Self-consistent treatment of the wake potential for fast ion pairs in metals

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The wake potential due to the exchange of virtual plasmons between two fast ions is calculated in a self-consistent manner by carrying out two canonical transformations. We show that the wake potential between two ions depends on each velocity in special way, and a deviation from the sine-type standing wave appears. It is also shown that the potential has a finite value along the ion path.

I. INTRODUCTION

Recently, the energy and angular distributions of ions transmitted through thin solids by the bombardment with molecular-ion beams have been measured[1] and interpreted in terms of the wake potential. The theoretical calculations of the wake potential have been performed according to a dielectric-response-function method by many authors,[2-5] in which only the wake field created by a leading ion is taken into account.

However, it is evident that a self-consistent treatment has to be developed for the calculation of the wake problem between two ions which have different masses and charges, and also different velocities (different speeds and directions). In this paper, we develop a theory of the wake potential in some systematic fashion (or self-consistent manner) between two ions, by carrying out the first and second canonical transformations.

In Sec. II, we develop the canonical calculations to get the wake potential due to an exchange of virtual plasmons between two fast ions. The wake potential is calculated in detail for different points and different velocities in Sec. III.

II. EFFECTIVE INTERACTION DUE TO AN EXCHANGE OF VIRTUAL PLASMONS BETWEEN TWO FAST IONS

We consider an effective interaction due to the exchange of virtual plasmons between two fast ions of charges Z_1e, Z_2e and momentum $\vec{p}_1, \vec{p}_2$, which are passing through a many-electron system. We regard the two ions as external charges, and define the charge densities of the ions and their Fourier components

$$\rho(\vec{r}) = \sum_{j=1}^{2} Z_j e \delta(\vec{r} - \vec{r}_j),$$
$$\rho(\vec{k}) = \frac{1}{V}\int \rho(\vec{r}) e^{-i\vec{k}\cdot\vec{r}}\, d\vec{r} = \frac{1}{V}\sum_{j=1}^{2} Z_j e\, e^{-i\vec{k}\cdot\vec{r}_j}, \tag{1}$$

where $\vec{r}_j$ denotes the coordinate of the jth ion and V the volume of our macrobox.

According to the Bohm-Pines method,[6] we write a Hamiltonian of our system,

$$H_T = H_0 + H_{pl} + H', \tag{2a}$$
$$H_0 = \sum_{j=1}^{2} \frac{\vec{p}_j^{\,2}}{2M_j} + \frac{Z_1 Z_2 e^2}{|\vec{r}_1 - \vec{r}_2|}, \tag{2b}$$
$$H_{pl} = \sum_{k<k_c} \left(\frac{\pi_k^\dagger \pi_k}{2} + \omega_k^2 \frac{q_k^\dagger q_k}{2} \right), \tag{2c}$$
$$H' = -\sum_{k<k_c} \left(\frac{4\pi V}{k^2} \right)^{1/2} \pi_k^\dagger \rho(\vec{k}), \tag{2d}$$

where H_0 denotes the kinetic energies of ions and the Coulomb interaction between them. In Eq. (2b), M_j is the mass of the jth ion. The Hamiltonian H_{pl} describes the energy of the plasmon field with a frequency ω_k, in which the dispersion relation is taken into account. The field operators π_k and q_k are the momentum variable and the coordinate variable which are conjugate to each other. In Eq. (2c), the term which corresponds to the Coulomb interaction of an electron with itself, $2\pi ne^2/k^2$, is omitted, where n is the electron density. H' is the characteristic term in such a Hamiltonian, which represents the coupling of the plasmon field to the density fluctuation of the ion. The value k_c is the cut-off wave number of the plasmon.

Now, by carrying out a canonical transformation, we rewrite our Hamiltonian in the form that an ion-plasmon interaction appears explicitly. Such a canonical transformation, $e^{-iS/\hbar} H_T e^{iS/\hbar}$, is worked by the following Hermitian operator S, which is defined as

$$S = -\sum_{k<k_c} \left(\frac{4\pi V}{k^2} \right)^{1/2} q_k \rho(\vec{k}). \tag{3}$$

Using the commutation relations, and performing the above canonical transformation, we obtain a new Hamiltonian (we drop the subscripts "new")

$$H_T = H_0 + H_{pl} + H_{int} + H_{pp}, \tag{4a}$$

$$H_0=\sum_{j=1}^{2}\frac{\vec{p}_j^{\,2}}{2M_j}+\frac{1}{V}\sum_{k>k_c}\frac{4\pi Z_1 Z_2 e^2}{k^2}e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}, \tag{4b}$$

$$H_{pl}=\sum_{k<k_c}\left[\frac{\pi_k^\dagger\pi_k}{2}+\left(\omega_k^2+\frac{1}{V}\sum_{j=1}^{2}\frac{4\pi Z_j^2e^2}{M_j}\right)\frac{q_k^\dagger q_k}{2}\right], \tag{4c}$$

$$H_{int}=-\frac{i}{V^{1/2}}\Big[\sum_{k<k_{c,j}}\frac{Z_j e}{k}\left(\frac{2\pi\hbar}{\omega_k}\right)^{1/2}(t_j^+ a_k e^{-i\vec{k}\cdot\vec{r}_j}-e^{i\vec{k}\cdot\vec{r}_j}a_k^\dagger t_j^+)\Big], \tag{4d}$$

$$H_{pp}=\sum_{k\neq k'<k_{c,j}}\frac{2\pi Z_j^2e^2}{VM_j}\frac{\vec{k}\cdot\vec{k}'}{kk'}q_k^\dagger q_{k'}e^{i(\vec{k}-\vec{k}')\cdot\vec{r}_j}. \tag{4e}$$

In Eq. (4d), we expressed q_k in terms of the plasmon creation and annihilation operators, $a_k^\dagger$ and a_k, by use of the relation

$$q_k=(\hbar/2\omega_k)^{1/2}(a_k+a_{-k}^\dagger). \tag{5}$$

for preparing the second canonical transformation, and also defined

$$t_j^\pm=(\vec{p}_j\cdot\vec{k})/M_j\pm\hbar k^2/2M_j. \tag{6}$$

In the above equations, the second term in H_0 denotes the screened interaction between the ions. In Eq. (4c), the frequency of the plasmon field is slightly corrected due to the ions, but is almost equal to ω_k, because the number of ions (external charges) is negligible in comparison with the number of electrons in our system, although, in fact, if we consider a many-ion system (ion gas), the term, $\sum_j 4\pi Z_j^2e^2/VM_j$, gives an ion-plasma frequency. H_{int} represents the ion-plasmon interaction in which the ion is scattered from a state $\vec{p}$ to $\vec{p}-\hbar\vec{k}$ ($\vec{p}+\hbar\vec{k}$) through the emission (absorption) of a plasmon of the momentum $\hbar\vec{k}$. H_{pp} represents the two-plasmon process. In the following, we omit H_{pp} because we do not discuss the two-plasmon process. In Eq. (4c), the term which corresponds to the Coulomb interactions of ions with themselves, $\sum_j 2\pi Z_j^2e^2/Vk^2$, is also omitted.

Next we consider the effective interaction due to the exchange of virtual plasmons between ions introduced by H_{int}. We can obtain such a type of the effective interaction by carrying out a second canonical transformation which eliminates the ion-plasmon coupling term in the transformed Hamiltonian. In carrying out the second canonical transformation, we get a new set of operators, $\vec{R}_j$, $\vec{P}_j$, $T_j^\pm$ and A_k, $A_k^\dagger$ (or Π_k, Q_k), which correspond to $\vec{r}_j$, $\vec{p}_j$, $t_j^\pm$ and a_k, $a_k^\dagger$ (or π_k, q_k), respectively. The canonical transformation discussed here is defined in the following form for $\vec{r}_j$, for example, under the representation of the new set of operators,

$$\vec{r}_j=e^{-iS/\hbar}\vec{R}_j e^{iS/\hbar}, \tag{7}$$

where

$$S=-\frac{1}{V^{1/2}}\sum_{k<k_{c,j}}\frac{Z_j e}{k}\left(\frac{2\pi\hbar}{\omega_k}\right)^{1/2}(U_j^+A_k e^{-i\vec{k}\cdot\vec{R}_j}+e^{i\vec{k}\cdot\vec{R}_j}A_k^\dagger U_j^+). \tag{8}$$

In the above, we defined

$$U_j^\pm=T_j^\pm/(\omega_k+T_j^\pm). \tag{9}$$

This transformation corresponds to one of the types of the intermediate coupling method which was first developed in the meson theory.[7] The result within the lowest order of the second canonical transformation on the Hamiltonian is expressed as follows (the method of the detailed calculations is cited in the work by Bohm and Pines[6]):

$$H_T^{(0)}=H_0+H_{pl}+H_1+H_{int}, \tag{10a}$$

$$H_0=\sum_{j=1}^{2}\frac{\vec{P}_j^{\,2}}{2M_j}+\frac{1}{V}\sum_{k>k_c}\frac{4\pi Z_1Z_2e^2}{k^2}e^{i\vec{k}\cdot(\vec{R}_1-\vec{R}_2)}, \tag{10b}$$

$$H_{pl}=\sum_{k<k_c}\left(\frac{\Pi_k^\dagger\Pi_k}{2}+\omega_k^2\frac{Q_k^\dagger Q_k}{2}\right), \tag{10c}$$

$$H_1=\frac{1}{V}\sum_{k<k_{c,j}}\frac{2\pi Z_j^2e^2}{k^2}\frac{(\vec{P}_j\cdot\vec{k}/M_j)^2}{(\vec{P}_j\cdot\vec{k}/M_j)^2-(\omega_k-\hbar k^2/2M_j)^2}, \tag{10d}$$

$$H_{int}=-\frac{1}{V}\Big(\sum_{k<k_{c,j\neq j'}}\frac{\pi Z_jZ_{j'}e^2}{k^2\omega_k}(e^{i\vec{k}\cdot(\vec{R}_{j'}-\vec{R}_j)}T_j^-U_{j'}^+ + U_{j'}^+T_j^-e^{-i\vec{k}\cdot(\vec{R}_{j'}-\vec{R}_j)})\Big), \tag{10e}$$

where H_{pl} denotes the independent plasmon field in the form of the new operator representation, in which we neglected the contribution from ions to the dispersion relation of plasmon frequency for the same reason mentioned before. H_1 represents the term which gives a correction of the ion mass (effective mass), resulting from the fact that the ions carry a plasmon cloud along with them. But, in our system, the contribution from H_1 is negligibly small. The Hamiltonian H_{int} denotes the effective interaction due to the exchange of virtual plasmons between ions, in which the dynamical effect between ions is included. The invariance of H_{int} for the space reflection, $\vec{R}_j\rightleftarrows\vec{R}_{j'}$, is easily confirmed in Eq. (10e). Here we can express our Hamiltonian in the form in which the ion-plasmon coupling term is eliminated and the effective interaction through virtual plasmons appears explicitly within the lowest-order approximation. This is our purpose in this chapter.

III. WAKE EFFECT

In this chapter, using the Hamiltonian given by Eq. (10a), we write down the Schrödinger equation,

and investigate the correspondence to the usual dielectric function method for the wake effect. Using the relation

$$U_{j'}^{+}T_{j}^{-}e^{-i\vec{k}\cdot(\vec{R}_{j'}-\vec{R}_{j})}=e^{-i\vec{k}\cdot(\vec{R}_{j'}-\vec{R}_{j})}T_{j}^{+}U_{j'}^{-}\quad(j\neq j') \tag{11}$$

and replacing $\vec{k}\to-\vec{k}$ at symmetrical terms for the summation $\vec{k}$, we rewrite Eq. (10e) in the following form:

$$H_{\rm int}=-\frac{1}{V}\sum_{k<k_c}\frac{2\pi Z_1 Z_2 e^2}{k^2}\times\left(e^{i\vec{k}\cdot(\vec{R}_1-\vec{R}_2)}T_2^{-}T_1^{+}\frac{1}{\omega_k^2-(T_1^{+})^2}+e^{i\vec{k}\cdot(\vec{R}_2-\vec{R}_1)}T_1^{-}T_2^{+}\frac{1}{\omega_k^2-(T_2^{+})^2}\right). \tag{12}$$

As is seen in Eq. (12), the dynamical effect in the effective interaction is introduced through momentum operators of ions in $T_j^{\pm}$, if we adopt the R representation on our Hamiltonian.

In considering the problem of the wake bound state, we introduce the wave function of two ions in the following form:

$$(\hat{H}_k+H_{\rm sc}+\hat{H}_{\rm int})\phi=E\phi\,, \tag{13a}$$

$$\phi=\phi_0(\vec{R}_1,\vec{R}_2)u_0(\vec{R}_1,\vec{R}_2)\,,$$

$$\phi_0(\vec{R}_1,\vec{R}_2)=\frac{1}{V}e^{(i/\hbar)(\vec{P}_1\cdot\vec{R}_1+\vec{P}_2\cdot\vec{R}_2)}\,, \tag{13b}$$

where operators are symbolized in terms of the caret, that is, $\hat{\vec{P}}_j=-i\hbar\nabla_{\vec{R}_j}$, $\hat{H}_k=-\sum_{j=1}^{2}(\hbar^2/2M_j)\nabla_{\vec{R}_j}^2$ and $H_{\rm int}$ is given by Eq. (12) for $\hat{\vec{P}}_j$. $H_{\rm sc}$ denotes the screened interaction between ions appearing in Eq. (10b), and we neglect $H_{\rm pl}$ and H_1 in Eq. (10a) because they have no important contributions to our problem.

The term $\hat{H}_{\rm int}(\phi_0u_0)$ is divided into the linear and nonlinear parts for u_0,

$$\hat{H}_{\rm int}(\phi_0u_0)=H_{\rm int}\phi_0u_0+[\hat{H}_{\rm int}(\phi_0u_0)]'\,, \tag{14}$$

where the Hamiltonian $H_{\rm int}$ in the first term is the velocity-dependent interaction derived from the contribution of the eigenstate for $\hat{\vec{P}}_j(\phi_0)$, and the second term is the nonlinear part for u_0.

Here, we note that, in the usual dielectric function method, the momentum of the external charge is treated as a c number through a transition frequency. This means that the dynamical effect of the effective interaction is introduced through the contribution from the plane wave only. Therefore, it can be seen that the equation corresponding to the usual one for the wake bound problem[3-5]

$$\left(-\frac{\hbar^2}{2\mu}\nabla_{\vec{R}}^2+V^{\rm eff}(\vec{R})\right)u(\vec{R})=(E-E_0)u(\vec{R})\,, \tag{15}$$

is obtained by using the free-particle approximation for the dynamical effect of the effective interaction. In the above, $V^{\rm eff}(\vec{R})=H_{\rm sc}(\vec{R})+H_{\rm int}(\vec{R})$, $u(\vec{R})=e^{(i/\hbar)\mu(\vec{V}_1-\vec{V}_2)\cdot\vec{R}}u_0$ and u_0 is taken as a function of $\vec{R}=\vec{R}_1-\vec{R}_2$. $\vec{V}_j$ and $\mu=M_1M_2/(M_1+M_2)$ are the velocity of the jth ion and the reduced mass, respectively. $E_0=(\vec{P}_1+\vec{P}_2)^2/2(M_1+M_2)$.

In Eq. (15), it is worth noting that the velocity correlation of two ions is included in $V^{\rm eff}$. In the dielectric-function method, the velocity correlation was not taken into account. Only the velocity dependence of the field created by the leading ion was taken into account.

Next, we calculate $H_{\rm int}$ in detail. In calculating it, we take into account that the recoil term $\hbar k^2/2M_j$ in $T_j^{\pm}$ can be neglected when $\vec{P}_j$ is large compared to $\frac{1}{2}\hbar\vec{k}$, and the contribution of the dispersion relation in the plasmon frequency is small.[5] Therefore, we may approximate

$$T_j^{\pm}=(\vec{k}/M_j)\cdot(\vec{P}_j\pm\tfrac{1}{2}\hbar\vec{k})\approx\vec{V}_j\cdot\vec{k}\,,\qquad \omega_k\approx\omega_p\,, \tag{16}$$

where ω_p is the usual plasmon frequency.

Furthermore, according to the experiments for the molecular-ion beams by Gemmell *et al.*,[1] we consider the case that the velocity directions of two ions are parallel in the direction Z;

$$\vec{V}_1=(0,0,V_1),\quad \vec{V}_2=(0,0,V_2). \tag{17}$$

Replacing $(1/V)\sum_{k<k_c}$ by $[1/(2\pi)^3]\int_{k<k_c}d\vec{k}$, and performing the rearrangement of $k_z\to-k_z$ at symmetrical terms in the integration of k_z, we obtain $H_{\rm int}(\vec{R})$:

$$H_{\rm int}(\vec{R})=\frac{Z_1Z_2e^2k_c}{\pi}\left\{\frac{2\,{\rm Si}(k_cR)}{k_cR}+\frac{1}{k_c}\int_{-k_c}^{k_c}dk_z\int_0^{(k_c^2-k_z^2)^{1/2}}dk_\perp\frac{k_\perp}{k_\perp^2+k_z^2}J_0(k_\perp R_\perp)\times\cos(k_zZ)\left[\frac{\omega_p^2}{V_1V_2}+k_z\omega_p(V_1-V_2)\left(\frac{1}{V_1V_2}+\frac{k_z^2}{2\omega_p^2}\right)\right]\Big/\left(k_z^2-k_z\omega_p\frac{V_1-V_2}{V_1V_2}-\frac{\omega_p^2}{V_1V_2}\right)\right\}, \tag{18}$$

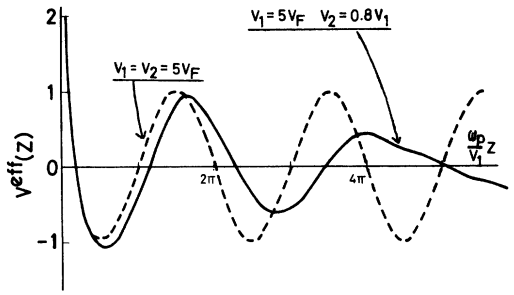

FIG. 1. Wake potential V^{eff} along the ion path ($R_\perp = 0$) in units of $Z_1Z_2e^2k_c/\pi$. The solid curve gives the results for the case that $V_1 = 5V_F$ and $V_2 = 0.8V_1$. The dashed curve denotes the results for the case of $V_1 = V_2 = 5V_F$.

where $J_0(t)$ is the Bessel function of the zeroth order, and $\mathrm{Si}(t)$ is the sine-integral function which is defined as

$$\mathrm{Si}(t) = \int_0^t \frac{\sin z}{z}\, dz. \tag{18'}$$

The first term in Eq. (18) denotes the long-range part of the Coulomb interaction between ions. The second term represents the oscillatory part of the effective interaction, which depends on $V_1 - V_2$ and V_1V_2. The self-consistent contribution from the partner ion is included in this term. Through such a velocity-correlation of ions, the deviation from the sine-type standing wave appears (see Fig. 1).

For the case that one ion moves with the velocity V_1 and the other is almost stationary in comparison with V_1, the integral part reduces to $-2\mathrm{Si}(k_cR)/k_cR$, and H_{int} becomes zero. In this case, there appears no wake field between ions. This is one of the differences between the self-consistent and non-self-consistent (the dielectric function method) treatments.

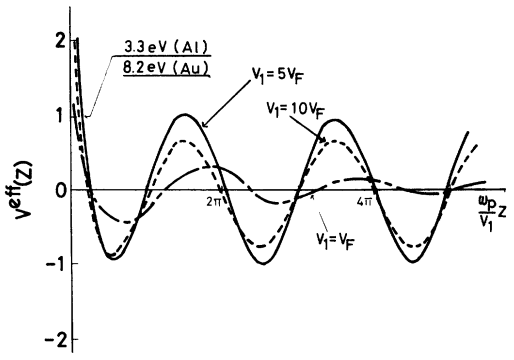

FIG. 2. Wake potential V^{eff} along the ion path ($R_\perp = 0$) in units of $Z_1Z_2e^2k_c/\pi$ for the case of $V_1 = V_2$. The values for Al and Au correspond to the case that two ions are proton.

For the case of $V_1 = V_2$, Eq. (18) reduces to

$$H_{\text{int}}(\vec{R}) = \frac{Z_1Z_2e^2k_c}{\pi}\left[\frac{2\,\mathrm{Si}(k_cR)}{k_cR} - \frac{\omega_p}{V_1k_c}(A^2+B^2)^{1/2} \times \sin\left(\frac{\omega_p}{V_1}Z + \alpha\right)\right], \tag{19}$$

where

$$\begin{aligned} A(R_\perp, Z) &= -\int_0^1 dt\, F_1(t, R_\perp)[\mathrm{Si}((k_ct + \omega_p/V_1)Z) \\ &\qquad + \mathrm{Si}((k_ct - \omega_p/V_1)Z)], \\ B(R_\perp, Z) &= -\int_0^1 dt\, F_1(t, R_\perp)[\mathrm{Ci}((k_ct + \omega_p/V_1)Z) \\ &\qquad - \mathrm{Ci}((k_ct - \omega_p/V_1)Z)], \\ \alpha(R_\perp, Z) &= \arctan(B/A), \\ F_1(t, R_\perp) &= \frac{d}{dt}\left(\int_0^{(1-t^2)^{1/2}} dt' \frac{t'}{t'^2+t^2} J_0(k_cR_\perp t')\right). \end{aligned} \tag{20}$$

In the above, $\mathrm{Ci}(t)$ is the cosine-integral function which is defined as

$$\mathrm{Ci}(t) = -\int_t^\infty \frac{\cos z}{z}\, dz. \tag{20'}$$

In Eq. (19), there appear the amplitude and the additional phase factor in the oscillatory part, which depend on the internuclear distance between ions ($R_\perp$ and Z). The deviation from the standing wave due to this dependence appears when the velocity of ion (V_1) is near the Fermi velocity, $V_F \approx \omega_p/k_c$, (see Fig. 2).

When $R_\perp = 0$, A and B reduce to

$$\begin{aligned} A(0, Z) &= \int_0^1 \frac{dt}{t}[\mathrm{Si}((k_ct + \omega_p/V_1)Z) \\ &\qquad + \mathrm{Si}((k_ct - \omega_p/V_1)Z)], \\ B(0, Z) &= \int_0^1 \frac{dt}{t}[\mathrm{Ci}((k_ct + \omega_p/V_1)Z) \\ &\qquad - \mathrm{Ci}((k_ct - \omega_p/V_1)Z)]. \end{aligned} \tag{21}$$

It is easily seen that the values of A and B become finite, although there is the singularity at $t = \omega_p/V_1k_c$ in $\mathrm{Ci}((k_ct - \omega_p/V_1)Z)$. Therefore, the amplitude of the oscillatory part in Eq. (19) has a finite value. If k_c becomes infinite, A becomes infinite and B becomes zero. Then, in this case, the amplitude and the additional phase factor of the oscillatory part in Eq. (19) become infinite and zero, respectively, and the wake potential along the ion path ($R_\perp = 0$) becomes infinite. This corresponds to the result obtained by Neelavathi *et al.*[3] qualitatively. The peak heights of the oscillatory part along the ion path become several eV for Au and Al in the case that the projectiles are proton (see Fig. 2). It is considered that this in-

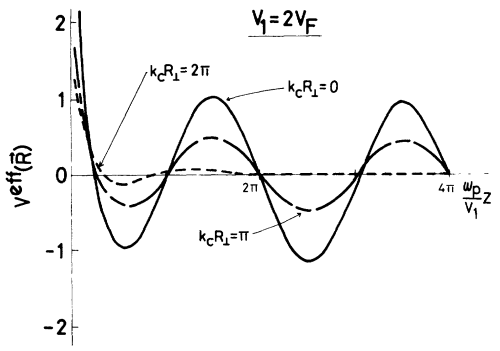

FIG. 3. Wake potential V^{eff} as a function of $R_\perp$ and Z for the case of $V_1 = V_2 = 2V_F$. The unit of the vertical axis is taken as $Z_1 Z_2 e^2 k_c/\pi$.

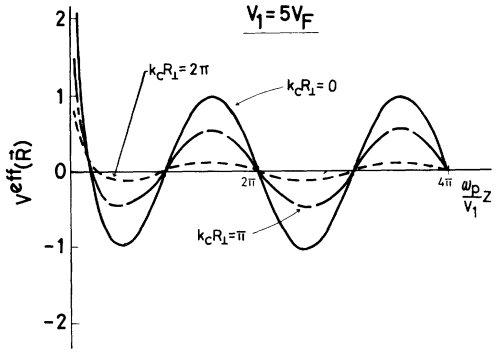

FIG. 4. Wake potential V^{eff} as a function of $R_\perp$ and Z for the case of $V_1 = V_2 = 5V_F$. The unit of the vertical axis is taken as $Z_1 Z_2 e^2 k_c/\pi$.

fluences the values of the binding energies of the trapped ion (or electron) in the wake bound state.

Figures 3 and 4 show the numerical results of $V^{\text{eff}}(\vec{R})$ for the cases of $V_1 (= V_2) = 2V_F$ and $5V_F$, respectively. The amplitude of the oscillatory part decreases with increasing $R_\perp$ of the order of π/k_c. For the case of $V_1 = 2V_F$, the phase shift appears at the small amplitude of the oscillatory part, although the phase shift is negligible for the case of $V_1 = 5V_F$.

IV. CONCLUDING REMARKS

We have calculated the wake potential due to the exchange of virtual plasmons between two fast ions in a self-consistent manner by carrying out two canonical transformations. It is worthy of note that, within the free-particle approximation about the dynamical effect of the effective interaction, the wake potential depends on the velocities of the two ions in a special way, and a deviation from the sine-type standing wave derived by the non-self-consistent methods (the dielectric-function methods) appears through the velocity correlation of two ions. For the case that the velocities of two ions are same, the deviation from the sine-type standing wave also appears when the velocity of the ions is near the Fermi velocity. Such a deviation was represented in the form of the amplitude function and the additional phase function. We could also show that the wake potential is not infinite along the line of the ion path.

From the above results for the wake potential, we may calculate in detail the explosion motion of the molecular beams with random orientations and vibrations. We can also calculate the binding energy in our method, in which, strictly speaking, the higher-order contribution from $\hat{H}_{\text{int}}$ is included in the nonlinear part for u_0. But such a calculation will be the same in principle as the Green's-function method recently discussed by Ritchie *et al.*[5]

[1]D. S. Gemmell, J. Remillieux, J.-C. Poizat, M. J. Gaillard, R. E. Holland, and Z. Vager, Phys. Rev. Lett. 34, 1420 (1975); see also Nucl. Instrum. Methods 132, 61 (1976).

[2]J. Neufeld and R. H. Ritchie, Phys. Rev. 98, 1632 (1955).

[3]V. N. Neelavathi, R. H. Ritchie, and W. Brandt, Phys. Rev. Lett. 33, 302 (1974).

[4]M. H. Day, Phys. Rev. B 12, 514 (1975).

[5]R. H. Ritchie, W. Brandt, and P. M. Echenique, Phys. Rev. B 14, 4808 (1976).

[6]D. Bohm and D. Pines, Phys. Rev. 92, 609 (1953); see also D. Pines, *Elementary Excitations in Solids* (Benjamin, New York, 1964).

[7]Z. Maki, M. Sato, and S. Tomonaga, Prog. Theor. Phys. 9, 607 (1953).