}$$

Let us introduce the retarded Green's function of two operators A and B defined as

$$G^R(A,B,\omega) = -i\int_{-\infty}^{\infty} dt\, e^{i\omega t}\theta(t)\langle[A(t),B]\rangle . \tag{A4}$$

Then from Eq. (16), $I_{\alpha\beta}(\omega)$ is given by

$$I_{\alpha\beta}(\omega) = -(e^2/m^2)G^R(U_\alpha, \pi_\beta, \omega) . \tag{A5}$$

The Green's function $G^R(P_x, \pi_y, \omega)$ satisfies the equation of motion

$$\omega G^R(P_x, \pi_y, \omega) = \langle [P_x, \pi_y] \rangle + G^R([P_x, H], \pi_y, \omega) = -iG^R(U_x, \pi_y, \omega) . \quad \text{(A6)}$$

We assume that the Green's function is regular or has a finite value at $\omega = 0$; then we obtain $G^R(U_x, \pi_y, 0) = 0$ from Eq. (A6). Using Eq. (A5) we have $I_{xy}(0) = 0$.

Similarly we have the equation of motion

$$\omega G^R(P_x, \pi_x, \omega) = \langle [P_x, \pi_x] \rangle + G^R([P_x, H], \pi_x, \omega) = -iG^R(U_x, \pi_x, \omega) . \quad \text{(A7)}$$

Letting $\omega \to 0$, we obtain $G^R(U_x, \pi_x, \omega) = 0$. Using Eq. (A5), we find $I_{xx}(0) = 0$.

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[21]The second reference in Ref. 1.

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