PHYSICAL REVIEW B VOLUME 16, NUMBER 11 1 DECEMBER 1977

Elastic constants of hcp ^{4}He

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(Received 18 July 1977)

Values of the shear elastic moduli for hcp ^{4}He are reported for molar volumes of 20.67 and 21.03 cm^3. These new measurements indicate that the value of C_{44} previously reported by the author is in error.

The complete set of elastic constants for hcp ^{4}He, based solely on sound-velocity measurements in crystals of known orientation, has been reported by Crepeau *et al.*[1] and by Greywall.[2] The two sets of moduli at molar volumes[3] of 20.97 and 20.55 cm^3, respectively, are in good agreement except for the value of C_{44} (which corresponds to the fast-transverse sound velocity along the *c* axis or in the basal plane). New measurements of the shear sound velocities in single crystals of hcp ^{4}He at molar volumes of 20.67 and 21.03 cm^3 indicate that the value of C_{44} reported previously by the author is in error.

A pulse-echo technique and 10-MHz *AC*-cut quartz transducers were used to measure the sound velocities in ^{4}He crystals grown under constant pressure. The plastic sample container and the x-ray system used in determining the crystal orientations were the same as those described previously.[4] The moduli C_{44} and C_{66} were determined mainly from measurements of sound propagating very nearly parallel or perpendicular to the *c* axis of the crystal. These elastic constants are shown as solid circles in Fig. 1 where they are compared with earlier results.[1,2,5] Not shown in Fig. 1 is the value of C_{44} at 20.55 cm^3 previously reported[2] (namely, 1.83×10^8 dyn/cm^2) since the present data demonstrate that this earlier result is considerably too large. The source of the discrepancy is not obvious. However, the errant modulus was determined on the basis of only three measurements of the fast-transverse velocity (T_2) and it is speculated that a significant portion of each of these three samples had an orientation different from that portion probed by the x-ray beam. In the present experiment a much larger fraction of the sample could be analyzed with the x rays and there were no experimental problems in observing the T_2 signals. In contrast to the earlier setup, the geometry of the present cell was arranged so that even the largest deviations of the sound beam from the wave normal[6] would not have prevented the detection of a sound signal.

Each of the straight lines in Fig. 1 (note the log-log scales) corresponds to a mode Grüneisen parameter, defined by

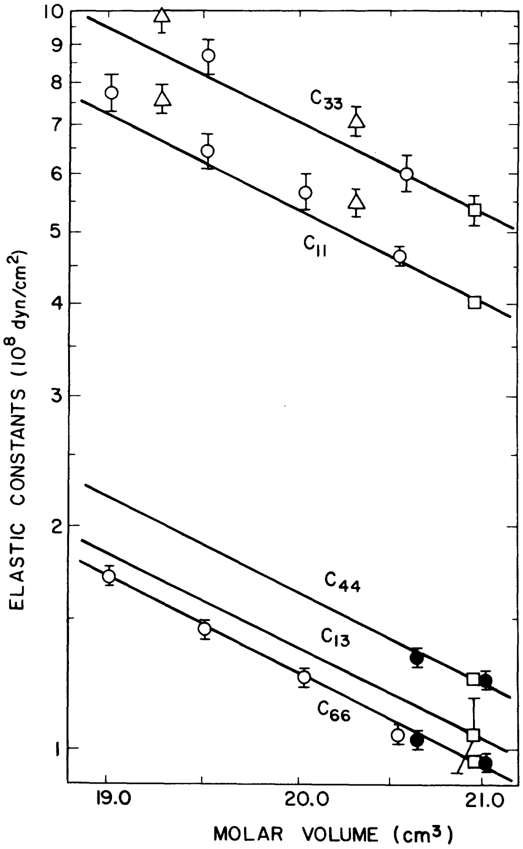

FIG. 1. Elastic constants for hcp ^{4}He as a function of molar volume plotted on log-log scales. The C_{ij} were determined from measurements of the sound velocities in single crystals of known orientation by Crepeau *et al.* (Ref. 1), squares; Wanner and Franck (Ref. 5), triangles; and by Greywall (Ref. 2), open circles. The solid circles are the present measurements. The straight lines correspond to mode Grüneisen parameters of 2.7.

$$\gamma_{ij} \equiv -\frac{1}{2}\frac{d\ln C_{ij}}{d\ln V} - \frac{1}{6} \;, \tag{1}$$

of 2.7 and has been drawn so that it passes through the data of Crepeau *et al.*[1] The C_{66} data are very well described by this parameter. The C_{11} and C_{33} moduli show systematic deviations from the straight lines, but since these moduli are less well determined, the deviations may not be significant.

The data of Crepeau *et al.*[1] satisfy, within the experimental uncertainty, the relation

$$C_{13} + C_{33} = 2(C_{11} - C_{66}) \;. \tag{2}$$

Equation (2) is due to Franck and Wanner[7] and is a consequence of the axial ratio c/a being independent of density. If all the γ_{ij} are taken to be 2.7, then the molar volume dependence of each of the moduli can be written

$$C_{ij}(V) = C_{ij}(V_0)(V_0/V)^{5.73} \;, \tag{3}$$

and Eq. (2) is satisfied within experimental uncertainty at all molar volumes. In Eq. (3), V_0 is 20.97 cm^3 and $C_{ij}(V_0)$ are the elastic constants determined by Crepeau *et al.*,[1] namely,

$$\begin{aligned} C_{11} &= (4.05 \pm 0.04) \times 10^8 \text{dyn/cm}^2 \;, \\ C_{13} &= 1.05 \pm 0.13 \;, \\ C_{33} &= 5.54 \pm 0.22 \;, \\ C_{44} &= 1.24 \pm 0.02 \;, \\ C_{66} &= 0.96 \pm 0.02 \;. \end{aligned} \tag{4}$$

Another consequence of taking all γ_{ij} to be equal is that the Grüneisen parameter defined by

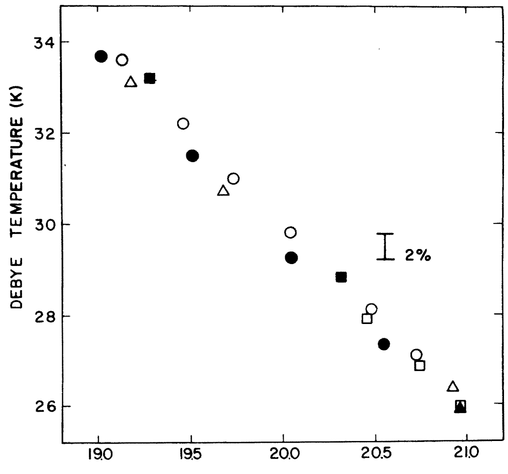

FIG. 2. Zero-temperature Debye Θ as a function of molar volume. Calorimetric data: open circles, Ahlers (Ref. 8); open triangles, Edwards and Pandorf (Ref. 10); open squares, Gardner *et al.* (Ref. 9). Elastic data: solid triangle, Crepeau *et al.* (Ref. 1); solid squares, Franck and Wanner (Ref. 7); solid circles, determined using the moduli given in Eq. (3).

$$\gamma \equiv -\frac{d\ln\Theta_0}{d\ln V} \;, \tag{5}$$

where Θ_0 is the Debye theta at zero temperature, is also equal to 2.7. This is in excellent agreement with calorimetric data which yield values of γ varying from 2.6 at 19 cm^3/mole in Ref. 8 to 2.8 at 21 cm^3/mole in Ref. 9.

A comparison of Debye temperatures determined from elastic and calorimetric data[8-10] is shown in Fig. 2.

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