Magnetodynamics of incommensurate charge-density waves*

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We present a microscopic derivation of the equations of motion in a magnetic field for the drift velocity of a charge-density wave (CDW) and the electron drift velocity for a three-dimensional system containing an unpinned, incommensurate CDW. We derive an expression for the effective mass of the CDW and show that the effective mass for acceleration by a magnetic field is the same as that for acceleration by an electric field. We examine the effects of the drifting CDW on the magnetotransport properties in the case of a simply-connected Fermi surface. We also discuss the theory of the induced-torque for a spherical sample containing a single CDW which is free to drift.

I. INTRODUCTION

In a previous paper,[1] hereafter referred to as I, we presented a set of equations describing electric transport in a three-dimensional system in the presence of an incommensurate unpinned charge-density wave (CDW). We discussed the dynamic effects of an applied electric field in accelerating the CDW and the electrons and derived an expression for the effective mass of the CDW. Our purpose in this paper is to continue the microscopic derivation of the equations presented in Paper I with a discussion of the dynamic effects of an applied magnetic field.

One of the primary goals of this work has been to discover the effect of a drifting CDW on electrical transport in a magnetic field. There are two types of effects which a CDW can have on magnetotransport: those associated with the drift of the CDW and those associated with the topology of the Fermi surface. This work is limited in that we choose not to include the possible topological effects (e.g., open orbits) in the discussion. Thus, in order to discover those effects associated with the drift of the CDW, we choose a model system in which the Fermi surface is simply-connected. We also choose to ignore any periodicities which would be associated with a real crystal lattice and will discuss the magnetotransport of a CDW in three-dimensional jellium. In particular, we discuss the effect of a drifting CDW on the magnetoresistance and Hall coefficient. The theory of the induced-torque experiment is discussed for a spherical sample of jellium containing a single CDW throughout.

II. EQUATIONS OF MOTION

In Paper I, we presented a set of equations describing the drift velocity of the CDW and the electron drift velocity for a three-dimensional jellium model containing a CDW. In this paper we will discuss the effect of an applied uniform magnetic field $\vec{H}$. In order that the goal of our arguments can be kept in mind throughout, we state the resulting equation of motion for the CDW velocity $\vec{D}$ in the absence of scattering for the case where the CDW wave vector $\vec{Q}$ is in the $\hat{x}$ direction and $\vec{H}$ is in the $\hat{z}$ direction

$$\frac{dD}{dt} = -\left(\frac{eH}{m^*c}\right)K_y , \tag{1}$$

where $\vec{D}$ is along $\vec{Q}$. The electronic charge is $-e$. m^* is the effective mass associated with the acceleration of the CDW. $\vec{K}$ is a quasivelocity of the electrons and is related to the average wave vector $\langle\vec{k}\rangle_{av}$ of the electron distribution by

$$\vec{K} \equiv \hbar\langle\vec{k}\rangle_{av}/m , \tag{2}$$

where m is the electron mass.

The equations of motion for the components of the electron quasivelocity parallel (K_x) and perpendicular (K_y, K_z) to $\vec{Q}$ are

$$\frac{dK_x}{dt} = -\left(\frac{eH}{mc}\right)K_y ,$$
$$\frac{dK_y}{dt} = \left(\frac{eH}{mc}\right)V_x , \quad \frac{dK_z}{dt} = 0 . \tag{3}$$

$\vec{V}$ is the average group velocity for the electron distribution, i.e.,

$$\vec{V} \equiv \langle\vec{v}_g\rangle_{av} = \langle\vec{p}/m\rangle_{av} , \tag{4}$$

and is related to $\vec{K}$ and $\vec{D}$ by

$$V_x = (1-\gamma)K_x + \gamma D, \quad V_y = K_y, \quad V_z = K_z . \tag{5}$$

The total electronic current is $\vec{J} = -ne\vec{V}$ where n is the number of electrons per unit volume. γ is a constant discussed in Paper I.

Equation (5) was discussed in Paper I. In this paper we will discuss Eqs. (1) and (3). It may seem surprising that a CDW could be accelerated by a magnetic field. Not only does this result

from a microscopic derivation, but if Eq. (1) did not have exactly this form, it would be easy to imagine an experiment (as in the Appendix) which violates the second law of thermodynamics.

Since jellium is isotropic, it may seem arbitrary that we have chosen to fix the direction of $\vec{Q}$ and to ignore any torques which might act to change the direction of $\vec{Q}$ in the presence of applied fields and currents. However, in a real metal the direction of $\vec{Q}$ would be some specific crystal direction and thus could not rotate. We therefore impose this property (artificially) while using the jellium model.

III. EFFECTS OF AN APPLIED MAGNETIC FIELD

In the presence of a CDW, the total self-consistent potential in the one-electron Hamiltonian for our jellium model is

$$V(\vec{r}) = G\cos\vec{Q}\cdot\vec{r}. \tag{6}$$

As in Paper I, we choose to ignore the velocity dependence of G for the sake of simplifying the discussion. This periodic potential leads to a periodic variation in electron density which also has a $\cos\vec{Q}\cdot\vec{r}$ dependence. There is an accompanying modulation of the positive ion background to preserve charge neutrality. In Paper I, it was demonstrated that an applied electric field has two effects on electrons in a periodic potential. First, it causes electrons to move in $\vec{k}$-space according to the equation of motion $d\vec{k}/dt = -e\vec{\mathcal{E}}/\hbar$. Second, the electric field polarizes the electron density in a manner analogous to the polarization of atomic wave functions by an electric field. This polarization of the electrons results in an electron density which is out of phase with that of the positive ion background. Since in jellium the ions are free to move under the nonzero net forces, the CDW will accelerate in the presence of an applied electric field.

The effects of an applied magnetic field are very similar. The magnetic field causes electrons to move in $\vec{k}$ space according to the Jones and Zener equation of motion[2]

$$\frac{d\vec{k}}{dt} = -\left(\frac{e}{\hbar c}\right)\vec{v}_g\times\vec{H}, \tag{7}$$

where $\vec{v}_g$ is the group velocity of the electron with wave vector $\vec{k}$. In addition, the magnetic field will, in general, polarize the electron density. The ions will move in response to the nonzero forces acting on them due to the polarized electron density, and the CDW will accelerate.

The one-electron Hamiltonian in the presence of an applied magnetic field and a periodic potential $V(\vec{r})$ is

$$\mathcal{H} = (\vec{p} + e\vec{A}/c)^2/2m + V(\vec{r}) - e\varphi, \tag{8}$$

where $\vec{A}$ is the vector potential and φ is the scalar potential. The complication which always arises in problems dealing with a magnetic field is in choosing a gauge to describe the magnetic field $\vec{H}$. Extreme care must always be exercised in the choice of a gauge because the physical significance of a wave function depends on the particular vector potential $\vec{A}(\vec{r})$ that is used.[3] Consider the expression for the quantum mechanical probability current $\vec{j}_p$. In the presence of a nonzero vector potential, $\vec{j}_p$ is given by

$$\vec{j}_p = \frac{\hbar}{2mi}\int d^3r[\Psi^*\vec{\nabla}\Psi - (\vec{\nabla}\Psi^*)\Psi] - \frac{e}{mc}\int d^3r\,\Psi^*\vec{A}\Psi. \tag{9}$$

Thus, the current associated with an electron in state Ψ is dependent upon the particular choice of vector potential $\vec{A}$.

In the absence of a magnetic field the group velocity, $\vec{v}_g$ associated with an electron with energy $E(\vec{k})$ is

$$\vec{v}_g = \vec{\nabla}_k E(\vec{k})/\hbar. \tag{10}$$

When we turn on a magnetic field, we want our (approximate) wave function to retain this physical property. Otherwise the question we hope to answer by perturbation theory would not be relevant to the physically characterized electron we had in mind.[3] In order that Eq. (10) be maintained, it is necessary to pick $\vec{A}$ such that the second term in Eq. (9) is identically equal to zero at all times. This means that we must pick a gauge such that the null point of $\vec{A}(\vec{r})$ travels with the center of the wave packet describing our chosen electron. This is the Jones and Zener gauge[2]

$$\vec{A} = \vec{H}\times\tfrac{1}{2}(\vec{r} - \vec{v}_g t),$$
$$\varphi = (1/2c)(\vec{H}\times\vec{v}_g)\cdot\vec{r}. \tag{11}$$

The scalar potential φ is required so that the electric field $-\vec{\nabla}\varphi - c^{-1}\partial\vec{A}/\partial t$ is zero. We have chosen without loss of generality that $\langle\Psi|\vec{r}|\Psi\rangle = 0$ at $t = 0$. It is only in the Jones and Zener gauge that Eq. (7) is valid for Bloch electrons in a magnetic field.

The Hamiltonian, Eq. (8) with the field on is explicitly time dependent. If we choose $\vec{H}$ in the $\hat{z}$ direction, then for $t = 0$ we have

$$\mathcal{H} = \frac{p^2}{2m} + V(\vec{r}) + \left(\frac{eH}{2mc}\right)(xp_y - yp_x) - e\varphi + \frac{e^2A^2}{2mc^2}. \tag{12}$$

To discuss the effects of the magnetic field terms for electrons in a periodic potential, we shall use the Bloch or crystal momentum representation as

in Paper I. First consider the problem of one electron in a periodic potential $V(\vec{r})$. The periodicity of $V(\vec{r})$ allows the Hamiltonian in zero magnetic field to have eigenfunctions of the Bloch form

$$\Phi_{n\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{n\vec{k}}(\vec{r}), \tag{13}$$

where $u_{n\vec{k}}(\vec{r})$ is periodic in $\vec{r}$ and n is the band index. If a general wave function Ψ can be written

$$\Psi(\vec{r}) \equiv \frac{1}{8\pi^3} \sum_n \int d^3k\, a_n(\vec{k}) e^{i\vec{k}\cdot\vec{r}} u_{n\vec{k}}(\vec{r}) \tag{14}$$

in the coordinate representation, then Ψ is

$$\Psi(\vec{k}) = [a_0(\vec{k})\, a_1(\vec{k})\, a_2(\vec{k}) \cdots] \tag{15}$$

in the Bloch representation. $\Psi(\vec{k})$ is an (infinite) column vector. In this representation the x component of $\vec{r}$ is[4]

$$x = iI \frac{\partial}{\partial k_x} + X_{nn'}(\vec{k}), \tag{16}$$

where I is an infinite unit matrix, and

$$X_{nn'}(\vec{k}) \equiv i \int d^3r\, u^{\dagger}_{n\vec{k}}(\vec{r}) \frac{\partial [u_{n'\vec{k}}(\vec{r})]}{\partial k_x}. \tag{17}$$

Similarly, the operator xp_y in the Bloch representation can be written[4]

$$xp_y = i(P_y)_{nn'} \frac{\partial}{\partial k_x} - \hbar L_{nn'}, \tag{18}$$

where

$$L_{nn'} \equiv \int d^3r\, e^{i\vec{k}\cdot\vec{r}} \frac{\partial \Phi^*_{n\vec{k}}}{\partial y} \frac{\partial u_{n'\vec{k}}}{\partial k_x} \tag{19}$$

and

$$(P_y)_{nn'} \equiv \int d^3r \Phi^*_{n\vec{k}} \left(-i\hbar \frac{\partial \Phi_{n'\vec{k}}}{\partial y}\right). \tag{20}$$

Let us consider an electron described by a wave packet in one band $E_0(\vec{k})$. The wave function describing this electron is

$$\Psi(\vec{k}) = [a_0(\vec{k})\ 0\ 0 \cdots], \tag{21}$$

where we assume $a_0(\vec{k})$ to be nonzero only in a small region of $\vec{k}$ space. We can then find the equation of motion in a magnetic field for the electron described by Eq. (21) by determining $a_0(\vec{k})$ such that $\Psi(\vec{r})$ satisfies the Schrödinger equation of motion for the Hamiltonian with Eq. (11) for the vector and scalar potentials. Using Eqs. (16) and (18) the terms with $\partial/\partial k_x$ lead to an equation of motion for $|a_0|^2$ which is satisfied by a general function $F(\vec{k} + e\vec{v}_g \times \vec{H}t/\hbar c)$, where $\vec{v}_g$ is the group velocity and is related to the energy by Eq. (10). Thus, the wave packet moves in $\vec{k}$ space such that Eq. (7) is satisfied. When this result is averaged over a distribution of electrons, Eq. (3) is obtained.

As in Paper I, the additional terms in Eqs. (16) and (18) introduce off-diagonal terms into the Hamiltonian matrix $\mathcal{H}_{nn'}$. These can be eliminated to first order by using a new set of basis functions given by first-order perturbation theory

$$u_{n\vec{k}}(\vec{r}) = u_{n\vec{k}}(\vec{r}) + \sum_{n'} \frac{u_{n'\vec{k}}}{E_n - E_{n'}} \left[\langle u_{n'\vec{k}} | \frac{-ie}{\hbar c} \vec{H} \times \vec{v}_g \cdot \vec{\nabla}_k | u_{n\vec{k}} \rangle - \left(\frac{eH}{2mc}\right) \int d^3r\, e^{i\vec{k}\cdot\vec{r}} \left(\frac{\partial \Phi^*_{n'\vec{k}}}{\partial y} \frac{\partial u_{n\vec{k}}}{\partial k_x} - \frac{\partial \Phi^*_{n'\vec{k}}}{\partial x} \frac{\partial u_{n\vec{k}}}{\partial k_y} \right) \right]. \tag{22}$$

The second term on the right-hand side in Eq. (22) comes from the scalar potential φ and the third term is due to the $\vec{A}\cdot\vec{p}$ term in the Hamiltonian. The A^2 term does not contribute to the out-of-phase part of the electron density, and we shall not consider it further.

As discussed in Paper I, the one-electron states below and above the energy gaps, respectively, in the presence of the CDW potential, Eq. (6), are

$$\begin{aligned} \Phi_{\vec{k}} &= \cos\xi\, e^{i\vec{k}\cdot\vec{r}} - \sin\xi\, e^{i(\vec{k}-\vec{Q})\cdot\vec{r}} \equiv e^{i\vec{k}\cdot\vec{r}} u_{\vec{k}}, \\ \Psi_{\vec{k}} &= \sin\xi\, e^{i\vec{k}\cdot\vec{r}} + \cos\xi\, e^{i(\vec{k}-\vec{Q})\cdot\vec{r}} \equiv e^{i\vec{k}\cdot\vec{r}} w_{\vec{k}}. \end{aligned} \tag{23}$$

Φ_k and Ψ_k are the states which result from a perturbation treatment of the part of the potential which produces the energy gap at $\vec{k} = \frac{1}{2}\vec{Q}$. The other part of the potential, which produces the gap at $\vec{k} = -\frac{1}{2}\vec{Q}$, can be treated separately as the effects are additive for our purposes as long as G is not too large compared with the Fermi energy. The energies for the states above and below the gap are

$$E_{\pm} = \tfrac{1}{2}(\epsilon_{\vec{k}} + \epsilon_{\vec{k}-\vec{Q}}) \pm \tfrac{1}{2}[(\epsilon_{\vec{k}} - \epsilon_{\vec{k}-\vec{Q}})^2 + G^2]^{1/2}, \tag{24}$$

and the coefficients obey the relation

$$\sin 2\xi = G/(E_+ - E_-) \equiv G/W. \tag{25}$$

For simplicity let us take $\vec{Q}$ in the $\hat{x}$ direction. In terms of the states in Eq. (23), the perturbed wave function $\Phi'_{\vec{k}}$ in a magnetic field is

$$\Phi'_{\vec{k}} = \Phi_{\vec{k}} - \frac{\Psi_{\vec{k}}}{W} \left\{ \langle w_{\vec{k}} | \frac{-ieH}{2mc} \left(\frac{V_x \partial}{\partial k_y} - \frac{V_y \partial}{\partial k_x} \right) | u_{\vec{k}} \rangle - \left(\frac{eH}{2mc}\right) \int d^3r\, e^{i\vec{k}\cdot\vec{r}} \left[\frac{\partial \Psi^*_{\vec{k}}}{\partial y} \frac{\partial u_{\vec{k}}}{\partial k_x} - \left(\frac{\partial \Psi^*_{\vec{k}}}{\partial x}\right)\left(\frac{\partial u_{\vec{k}}}{\partial k_y}\right) \right] \right\}. \tag{26}$$

The change in electron density Δn_k induced by the magnetic field for a state $\Phi_{\vec{k}}$ below the gap is

$$\Delta n_{\vec{k}} = |\Phi'_{\vec{k}}|^2 - |\Phi_{\vec{k}}|^2 = \frac{\hbar^3 eHGQk_y}{m^2 cW^3} \sin\vec{Q}\cdot\vec{r}\,. \quad (27)$$

Both terms in the bracketed expression in Eq. (26) give the same contribution. The corresponding change in electron density for a state above the gap is opposite in sign. Note that the induced electron density is 90° out-of-phase with the $\cos\vec{Q}\cdot\vec{r}$ dependence of the unperturbed electron density. In general, the out-of-phase electron density depends on the product $\vec{H}\cdot\vec{Q}\times\vec{k}$ so that only the components of $\vec{H}$ and $\vec{k}$ perpendicular to $\vec{Q}$ are effective in inducing the out-of-phase electron density.

The total out-of-phase part of the electron density $\Delta N_H \sin\vec{Q}\cdot\vec{r}$ induced by the magnetic field is found by summing up all the contributions such as Eq. (27) weighted by the appropriate Fermi-Dirac distribution function. At zero temperature only states below the gaps are filled (since we have chosen to discuss the case of a simply-connected Fermi surface). From Eq. (27) it is easy to see that Δn_k is an odd function of k_y. Thus, unless the electrons have a net quasivelocity in the $\hat{y}$ direction, the total ΔN_H will be zero. Let us take the electrons to have a net quasivelocity K_y in the $\hat{y}$ direction, where K_y is defined in Eq. (2). The net contribution to ΔN_H comes from a shell of thickness $mK_y k_y/\hbar k_F$ at the Fermi surface. The total out-of-phase electron density is then conveniently written in cylindrical coordinates with k_x the axis of the cylinder,

$$\Delta N_H = \frac{2\pi\hbar^2 eHGQK_y}{mc} \int \frac{dk_x}{4\pi^3} \frac{\pi k_\perp^2}{W^3}\,. \quad (28)$$

$k_\perp$ is the radius of a cross section of the cylindrically symmetric Fermi surface measured from the k_x axis.

Having found ΔN_H, a treatment similar to that in Sec. VI of Paper I leads to the expression for the acceleration dD/dt of the CDW in a magnetic field

$$\frac{dD}{dt} = -\left(\frac{eH}{m_H^* c}\right) K_y\,, \quad (29)$$

where

$$\frac{m_H^*}{m} = 1 + Mp\epsilon_1/8\pi e^2\hbar^2 G \int \frac{dk_x}{4\pi^3} \frac{\pi k_\perp^2}{W^3}\,. \quad (30)$$

p if the fractional modulation of the electron density by the CDW and ϵ_1 is the electron-gas dielectric function for wave vector Q which results from including electron-electron interactions self-consistently and is discussed in Paper I. M is the mass of an ion.

To demonstrate that m^* is the same for electric and magnetic fields we restate the effective mass $m_{\mathcal{E}}^*$ for acceleration by an electric field found in Paper I

$$\frac{m_{\mathcal{E}}^*}{m} = 1 + Mp\epsilon_1/8\pi e^2\hbar^2 G \sum_k \frac{1}{W^3}\,, \quad (31)$$

where the sum is over states below the Fermi surface. For $\vec{Q}$ in the $\hat{x}$ direction we write the sum as an integral in cylindrical coordinates where the volume element is $2\pi k_\perp dk_\perp dk_x$ for an axially symmetric function. The integration over $k_\perp$ is trivial since W is a function only of k_x. Therefore,

$$\sum_k \frac{1}{W^3} = \int \frac{dk_x}{4\pi^3} \frac{\pi k_\perp^2}{W^3}\,, \quad (32)$$

and $m_{\mathcal{E}}^* = m_H^*$. These arguments are easily extended to finite temperatures where states above the Fermi energy are occupied as well.

IV. MAGNETORESISTANCE AND HALL COEFFICIENT

In this section, we wish to discuss the Hall coefficient and the magnetoresistance for "jellium" containing an incommensurate CDW. We continue to assume that the Fermi surface is simply connected.

As shown in Fig. 1, let us imagine the Hall experiment with an applied electric field $\vec{\mathcal{E}}_{x'}$ along the $\hat{x}'$ direction of the sample. We take $\vec{Q}$ to be along the $\hat{x}$ direction and the magnetic field $\vec{H}$ to point in the $\hat{z}$ direction. The angle between $\vec{Q}$ and $\hat{x}'$ is θ.

From Paper I, the equations describing the drift velocity $\vec{D}$ of the CDW and the electron quasivelocity $\vec{K}$ in the presence of electric and magnetic fields and scattering processes are

$$\begin{aligned}
\frac{dD}{dt} &= \frac{1}{m^*}\left(-e\mathcal{E}_x - \frac{eH}{c}K_y\right) - \frac{D-\beta K_x}{\tau_D}\,,\\
\frac{dK_x}{dt} &= \frac{1}{m}\left(-e\mathcal{E}_x - \frac{eH}{c}K_y\right) - \frac{K_x - \alpha D}{\tau_x}\,,\\
\frac{dK_y}{dt} &= \frac{1}{m}\left(-e\mathcal{E}_y + \frac{eH}{c}V_x\right) - \frac{K_y}{\tau_y}\,.
\end{aligned} \quad (33)$$

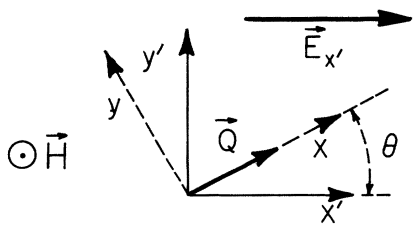

FIG. 1. Sketch of the Hall experiment described in text. The applied electric field $\vec{\mathcal{E}}_{x'}$ is along the $\hat{x}'$ direction. $\vec{Q}$ is along $\hat{x}$ and makes an angle θ with $\hat{x}'$. The magnetic field $\vec{H}$ points along $\hat{z}$ and out of the plane of the figure.

τ_D, τ_x, and τ_y are relaxation times for the CDW velocity and the components of the electron drift velocity parallel (τ_x) and perpendicular (τ_y) to $\vec{Q}$. α is a constant which arises from scattering because the moving CDW tries to pull along the electrons in a manner analogous to the acoustoelectric effect. Similarly the βK_x term tries to pull the CDW along with the drifting electrons. The discussion of scattering effects and expressions for α, β, and the relaxation times will be given in a subsequent paper. $\vec{V}$ is the mean group velocity of the electrons as defined in Eqs. (4) and (5), and the total current is $\vec{J} = -ne\vec{V}$, where n is the number of electrons per unit volume.

It is convenient to work in the x-y coordinate system. In terms of the applied electric field $\mathcal{E}_{x'}\hat{x}'$ and the Hall field $\mathcal{E}_{y'}\hat{y}'$, we have $\mathcal{E}_x = \mathcal{E}_{x'}\cos\theta + \mathcal{E}_{y'}\sin\theta$ and $\mathcal{E}_y = -\mathcal{E}_{x'}\sin\theta + \mathcal{E}_{y'}\cos\theta$. In equilibrium the equations of motion become, using Eq. (5),

$$
\begin{aligned}
&D - \beta K_x + \frac{e\tau_D}{m^*}(\mathcal{E}_{x'}\cos\theta + \mathcal{E}_{y'}\sin\theta) + \frac{eH\tau_D}{m^*c}K_y = 0\,,\\
&-\alpha D + K_x + \left(\frac{e\tau_x}{m}\right)(\mathcal{E}_{x'}\cos\theta + \mathcal{E}_{y'}\sin\theta) + \left(\frac{eH\tau_x}{mc}\right)K_y = 0\,,\\
&-\left(\frac{eH\tau_y\gamma}{mc}\right)D - (1-\gamma)\left(\frac{eH\tau_y}{mc}K_x\right)\\
&\qquad + \frac{e\tau_y}{m}(-\mathcal{E}_{x'}\sin\theta + \mathcal{E}_{y'}\cos\theta) + K_y = 0\,.
\end{aligned}
\tag{34}
$$

In addition, there is the condition that no transverse current can flow, i.e., $V_{y'} = 0$, which becomes

$$\gamma D\sin\theta + (1-\gamma)K_x\sin\theta + K_y\cos\theta = 0\,. \tag{35}$$

Equation (34) and (35) are four equations which are readily solved for the unknowns K_x, K_y, D, and $\mathcal{E}_{y'}$. The current along $\hat{x}'$ is $J_{x'} = -neV_{x'}$, and

$$V_{x'} = (1-\gamma)K_x\cos\theta + \gamma D\cos\theta - K_y\sin\theta\,. \tag{36}$$

The resulting expression for $J_{x'}$ is

$$J_{x'} = ne^2\tau_y\mathcal{E}_x\Delta_1/m\Delta\,, \tag{37}$$

where

$$\Delta = \frac{\tau_y}{m}(1-\alpha\beta)\cos^2\theta + \Delta_1\sin^2\theta \tag{38}$$

and

$$\Delta_1 = (\tau_x/m)(1-\gamma+\beta\gamma) + (\tau_D/m^*)(\alpha+\gamma-\alpha\gamma)\,. \tag{39}$$

The transverse electric field $\mathcal{E}_{y'}$ is

$$\mathcal{E}_{y'} = -\frac{\mathcal{E}_{x'}}{\Delta}\left[\left(\Delta_1 - \frac{\tau_y}{m}(1-\alpha\beta)\right)\sin\theta\cos\theta + \omega\tau_y\Delta_1\right], \tag{40}$$

where $\omega = eH/mc$. Note that there is a transverse electric field $\mathcal{E}_{y'}$ even in the absence of a magnetic field since the drifting CDW will, in general contribute to the current in the $\hat{y}'$ direction. In zero field the sign of $\mathcal{E}_{y'}$ is dependent on the quadrant in whch $\vec{Q}$ lies. Thus, a plot of the transverse electric field versus $\omega\tau$ would give a straight line with an intercept which is positive or negative depending on the quadrant in which $\vec{Q}$ lies. The Hall coefficient R_H is found from Eqs. (37) and (40) with the result in all fields

$$R_H = -1/nec\,. \tag{41}$$

The magnetoresistance $\rho_{x'x'} = \mathcal{E}_{x'}/J_{x'}$ is

$$\rho_{x'x'} = m\Delta/ne^2\tau_y\Delta_1 \tag{42}$$

and is independent of magnetic field for all field strengths.

It is interesting to note the result of ignoring the acceleration of the CDW by the magnetic field or of taking a different effective mass for acceleration by electric and magnetic fields. In this case, the high-field magnetoresistance is linearly dependent on H and changes sign with H and with the quadrant of $\vec{Q}$, which leads to a violation of the second law of thermodynamics. This is discussed in greater detail in the Appendix in the form of a theorem which states that the effective mass for acceleration of a CDW by a magnetic field must be equal to that for acceleration of the CDW by an electric field.

V. THEORY OF THE INDUCED TORQUE

One of the frequently used tools for studying the topology of the Fermi surface in metals is the induced-torque experiment.[5] It is useful because the theory for a single-crystal sphere predicts that the high-field behavior of the torque gives a dramatic and unambiguous determination of the presence of open orbits. In this section, we discuss the results to be expected for a spherical sample of jellium containing a single CDW which is free to drift.

In an induced-torque experiment the spherical sample is suspended by a rod along the $\hat{y}$ axis in a magnetic field. The magnetic field, which is held constant in strength, rotates in the x-z plane at a constant frequency (typically about 0.01 sec^{-1}). The changing magnetic field induces a current in the sample. The $\hat{y}$ component of the torque on the sample, which results from the interaction of the induced current and the magnetic field, is then measured. In general, the torque depends in a fairly complicated way upon the components of the conductivity tensor.

The problem of interest here is the torque to be expected for a spherical sample of jellium contain-

ing a single CDW which is free to drift. In the usual derivation of the induced torque, it is assumed that the electric field and the current are related through a local conductivity tensor. However, in case of a sample containing a single CDW, this local relationship cannot be valid since the CDW can have only one drift velocity throughout the sample since planes of constant phase cannot be bunched together. A local relationship between the electric field and the CDW drift velocity implies that the CDW can have a different velocity at each point in the sample. In general, it is by no means necessary that a sample be characterized by a single $\vec{Q}$ throughout. There could be a domain structure with different $\vec{Q}$'s in different domains. The CDW drift velocity in each domain could then be different. At this time we choose to discuss only the case of a sample with a single $\vec{Q}$. In order that planes of constant phase not be bunched together, we take the CDW drift velocity to be a function of the spatial average of the electric and magnetic fields. For the sake of brevity we do not include the details of the solution to the equations of motion, Eq. (33), of the CDW and the appropriate boundary conditions and Maxwell's equations.[5] If an initial value for $\vec{D}$ is assumed, it is found that $\vec{D}$ decays in time due to a buildup of charge at the surface of the sample. Thus the steady state value for $\vec{D}$ is zero. In addition, any currents associated with the drifting CDW and the electric fields produced by the charge on the surface are uniform (since we take $\vec{D}$ the same everywhere in the sphere). Since there is no torque on a uniform current, the drifting CDW causes no torque on the sample while it has a nonzero velocity. Thus, the induced torque for a spherical sample of jellium with one CDW throughout is given by the usual result of Visscher and Falicov.[5]

VI. CONCLUSIONS

In this paper, we have microscopically derived the equations of motion in a magnetic field for the drift velocity of an unpinned incommensurate CDW and the electron drift velocity for a three-dimensional jellium model. The effective mass characterizing the acceleration of the CDW by the magnetic field was shown to be equal to that for the electric-field case previously considered in Paper I. We examined the magnetotransport effects associated with the drift of the CDW for the case of a simply-connected Fermi surface. The magnetoresistance was found to be independent of magnetic-field strength and the Hall coefficient was equal to the free-electron value for all values of the magnetic field. The transverse electric field was found to contain a term which is independent of magnetic field strength and changes sign with the quadrant of the CDW wave vector $\vec{Q}$. Thus, a plot of transverse electric field versus magnetic field would give a straight line with an intercept, the sign of which depends on the quadrant of $\vec{Q}$. We also discussed the theory of the induced torque experiment for a spherical sample of jellium with one CDW throughout which was free to drift and found that the torque did not depend on the CDW drift and that the steady-state CDW drift velocity was zero.

Because the CDW model has been successful at quantitatively explaining several of the highly anomalous properties of potassium including the Mayer–El Naby optical absorption,[6] the conduction-electron spin-resonance measurements,[7] and the anisotropic residual resistance anomaly,[8] one of the motivating factors behind this work was the question whether the CDW model could explain the linear magnetoresistance[9] and the anomaly of the high-field induced-torque anisotropy.[10] It is clear that the results presented in this paper concerning the effects associated with the drift of the CDW in the case of a simply connected Fermi surface cannot offer an explanation of these phenomena. The present discussion has not, however, included a study of the possible topological effects of the Fermi surface on the magnetotransport properties. Although for the discussion here we have taken the Fermi surface to be simply connected in order to examine the effects associated with the drift of the CDW, the actual Fermi surface in jellium in the presence of a CDW is not known and effects associated with the topology of Fermi surface must be investigated. Thus, it remains an open question whether the CDW hypothesis can offer an explanation of the magnetoresistance and high-field induced-torque anisotropy in potassium.

APPENDIX

In the text it was stated that the effective mass of a CDW characterizing the acceleration by an electric field must be the same as that for acceleration by a magnetic field. The purpose of this appendix is to present a proof of this theorem based on thermodynamic arguments. In Sec. III of the text, a microscopic proof was presented for the case where the CDW potential was taken to be a local non-velocity-dependent function. Because the very existence of CDW's depends crucially on the velocity dependence of electron-electron interactions, the proof on the basis of general thermodynamic arguments is very useful since a microscopic treatment of velocity dependent effects would by highly intricate.

As in Sec. IV, we imagine the Hall experiment shown in Fig. 1. Let us assume that the effective

mass of the CDW for acceleration by an electric field $m_{\mathcal{E}}^*$ is different from that for acceleration by a magnetic field m_H^*.

Then the first of the equations in (34) becomes

$$D - \beta K_x + \frac{e\tau_D}{m_{\mathcal{E}}^*}(\mathcal{E}_{x'}\cos\theta + \mathcal{E}_{y'}\sin\theta) + \frac{eH\tau_D}{m_H^* c}K_y = 0 . \tag{A1}$$

Solving as in Sec. IV for the magnetoresistance $\rho_{x'x'}$ obtains

$$\rho_{x'x'} = m(\Delta + \Delta_H)/ne^2\tau_y\Delta_1 , \tag{A2}$$

where Δ and Δ_1 are given by Eqs. (38) and (39) with m^* replaced by $m_{\mathcal{E}}^*$ and

$$\Delta_H = \omega\tau_D\tau_y(\alpha + \gamma - \alpha\gamma)\sin\theta\cos\theta(1/m_{\mathcal{E}}^* - 1/m_H^*) , \tag{A3}$$

where $\omega \equiv eH/mc$.

Thus, for $m_{\mathcal{E}}^* \neq m_H^*$ the magnetoresistance is a linear function of magnetic field. Note, however, that depending on the direction of $\vec{H}$ and the quadrant of angle θ, the high-field magnetoresistance can be negative. This clearly violates the Onsager relations and the second law of thermodynamics. Thus, it must be that $m_H^* = m_{\mathcal{E}}^*$.

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