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High-field Righi-Leduc effect and lattice thermal conductivity of potassium*

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We report measurements of the high-field Righi-Leduc coefficient of polycrystalline specimens of potassium. The measurements were made between 2 and 9 K for magnetic fields up to 9 T on specimens having residual-resistance ratios ranging from 2100 to 7300. We find that the lattice thermal conductivity causes the low-temperature Righi-Leduc coefficient to be field dependent. Using simple theoretical ideas, we are able to determine the magnitude and temperature dependence of the lattice conductivity and demonstrate that it is not large enough to be responsible for the previously observed field dependence in the transverse thermal magnetoconductivity of potassium. After correcting the measured Righi-Leduc coefficient for the lattice contribution, we find it close to the theoretical predictions. When compared to the Hall coefficient, we find a substantial deviation from the Wiedemann-Franz law, in direct contrast to the theoretical predictions. By combining measurements of the thermal gradients in both directions perpendicular to the magnetic field we are able to operationally define an average thermal scattering time τ_{th}. We find our values of τ_{th} are consistent with theoretical predictions.

I. INTRODUCTION

During the past several years it has become firmly established that the magnetotransport coefficients of potassium are anomalous in that their magnetic field dependence does not follow that predicted by the semiclassical theory of Lifshitz, Azbel, and Kaganov (LAK).[1,2] This theory makes several unambiguous predictions concerning the magnetic field dependence of the various transport coefficients; most, if not all, of these predictions are not borne out by experiment. The following paragraph is a brief outline of the conflicts; for a more complete discussion, including a discussion of several theoretical explanations, see Refs. 3 and 4.

For an uncompensated metal such as potassium, which has a closed nearly spherical Fermi surface[5] the LAK theory predicts that in the high-field limit ($\omega_c\tau \gg 1$), the diagonal components of the two resistivity (thermal and electrical) tensors will saturate (become independent of the magnetic field), and the off-diagonal components will tend to their free-electron values. Experiment has shown several of these predictions to be untrue. The transverse electrical magnetoresistivity is linear in the applied field, the transverse thermal magnetoresistivity has a term that is quadratic in the field,[4,6,7] and the two longitudinal magnetoresistivities appear to be linear in the applied field.[8,9] On the other hand, the measured Hall coefficient of potassium agrees, within a few percent, with the semiclassical predictions[10] (however, see below, Sec. V). This paper presents the results of measurements of the Righi-Leduc (thermal Hall) coefficient of potassium. These measurements were performed with two objectives in mind: (i) to determine if the off-diagonal elements of the thermal-resistivity tensor obeyed the LAK theory, and (ii) to investigate the possibility that the lattice conductivity plays a large role in the quadratic term in the transverse thermal magnetoresistivity. We may briefly summarize our most important findings: (a) the measured Righi-Leduc coefficient is within a few percent of the predicted value and (b) our measurements show conclusively that the thermal conductivity of the lattice cannot be as large as necessary to explain the field dependence of the thermal magnetoresistivity. An inconsistency arises if the lattice thermal conductivity is used to explain the field dependence of both the Righi-Leduc effect and the transverse thermal magnetoresistance. We note that extending the measurements of the thermal transport coefficients to fields well in excess of 1 T has contributed remarkably to our ability to unfold the various processes contributing to these coefficients.

Section II is a brief discussion of the effects of a finite lattice conductivity on the transport coefficients of a metal. Section III briefly discusses sample preparation and measurement techniques. Section IV presents the actual experimental data with several brief comments and the data are analyzed in Sec. V where our conclusions are presented.

II. LATTICE THERMAL CONDUCTION AND ITS EFFECT ON THE THERMAL TRANSPORT COEFFICIENTS

This section is a discussion of the effect that a finite lattice thermal conductivity has in determining the measured values of the thermal transport coefficients of a metal; in particular, we are interested in the effect of the lattice conductivity κ_g on the behavior of the transverse thermal magnetoresistivity and on the Righi-Leduc coefficient in the high-field limit. Only the case of an uncompensated metal with a closed Fermi surface is considered; for a discussion of the results for other cases see Ref. 11.

Ignoring magnetothermoelectric effects,[11] the thermal current density $\vec{J}_Q$ may be written

$$\vec{J}_Q = -\overleftrightarrow{K} \cdot \vec{\nabla} T ,$$

where T is the absolute temperature and $\overleftrightarrow{K}$, the thermal-conductivity tensor, is a function of temperature, magnetic field, and purity. Experimentally, one usually directs the heat current along the specimen and measures the resultant gradient:

$$-\vec{\nabla} T = \overleftrightarrow{W} \cdot \vec{J}_Q ,$$

where $\overleftrightarrow{W} = \overleftrightarrow{K}^{-1}$ is the thermal-resistivity tensor. For the purposes of this discussion the thermal current may be considered to consist of two types of carriers: (i) electrons, whose trajectories are affected by the magnetic field, and (ii) phonons, whose trajectories are unaffected by the magnetic field. We represent the thermal conductivity of each of these systems of carriers by the tensors $\overleftrightarrow{K}_e$ and $\overleftrightarrow{K}_g$, respectively, and assume they add linearly to determine the total thermal conductivity:

$$\overleftrightarrow{K} = \overleftrightarrow{K}_e + \overleftrightarrow{K}_g .$$

With the magnetic field $\vec{H} = H\hat{z}$ and parallel to a high-symmetry direction, the two tensors may be written

$$\overleftrightarrow{K}_e = \begin{pmatrix} K_{xx} & K_{xy} & 0 \\ -K_{xy} & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{pmatrix}$$

and $\overleftrightarrow{K}_g = \kappa_g \overleftrightarrow{I}$. Adding these two tensors and inverting the result yields the thermal resistivities

$$W_{xx}^M = (K_{xx} + \kappa_g)/[(K_{xx} + \kappa_g)^2 + K_{xy}^2] , \tag{1}$$

$$W_{xy}^M = -K_{xy}/[(K_{xx} + \kappa_g)^2 + K_{xy}^2] , \tag{2}$$

$$W_{zz}^M = 1/(K_{zz} + \kappa_g) \tag{3}$$

(we have assumed cubic symmetry).

These are the transport coefficients one would *measure* in the presence of a nonzero lattice thermal conductivity. The quantities of most interest to us are the electronic contributions to these coefficients, defined when $\kappa_g = 0$:

$$W_{xx}^e = \frac{K_{xx}}{K_{xx}^2 + K_{xy}^2} ; \quad W_{xy}^e = \frac{-K_{xy}}{K_{xx}^2 + K_{xy}^2} ; \quad W_{zz}^e = 1/K_{zz} . \tag{4}$$

Using Eq. (4), we may rewrite Eqs. (1)–(3):

$$W_{xx}^M = \frac{W_{xx}^e \{1 + \kappa_g W_{xx}^e + \kappa_g [(W_{xy}^e)^2 / W_{xx}^e]\}}{(1 + \kappa_g W_{xx}^e)^2 + (\kappa_g W_{xy}^e)^2} , \tag{5}$$

$$W_{xy}^M = W_{xy}^e / [(1 + \kappa_g W_{xx}^e)^2 + (\kappa_g W_{xy}^e)^2] , \tag{6}$$

$$W_{zz}^M = W_{zz}^e / (1 + \kappa_g W_{zz}^e) . \tag{7}$$

These results have been confirmed in the low-field limit.[12] In that limit, one obtains for "reasonably" pure materials, $W_{xx}^M \cong W_{xx}^e$ and $W_{xy}^M = W_{xy}^e (1 + \kappa_g W_{xx}^e)^{-2}$, the measured Righi-Leduc coefficient being less than the electronic coefficient by the factor $(\kappa_{\text{electronic}}/\kappa_{\text{total}})^2$.

In this paper we are concerned about the field and temperature dependence of these results [Eqs. (5)–(7)] in the high-field limit. For an uncompensated cubic metal with a closed Fermi surface, the high-field limit ($\omega_c \tau \gg 1$ where ω_c is the cyclotron frequency and τ is the mean time between scattering events) of the electronic thermal-conductivity tensor may be shown to be[2]:

$$\overleftrightarrow{K}_e = \begin{pmatrix} A_{xx}/H^2 & L_0 T n e/H & 0 \\ -L_0 T n e/H & A_{xx}/H^2 & 0 \\ 0 & 0 & A_{zz} \end{pmatrix} .$$

The coefficients A_{ii} depend on temperature, scattering, etc., but are independent of the magnetic field. This, of course, leads to the well-known prediction of a saturating magnetoresistivity and a "free-electron" Righi-Leduc effect:

$$W_{xx}^e = A_{xx}/(L_0 T n e)^2 ,$$

and

$$W_{xy}^e = H/L_0 T n e = H R_H / L_0 T .$$

(L_0 is the Lorenz number. It should be also noted that these results do not depend on the relaxation-time approximation.) Comparing these results with similar results for the electrical magnetoresistivity[1] shows that in the high-field limit the ratio of the Hall resistivity to the electronic contribution to the Righi-Leduc resistivity is $L_0 T$; that is, the theory predicts that for the off-diagonal terms, the Wiedemann-Franz law is obeyed exactly in the high-field limit.

To proceed further we first define a "very-high-field limit," which occurs when the magnetic field is so large that $(\kappa_g W_{xy}^e)^2 > 1$ or $H > L_0 T / \kappa_g R_H$. On the other hand, if we roughly determine the relaxation time from the free-electron resistivity,

$\rho_0^{-1} = ne^2\tau/m$, $\omega_c\tau > (L_0T/R_H\kappa_g)(\omega_c\tau/H)$. The asymptotic behavior of W^M_{xy} and W^M_{xx} will be discussed for two cases, either $\kappa_g W^e_{xx}$ is negligible or it is not negligible compared to one. Note that for a particular metal either of these cases can occur, depending on the variation of W^e_{xx} with the magnetic field.

A. High-field limit: $1 << \omega_c\tau << (L_0T/R_H\kappa_g)(\omega_c\tau/H)$

Case (i): $\kappa_g W^e_{xx} \gtrsim 1$. From Eqs. (5) and (6), with appropriate approximations, it may be shown that

$$W^M_{xx} = W^e_{xx}/(1+\kappa_g W^e_{xx}) \tag{8}$$

and

$$W^M_{xy} = W^e_{xy}/(1+\kappa_g W^e_{xx})^2 \ . \tag{9}$$

The presence of the nonzero lattice conductivity substantially reduces the experimentally measured magnitudes of both transport coefficients from the magnitude due to electronic conduction. Experimentally this situation will occur whenever the electronic thermal conductivity is substantially reduced by alloying or when the saturation value of W^e_{xx} is large. [We note that there is a practical limit to increasing W^e_{xx} by alloying since we must be in the high-field limit for Eqs. (8) and (9) to be valid.]

Case (ii): $\kappa_g W^e_{xx} \ll 1$. In this case, from Eqs. (5) and (6):

$$W^M_{xx} = W^e_{xx} + \kappa_g (W^e_{xy})^2 \quad \text{and} \quad W^M_{xy} = W^e_{xy} \ . \tag{10}$$

In this limit the lattice conductivity has essentially no effect upon the Righi-Leduc term, but a quadratic magnetic field dependence is introduced into the thermal magnetoresistivity ($W^e_{xy} \propto H$). (As mentioned, such a quadratic field dependence has been observed in the thermal magnetoresistivity of potassium[4, 6, 7] and indium.[13] The application of this result to potassium is discussed in Sec. V.)

B. Very-high-field limit: $\omega_c\tau >> (L_0T/R_H\kappa_g)(\omega_c\tau/H)$

In this case Eqs. (5) and (6) yield

$$W^M_{xx} = 1/\kappa_g \quad \text{and} \quad W^M_{xy} = 1/W^e_{xy}\kappa_g^2 \ . \tag{11}$$

The thermal magnetoresistivity again saturates, at a different but obvious value, but now the Righi-Leduc term decreases with increasing field.

In general this very-high-field limit is experimentally unattainable. For example, aluminum with a Hall coefficient[14] that varies between $R_H = 1.02\times10^{-10}$ and -0.33×10^{-10} m^3/C and with a lattice conductivity[15] $\kappa_g \approx 0.05$ W/m K yields (at a temperature of 3 K) a very-high-field limit in excess of 10^4 T, a field that is clearly unattainable in the laboratory. However, in potassium, the high-field Hall coefficient is over 10 times as large and κ_g has been postulated to be as much as 200 times as large.[7] This reduces the very-high-field limit to about 12 T. This is an attainable field and the effects of nearing this very-high-field region should be noticeable at fields considerably lower than this value. We discuss this further in Sec. V.

III. EXPERIMENTAL DETAILS

Most of the details concerning specimen preparation and mounting procedures and the measuring apparatus have been reported previously.[4, 6, 16] Standard linear heat-flow techniques were employed, with the temperature gradients in the specimen being measured using four germanium resistance thermometers calibrated as a function of the applied magnetic field as well as the temperature. The resistance of the measuring thermometers was determined using four-terminal ac potentiometric techniques. The data were obtained in two groups, using two different superconducting solenoids. For specimens K-19 and K-22, a 1.8-T 2-in.-bore solenoid was employed and, for specimens KHF 5, 7, and 8, a 1.5-in.-bore solenoid, nominally rated at 8 T at 4.2 K was employed. (This magnet was capable of 10 T at 2 K.) For the runs in the 8-T magnet, the germanium thermometers were calibrated versus field during each run, using a capacitance thermometer[17] as a transfer standard. The capacitance thermometer is wired into one arm of an active-bridge temperature controller.[18] The temperature is chosen in zero magnetic field and the controller balanced; as the magnetic field is increased, the resistances of the germanium thermometers change rapidly, but the capacitance thermometer is unaffected by the field, enabling the controller to maintain the temperature. We found we are able to maintain the temperature to within 5 mK or less during the 40-min calibration run, the error being due to drift in the capacitive sensor. The resistance of the germanium thermometers in zero field (at the same capacitance thermometer value) is measured before and after the calibration run and a linear correction applied to account for the drift. Once the calibration data is obtained, we proceed to obtain the thermal-resistivity and Righi-Leduc data. During all runs the 25-Hz thermometer current is adjusted to keep the power dissipation in the thermometers below 1 nW.

All resistance, magnetic field, and heater power data are measured using a digital data acquisition system[19] with paper tape output; the tape is then read into a computer which calculates the temperatures from the field and resistance values and performs the subsequent data analysis.

Data was taken with the magnetic field in both directions normal to the large surfaces of the flat-plate specimens and that component of the transverse temperature gradient that is odd in the applied magnetic field is used to determine the Righi-Leduc coefficient.

The magnetic field is determined by measuring the magnet current. The current in the 1.8-T solenoid was calibrated versus the magnetic field using NMR techniques,[20] the 8-T coil current was calibrated using a rotating coil gaussmeter.[21] In the 1.8-T coil and in the 8-T coil below 1.5 T, a small current-field hysteresis was noted; to ensure that the magnitude of the normal and reverse magnetic fields agreed to with 0.05%, a bismuth magnetoresistance probe[22] was placed near the specimen, and the magnet current adjusted until the field up and field down magnetoresistances matched.

Since the changes in the thermal resistance of potassium with field are very large, the heat current used is gradually reduced as the field is increased to keep the average specimen temperature within 50 mK of the chosen value during the field sweep.

The specimens were fabricated and mounted as previously described; their physical characteristics are shown in Table I. Following our previous procedures,[4,16] the zero-magnetic-field thermal resistivity was measured for each specimen and plotted vs T^3. The zero-temperature intercept and slope are shown in Table I; the zero-temperature intercept is used to determine the residual resistivity ratio (RRR) as previously discussed.[16]

All the specimens were annealed under vacuum, at room temperature, in a bath of paraffin oil for at least two days and then carefully mounted. The specimens were then slow-cooled[23] to liquid-N_2 temperatures over a period of 45 min. The precise details of the sample mounting techniques and details of the cryostat have been described previously.[4,16]

TABLE I. Physical characteristics of samples.

Specimen[a]	RRR[b]	Width[c] (mm)	Thickness[c] (mm)
K-19	2350	7.0	1.0
K-22	3920	7.0	1.0
KHF-5	2900	8.0	1.2
KHF-7	2090	8.0	1.0
KHF-8	7300	8.0	1.3

[a] Our potassium was purchased from MSA Corp., Evans City, Pa.

[b] The RRR was calculated using the Wiedemann-Franz law as described in Ref. 16.

[c] The thickness measurements are accurate to ±0.05 mm while the width measurements are accurate to ±0.1 mm.

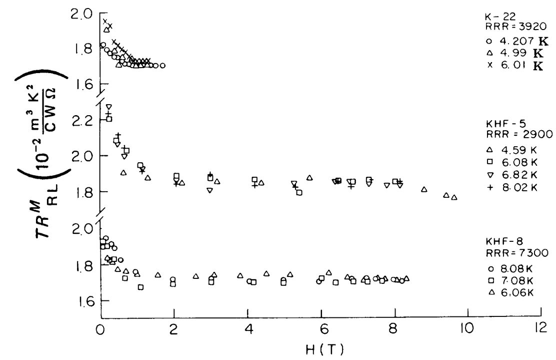

FIG. 1. Measured Righi-Leduc coefficient of potassium times the temperature as a function of the applied magnetic field; only the higher temperatures are displayed The specimens shown are representative of the range of purity studied.

IV. RESULTS

In order to keep the actual experimental results separate from the interpretation and analysis, this section discusses the experimental quantities as measured along with several brief comments. Section V contains a detailed analysis of the data.

Figure 1 shows high-temperature Righi-Leduc data from two representative specimens KHF-8 over the entire field range and similar data for K-22, over a lower field range. In the interest of clarity not all of the data have been plotted. Figure 2 shows some of the lower temperature data for KHF-5 and -8. (The curves in the figure are explained in Sec. V.) These figures show the Righi-Leduc coefficient times temperature as a

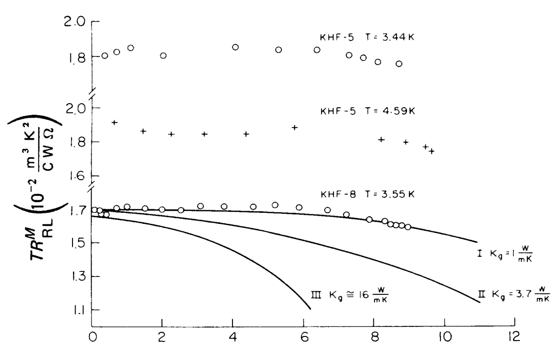

FIG. 2. Measured Righi-Leduc coefficient of potassium times the temperature as a function of the applied magnetic field, for several low temperatures. These data illustrate the field dependence of TR^M_{RL} at high fields. The curves are discussed in the text.

function of the applied magnetic field, where the measured Righi-Leduc coefficient is determined from the transverse thermal gradient $\nabla_y T$ and the thermal current J_Q:

$$TR^M_{RL} = (-\nabla_y T)T/HJ_Q$$

(We plot TR_{RL} as it is the thermal analog of R_H.) The pertinent parameters for the other specimens are shown in Table II. (The thermal magnetoresistivity data of these as well as several other specimens are reported elsewhere.[24]) There are several points to be noted in the data. At low fields there is considerable temperature and field dependence to TR^M_{RL}. For magnetic fields on the order of 1.2 to 2 T, at the higher temperatures investigated, TR_{RL} settles down to a constant value independent of field and temperature (at least to fields of the order of 8 T). From Fig. 2, however, it may be observed that the lower-temperature curves show a noticeable decrease in TR^M_{RL} at higher fields. This decrease is discussed in Sec. V in the light of the results of Sec. II.

It should also be noted that TR^M_{RL} remains temperature dependent to much higher fields than does the Hall coefficient.[10]

V. DISCUSSION

This section presents the interpretation and analysis of the data and is divided into three parts. It begins in Sec. VA with a discussion of the process used to determine when the high-field limit for thermal phenomena has been reached. Section VB discusses the Righi-Leduc coefficient and the effects of the lattice thermal conductivity. Section VC discusses the high-field results for the Righi-Leduc coefficient, compares the results with the predictions of the semiclassical theory and discusses the magnetoresistance anomalies in general.

A. High-field limit

As discussed in a previous article (Ref. 4), when thermal transport is considered, what is meant by the high-field limit is somewhat unclear. The problem lies in deciding on a scattering time τ_{th} appropriate for thermal transport processes; it is not possible from the Boltzmann equation to mathematically define an average "relaxation time" for small-angle inelastic scattering events. However, it is possible to obtain a reasonable determination of $\omega_c\tau_{th}$ from simultaneous measurements of the two temperature gradients ($\nabla_y T$ and $\nabla_x T$) in a specimen, following a procedure analogous to the electrical case. We consider first the electrical transport coefficients. It is easy to show using simple ideas, that

$$\omega_c\tau_{el} = HR_H/\rho_0 = E_y/E_x ,$$

where τ_{el} is the mean scattering time for determining electrical conduction, and E_x and E_y are the components of the electric field parallel and perpendicular to the current flow. Thus, in the electric case, $\omega_c\tau_{el} \gg 1$ implies $E_y \gg E_x$. Proceeding by analogy for the thermal case, an operational definition of $\omega_c\tau_{th}$ is

$$\omega_c\tau_{th} = \nabla_y T/\nabla_x T .$$

We will assume the specimen is well into the high-field limit when the Hall gradient is dominant and $\nabla_y T \gg \nabla_x T$. In Fig. 3, $\nabla_y T/\nabla_x T$ vs H is plotted for specimen KHF-8 at 3.55 K. $\nabla_y T/\nabla_x T$ rapidly increases, reaching 5.2 at 0.7 T and then drops down approximately as H^{-1}, the decrease being due to the rapid onset of the quadratic field-dependent term in the thermal magnetoresistance. Concentrating on the rising portion of the curve we may draw a straight line in a manner that ignores the onset of the linear and quadratic term in the thermal resistivity. (This is simply making the usual assumption of the semiclassical theory, that electron scattering rates are field independent.) From this line the field at which $\omega_c\tau_{th} = 1$ may be determined. In this case, $(\omega_c\tau_{th}) = 1$ at $H \cong 0.05$ T. We have done this for all our specimens at each tem-

TABLE II. Experimental Righi-Leduc coefficients.

Specimen	Field range	$\langle TR_{RL}\rangle_{av}$
K-19	1.8 T	1.75×10^{-2} m^{3}K^2/C WΩ
K-22	1.8 T	1.73
KHF-5	9.3 T	1.84
KHF-7	9.3 T	1.75
KHF-8	9.3 T	1.72

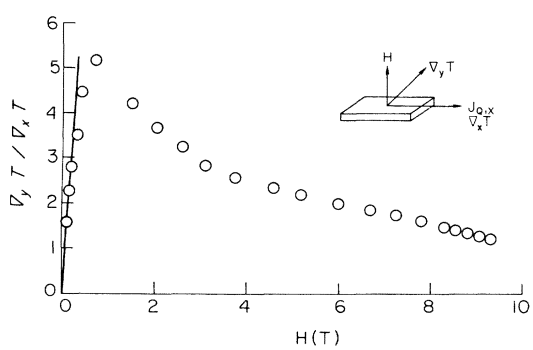

FIG. 3. Ratio of the thermal gradient perpendicular to the heat current to the thermal gradient parallel to the heat current (see inset) as a function of the applied magnetic field. The line drawn at low fields is discussed in the text.

perature; Fig. 4 is a plot of H_1 vs T for specimens KHF-5, 7, and 8 where H_1 is the field at which $\omega_c\tau_{th}=1$. (The electrical analog is the field at which $R_H/\rho=1$.) The data are plotted after subtracting an impurity component as follows. A plot of H_1 vs T is essentially a plot of $1/\tau_{total}$ vs T. Noting that

$$1/\tau_{tot}=1/\tau_{imp}+1/\tau_{ph} \quad ,$$

and estimating $1/\tau_{imp}$ or, rather, $H_{1,imp}$ from $W(H=0,T)T|_{T=0}$, leads to

$$H_{1,imp}=[W(0,T)T]_{T=0}L_0/R_H \; .$$

This value was determined for each specimen and was then subtracted from the temperature-dependent values. The results are plotted in Fig. 4; this figure may also be viewed as a plot of $1/\tau_{ph}$ vs T. Simple theoretical models and measurements[25] of $1/\tau_{ph}$ for potassium have indicated that $1/\tau_{ph}$ should be cubic in the temperature. The solid curve in Fig. 4 is cubic in the temperature and is seen to be quite a good fit. The coefficient of T^3 for this curve is 1.2×10^{-3} T/K^3, or scaling by e/m, we have at 4 K

$$1/\tau_{ph}\cong 14\times10^9 \text{ rad/sec.}$$

This may be compared to the results of Wagner and Albers,[25] who have calculated the average scattering rate at 4 K:

$$1/\tau_{ph}=3.2\times10^9 \text{ rad/sec.}$$

Thus the magnitude of the experimental value of τ_{th} is in fair agreement with calculated values and the temperature dependence is in excellent agreement; this gives additional support to our operational method of obtaining $\omega_c\tau_{th}$. We consider that the curve in Fig. 4 represents the transition field between low- and high-field behavior. It is important to note that the high-field limits determined in this manner are in general significantly higher than those determined from the electrical resistivity. For example, for specimen KHF-5, $\rho_{imp}=2.46\times10^{-9}$ Ω-cm and the impurity contribution to the electrical resistivity is at *least* 10 times as large as the temperature-dependent portion of the electrical resistivity[23] to temperatures as high as 4 K. Thus, $\omega_c\tau_{el}\approx1$ at fields of the order of 0.05 T, and is essentially independent of temperature to temperatures in excess of 4 K. From Fig. 4, when the impurity term is added in, it may be seen that at 4 K, $\omega_c\tau_{th}\approx1$ at about 0.15 T, a field nearly 3 times as large.

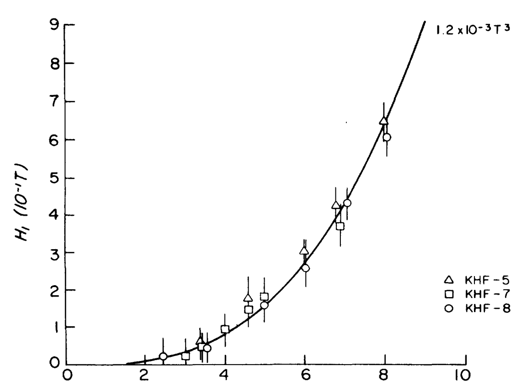

FIG. 4. H_1, the field at which $\omega_c\tau_{th}=1$ as a function of the temperature. The impurity contribution to τ_{th} has been substracted from H_1.

In the discussion to follow we will distinguish between the high- and low-field limits according to the curve in Fig. 4. It is also worth noting that for our purest specimens at low temperatures $\omega_c\tau_{th}$ is in excess of 200.

B. Effects of the lattice conductivity on TR^M_{RL}

This section examines the effect of a finite lattice thermal conductivity on the high-field values of TR^M_{RL}. To discuss the data in terms of the results from Sec. II it is necessary to determine $\kappa_g W^e_{xx}$ and $\kappa_g W^e_{xy}$. Unfortunately, a good experimental value for κ_g is not available for specimens of the purity used in this work. For such high-purity specimens there are in the literature two estimates of κ_g: Ekin,[26] using a variational calculation and only considering phonon-electron scattering, has determined values for κ_g over a wide temperature range, and, Fletcher[7] in an attempt to explain the quadratic field dependence of the thermal magnetoresistance of potassium has determined possible values of κ_g. These two results will be used to examine the high-field Righi-Leduc coefficient. For discussion purposes, we will concentrate on specimen KHF-8, using the $T=3.55$ K data, plotted in Fig. 2. It will be demonstrated that the large κ_g necessary to explain the thermal magnetoresistivity data is inconsistent with the Righi-Leduc data (and also inconsistent with the approximations one makes to obtain it).

Both the electrical and thermal magnetoresistivity of potassium are highly anomalous[3,4,6,7]; neither one shows any signs of saturation. Thus it is not clear if semiclassical magnetoconductivity theory is applicable to this seemingly simple metal. However, the off-diagonal terms in the resistivity tensors do appear to approach the values predicted by LAK: the Hall coefficient is within (4–6)% of the predicted value and the Righi-Leduc coefficient is within 5%. (Figure 1 and Table II.) Thus, a negligible error will be introduced by using free-electron values for W^e_{xy} to determine the relative magnitudes of the terms in the denominators of Eqs. (5)–(11).

Fletcher[7] made the interesting speculation that the entire quadratic term in the thermal magnetoresistivity of potassium was due to the lattice term, i.e., he assumed that both $\kappa_g W_{xx}^e$ and $\kappa_g^2 (W_{xy}^e)^2$ could be neglected and analyzed his data using Eq. (10). Fitting our thermal resistivity[24] data to Eq. (10), we obtain $\kappa_g = 15.9$ W/m K; using $R_{H,\mathrm{fe}} = 4.45 \times 10^{-10}$ m^3/C and this value for κ_g, $\kappa_g^2 (W_{xy}^e)^2 \approx 0.54$ at $H = 9$ T and $T = 3.55$. This is certainly *not* negligible compared to 1. Similarly, with $W_{xx}^e = W_{xx}^m - \kappa_g (W_{xy}^e)^2$, we obtain $\kappa_g W_{xx}^e \cong 0.06$. Both these terms contribute significantly to the denominator in Eqs. (5) and (6). We are led to two conclusions: (i) the large κ_g implies that we are at magnetic field values intermediate between the high- and very-high-field limit and that we must use Eq. (5) to determine W_{xy}^e and (ii) Fletcher's argument is internally inconsistent. Inserting the above values for $\kappa_g W_{xx}^e$ and $\kappa_g W_{xy}^e$ into Eq. (6) we obtain

$$W_{xy}^M / W_{xy}^e = TR_{\mathrm{RL}}^M / TR_{\mathrm{RL}}^e \cong 0.6$$

at 3.55 K and 9 T. As may be seen from Fig. 2 there are no such large decreases in TR_{RL}^M. Thus the assumption of a large anomalous κ_g being the cause of the quadratic term in the thermal magnetoresistivity leads to a prediction of a substantial decrease in the measured Righi-Leduc coefficient. Experimentally this does not occur and we conclude that Fletcher's speculation is not correct.

From Fig. 2 we note that there is a (5–10)% decrease in TR_{RL}^M as the field increases. Using Eqs. (5) and (6), we have attempted to fit this data and in the process determine κ_g. The results are the solid curves in Fig. 2. To obtain these curves we (a) "guessed" a value for κ_g; (b) calculated W_{xx}^e from Eq. (5) (all terms were used); (c) used the free-electron value of W_{xy}^e in the bracket of Eq. (6) and calculated values of W_{xx}^e and κ_g in Eq. (6) to determine W_{xy}^M; and (d) fit the result to the "intermediate" field values. Curve I uses Fletcher's speculation for κ_g, curve II uses Ekin's κ_g, and curve III is a "best" fit to the data.

We again observe that these results definitely rule out a large κ_g. Curve II, using Ekin's value (= 3.7 W/m K) is similarly seen to be a poor fit, the "best" fit being obtained with $\kappa_g = 1$ W/m K. We have performed similar fits to the Righi-Leduc data for other temperatures and specimens and we are able to estimate κ_g as a function of temperature. Before discussing the results, a few words of caution are in order. First, and most importantly, we are assuming that the decrease in TR_{RL}^M at high fields is due to κ_g. This may not be so; it is certainly possible that this may be related to the magnetoresistive anomaly of potassium. Secondly, the effects of κ_g on TR_{RL} are small and measurement imprecision precludes an accurate determination. Thus, while we are able to rule out large values of κ_g, the most we are able to say about the magnitude of κ_g is that the results are consistent with our data and seem to show the expected temperature dependence.

Figure 5 is a plot of the values determined for κ_g for specimen KHF-5, plotted as κ_g vs T^2. The line in the figure is a "free-hand" fit, and we may say that $\kappa_g = (3.3 \pm 1) \times 10^{-2} T^2$ is consistent with our Righi-Leduc data. Within the errors, κ_g appears to have the temperature dependence expected for electron-phonon limited phonon conduction, but has a magnitude that is roughly 4 times less than that calculated by Ekin. This is quite reasonable considering the approximations made in that calculation.

There are two additional points to be mentioned:

(a) In using Eq. (5) to determine W_{xx}^e, a problem arises. For any nonzero value of κ_g, if W_{xx}^M has a quadratic field dependence, Eq. (5) yields a nonphysical result for W_{xx}^e at sufficiently high fields. This is a result of the mathematics and is not physical. W_{xx}^M will ultimately saturate at a value of $1/\kappa_g$ no matter how large W_{xx}^e gets.

(b) The free-electron value of W_{xy}^e was used in the denominator of Eq. (6) to determine W_{xy}^M. This leads to a few percent error in the value for κ_g and is unimportant.

We are therefore led to several conclusions:

(i) The lattice thermal conductivity cannot be the sole cause of the quadratic term in the magnetothermal resistivity, but can only account for a small percentage of it;

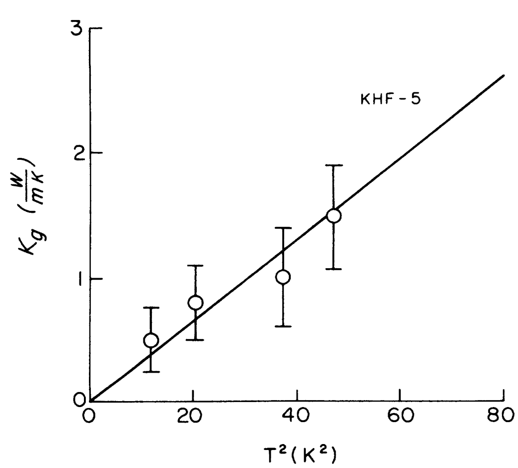

FIG. 5. Lattice thermal conductivity of the potassium as a function of the square of the temperature for specimen KHF-5. The errors are determined by the fit to the Righi-Leduc data.

(ii) The terms $\kappa_g W^e_{xx}$ and $(\kappa_g W^e_{xy})^2$ in Eqs. (5) and (6) are not negligible in potassium;

(iii) The values obtained for κ_g are 3 to 5 times less in magnitude than those calculated by Ekin. This is not unreasonable since Ekin's calculation depends strongly on the pseudopotential and the variational function chosen for the distribution function. Even so, we consider our results to be in reasonable agreement with Ekin's; and the results for κ_g appear to have the predicted temperature dependence—at least over the narrow temperature range we observed.

C. High-field Righi-Leduc coefficient

We have determined the high-field limiting value of TR^e_{RL} for each of our specimens. This was accomplished at each temperature by (a) determining from Fig. 4, the appropriate field for which $\omega_c\tau_{\mathrm{th}} \gg 1$, (b) roughly determining the field at which κ_g begins to have an effect, (c) fitting a straight line to the data between these limits, and (d) averaging all the data for each specimen. The results are listed in Table II. We note that the spread in the values is about 7%, which is approximately the error expected due to geometry errors. The average value of TR^e_{RL} is 1.76×10^{-2} $\mathrm{m^3\,K^2/C\,W\,\Omega}$; this may be compared to Fletcher and Friedman's[27] value, $TR^e_{\mathrm{RL}} = 1.79\times10^{-2}$ $\mathrm{m^3 K^2/C\,W\Omega}$. (This was obtained from their Fig. 2, at a temperature at which their specimens should be in the high-field limit; note that their geometrical errors are the same as ours.) These results are to be compared with the predictions of the semiclassical theory.

The LAK theory[1,2] predicts that, regardless of the number of type of scattering mechanisms present in a metal such as potassium, the off-diagonal components of the electrical and thermal resistivity tensor are given by

$$W^e_{xy} = HR_{\mathrm{RL}} = -H/L_0Tne, \quad \text{and } \rho_{xy} = HR_H = -H/ne ,$$

where n is the electron density. Thus, according to this theory, in the limit $\omega_c\tau \gg 1$, only the static parameters of the metal and *not* the number or type of scattering mechanisms determine the Hall and Righi-Leduc coefficients. Babiskin and Siebenman[28] and Chimenti and Maxfield[10] have shown that the Hall coefficient of potassium is independent of field and nearly equal to the predicted free-electron value.

We observe that our average value of $TR^e_{\mathrm{RL}} = 1.76 \times 10^{-2}\,\mathrm{m^3\,K^2/C\,W\,\Omega}$ is within 3% of the free-electron value $TR_{\mathrm{RL,\,fe}} = 1.82\times10^{-2}$ $\mathrm{m^3\,K^2/C\,W\,\Omega}$.

Thus, in the high-field limit, the off-diagonal terms in the two resistivity tensors have magnitudes within a few percent of the theoretical predictions and are field and temperature independent. Since, theoretically, these off-diagonal terms are independent of the number or nature of scattering mechanisms, and are determined solely by the electronic structure, whereas the diagonal tensor elements vanish in the absence of scattering processes, it may be concluded that an understanding of the magnetoresistance is to be found either in the scattering processes or in the "curvature" of the orbits.

Our results, together with the Hall coefficient data appear to indicate that there are no gross effects due to band structure or the Fermi surface in the field range from 0.1 to 10 T, or $1 \leq \omega_c\tau_{\mathrm{th}} \leq 290$.

There are however, two weak links in this argument. In a study of the Hall coefficient, Chimenti and Maxfield[10] deduced R_H from high-frequency helicon wave transmission measurements on single and polycrystalline specimens of potassium. They observe a small (~4%) directional anisotropy in R_H which is well outside their experimental error. Such anisotropy is definitely not expected. Secondly, three very different measurements of R_H, each having an overall accuracy of better than 3%, all show R_H to be (4–6)% larger than $R_{H,\,\mathrm{fe}}$. (These are the high-frequency helicon transmission measurements[10] just mentioned, helicon absorption edge measurements,[29] and low-frequency helicon-like electromagnetic resonances in spheres.[30]) (Only one measurement[28] finds 1% agreement with $R_{H,\,\mathrm{fe}}$ and they do not adequately discuss their errors.) This is potentially a very important deviation as the semiclassical theory predicts $R_H \equiv 1/ne$.

It is also possible that TR^e_{RL} may differ from $TR_{\mathrm{RL,\,fe}}$ by (3–4)%. Additionally, we note $R_H > R_{H,\,\mathrm{fe}}$ whereas R^e_{RL} may be less than $R_{\mathrm{RL,\,fe}}$; thus, there could be, depending on the crystal orientation, as much as an 11% increase in the Lorenz ratio. A deviation of this magnitude is very important; LAK unambiguously predicts $R_H/TR^e_{\mathrm{RL}} \equiv L_0$. One possibility is that a small group of carriers is not in the high-field limit and thus R_H and R_{RL} have not yet reached their true asymptotic values. This is an unlikely explanation; Chimenti and Maxfield's measurements show no significant variation in the magnitude or field dependence of R_H between 2.2 K and 4.2 K and our measurements show no real changes in TR_{RL} with temperature and field (disregarding the effects of the lattice conduction). To verify this simultaneous measurements of the Hall and Righi-Leduc coefficients must be made.

We may conclude this discussion by summarizing the several major points we feel are evident in our data.

(i) In the high-field limit the Righi-Leduc coefficient is nearly equal to the prediction of the semiclassical theory.

(ii) It is possible that the high-field Righi-Leduc coefficient is (3–4)% less than the semiclassical prediction. Coupled with the results for the Hall coefficient, we note that the Lorenz ratio is too large. Since the theory unambiguously predicts $R_H/TR^e_{\rm RL} \equiv L_0$, these small deviations may be of considerable importance.

(iii) Because the Righi-Leduc coefficient of potassium is relatively large and because the change in electronic thermal resistivity is very large, great care must be exercised in determining the effects of the lattice thermal conductivity on the magnetothermal transport coefficients.

(iv) A large anomalous lattice conductivity is not consistent with our data.

(v) A lattice conductivity consistent with our data can be extracted and has a magnitude that is in fair agreement with a simple theoretical calculation and has the correct temperature dependence.

(vi) By using the ratio of the transverse to be longitudinal temperature gradient we are able to determine when $\omega_c\tau_{\rm th} \geqslant 1$. The value of $\tau_{\rm th}$ thus measured agrees well with calculated values.

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