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Determination of the spin-dependent electron momentum distribution by double Compton scattering*

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The method of double Compton scattering for obtaining the electron spin-dependent momentum distribution is analyzed. It is shown that the effect, although small, is sufficiently large to be measured by present Compton-scattering experiments.

There is considerable recent interest in the determination of electron momentum distributions in solids by the Compton scattering of high-energy nuclear γ rays.[1] It would be of particular interest if the spin-dependent electron momentum distribution from ferromagnetic materials could be determined.

Some time ago Platzman and Tzoar[2] pointed out that the scattering of circularly polarized x-rays from ferromagnetic materials could be used to determine the spin-dependent electron distribution function. Recently Sakai and Ono[3] measured the Compton profile due to magnetic electrons in ferromagnetic iron with circularly polarized γ rays using a 10-mCi source of ^{57}Co which was cooled down to 40 mK. A superconducting magnet served to saturate the magnetization of the iron foil in which the ^{57}Co source was diffused.

The primary difficulty in the widespread use of the above technique are the very low temperatures required and the limitations to sources which can easily be polarized.

Synchrotron radiation could also be used in principle.[4] The radiation emitted is in general elliptically polarized. The radiation emitted exactly in the plane of the synchrotron orbit has zero circular polarization and 100% linear polarization. A circular polarization component can be obtained by using the radiation above or below the plane of the synchrotron orbit.

The emitted radiation, however, is strongly peaked about the plane of the synchrotron orbit. For example in the DESY synchrotron at 6.3 GeV for emitted 100-keV photons, the intensity drops by an order of magnitude within an angle of 7×10^{-2} mrad from the orbital plane.[5] In the laboratory area this is a beam height of ~2 mm. Thus in order to extract a beam with a circular polarization component, most of the primary beam must be intercepted and only a fraction of the upper- or lower-half of the beam be transmitted. There are, however, considerable difficulties involved in such an experiment.

It is very desirable to be able to measure the spin-dependent electron momentum distribution without the use of very low temperatures and easily polarizable nuclei[3] or of synchrotron radiation.[4,5] In the present paper we describe the method of double Compton scattering for obtaining the electron spin-dependent momentum distribution. The method uses approximately the same experimental equipment now being used for single Compton scattering.[1]

Since the coupling of a circularly polarized photon to the spin of an electron is proportional to the ratio of the energy of the photon to the rest mass of the electron, high-energy photons are needed in order to obtain an appreciable effect. We shall thus present here a relativistic analysis and will evaluate the effect for 660- keV photons readily available from a ^{137}Cs source.

The polarization of the scattered photons (P') is related to the polarizations of the target electrons ($\vec{\xi}$) and the incident photons (P) by

$$P' = A\,|\vec{\xi}\,| + BP + C\ . \tag{1}$$

For example, the circular polarization of the scattered photons (P_c') is given by[6]

$$P_c' = A_c\,|\vec{\xi}\,| + B_c P_c + C_c\ , \tag{2}$$

with $C_c = 0$, and

$$A_c = -\frac{(1-\cos\theta)(\vec{k}\cos\theta + \vec{k}')\cdot\vec{\xi}}{m(\omega/\omega' + \omega'/\omega - \sin^2\theta)\,|\vec{\xi}\,|}\ , \tag{3}$$

$$B_c = \frac{(\omega/\omega' + \omega'/\omega)\cos\theta}{\omega/\omega' + \omega'/\omega + (P_\perp - 1)\sin^2\theta}\ , \tag{4}$$

where $P_\perp$ is the initial linear polarization perpendicular to the scattering plane (in the direction of $\vec{k}\times\vec{k}'$), $\vec{k}$ and $\vec{k}'$ are the initial and final photon momenta, θ is the angle between $\vec{k}$ and $\vec{k}'$, and ω and ω' are the initial and final photon energies.

The linear polarization $P_\perp'$ of the scattered photon in the direction $\vec{k}\times\vec{k}'$ is given by[6]

$$P_\perp' = A_\perp\,|\vec{\xi}\,| + B_\perp P_\perp + C_\perp, \tag{5}$$

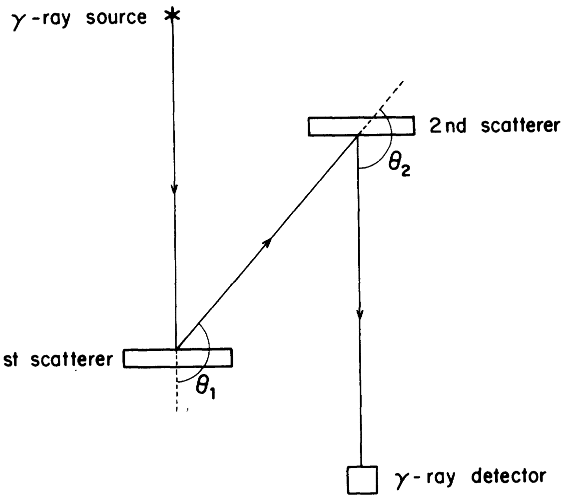

FIG. 1. Experimental configuration.

with $A_{\perp}=0$, and

$$B_{\perp}=\frac{1+\cos^2\theta}{\omega/\omega'+\omega'/\omega+(P_{\perp}-1)\sin^2\theta}, \tag{6}$$

$$C_{\perp}=\frac{\sin^2\theta}{\omega/\omega'+\omega'/\omega+(P_{\perp}-1)\sin^2\theta}. \tag{7}$$

The linear polarization P'_V of the scattered photon in a direction 45° with respect to the direction $\vec{k}\times\vec{k}'$ is given by[6]

$$P'_V=A_V|\vec{\zeta}|+B_V P_V+C_V, \tag{8}$$

with $A_V=0$, $C_V=0$, and

$$B_V=\frac{2\cos\theta}{\omega/\omega'+\omega'/\omega+(P_{\perp}-1)\sin^2\theta}. \tag{9}$$

The cross section for the scattering is given by[6]

$$\frac{d\sigma}{d\Omega'}=\frac{1}{2}r_0^2\left(\frac{\omega'}{\omega}\right)^2\left[\left(\frac{\omega}{\omega'}+\frac{\omega'}{\omega}-\sin^2\theta\right)+P_{\perp}\sin^2\theta - P_c\frac{(1-\cos\theta)}{m}(\vec{k}\cos\theta+\vec{k}')\cdot\vec{\zeta}\right]. \tag{10}$$

Let us now examine the case of double Compton scattering. If the initial photons are unpolarized, we have for the first scattering just the first term in Eq. (10) or

$$\frac{d\sigma_1}{d\Omega'}=\frac{1}{2}r_0^2\left(\frac{\omega_1}{\omega_0}\right)^2\left(\frac{\omega_0}{\omega_1}+\frac{\omega_1}{\omega_0}-\sin^2\theta_1\right), \tag{11}$$

where θ_1 is the scattering angle for the first scattering, ω_0 is the incident photon energy, and ω_1 is the scattered photon energy.

The photons resulting from this first scattering are polarized with polarization from Eqs. (2) and (5):

$$P_{1,\perp}=\frac{\sin^2\theta_1}{\omega_0/\omega_1+\omega_1/\omega_0-\sin^2\theta_1},$$
$$P_{1,c}=-\frac{(1-\cos\theta_1)(\vec{k}_0\cos\theta_1+\vec{k}_1)\cdot\vec{\zeta}}{m(\omega_0/\omega_1+\omega_1/\omega_0-\sin^2\theta_1)}. \tag{12}$$

From Eq. (10) the cross section for the second scattering is given by

$$\frac{d\sigma_2}{d\Omega'}=\frac{1}{2}r_0^2\left(\frac{\omega_2}{\omega_1}\right)^2\left[\left(\frac{\omega_1}{\omega_2}+\frac{\omega_2}{\omega_1}-\sin^2\theta_2\right)+P_{1,\perp}\sin^2\theta_2 - P_{1,c}\frac{(1-\cos\theta_2)}{m}(\vec{k}_1\cos\theta_2+\vec{k}_2)\cdot\vec{\zeta}\right]. \tag{13}$$

We shall apply the above equations to the experimental configuration described in Fig. 1. The first and second scatterers in Fig. 1 are assumed to be magnetized iron and sufficiently thin so that no appreciable double scattering occurs in either scatterer. Two situations were compared, namely, one in which the magnetizations of the two scatterers were both in the direction of the initial incoming photons, and the other in which the magnetization of the second scatterer was reversed with respect to the magnetization of the first scatterer. The computational procedure is a modification of the Monte Carlo code developed recently for the calculation of multiple scattering of photons in Compton profile measurements, described by Felsteiner *et al.*[7,8] An unpolarized beam of photons of energy 660 keV strikes the first scatterer at 90° with respect to its surface. For simplicity, each photon is forced to Compton scatter at the same backward angle θ_1. After the first

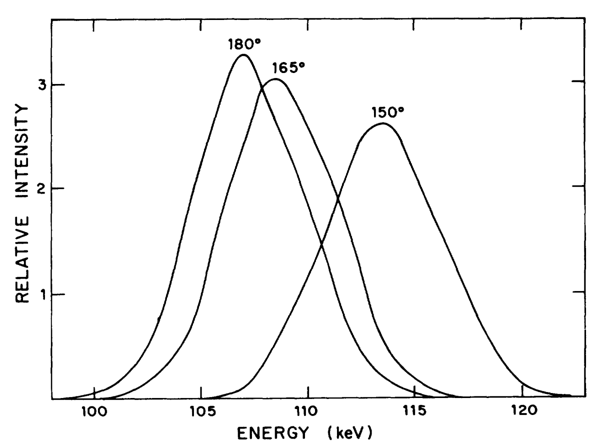

FIG. 2. Difference of the photon energy distributions of (i) both scatterers magnetized in the same direction, and (ii) both scatterers magnetized in opposite directions. The energy distribution for the difference of the above two situations is shown for the three cases $\theta_1=\theta_2=180°$, 165°, and 150°.

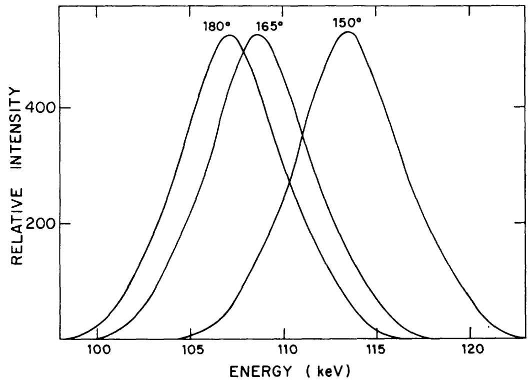

FIG. 3. Energy distributions for $\theta_1=\theta_2=180°$, 165°, and 150° of all of the photons for the case in which the magnetizations of the scatterers are in opposite direction to each other. The same relative intensity scale is used as in Fig. 2, thus the ratio of the areas of Figs. 2 to 3 are absolute and not relative.

scattering the photon is forced to strike the second scatterer and to Compton scatter from it at an angle $\theta_2=\theta_1$ so that the double scattered photons reach the detector parallel to the initial incoming beam. The orbital of the electron for a given scattering is determined according to its weight, the weight being given by the ratio of the number of electrons in the orbital to the total number of electrons in the atom. The differential cross sections given in Eqs. (11) and (13) were used for the first and second scatterings, respectively. They were modified according to the Compton profile of the particular orbital in the way described in Ref. 8. The core profiles for iron were taken from the calculations of Ref. 9. The Compton profile for the outer electrons was then deduced from the experimental data of Ref. 10. The number of $3d$ electrons was taken from Ref. 10 to be 6.5. The net number of polarized electrons was taken from Ref. 11 to be 2.22.

The difference of the data of the two situations analyzed [(i), both scatterers magnetized in the same direction, and (ii), the scatterers magnetized in opposite directions] is proportional to the number of photons scattered from $3d$ electrons both in the first and in the second scatterer. The energy distributions of the data for the difference of the above two situations for the three cases $\theta_1=\theta_2=180°$, 165°, and 150° are shown in Fig. 2. In an experiment in which scattering can take place for $180°\geq\theta_1,\theta_2\geq165°$, the resultant energy distribution of the emitted photons will be smeared between the two curves 180° and 165°; similarly, in an experiment in which scatterings take place for $180°\geq\theta_1,\theta_2\geq150°$, the emitted photons will be smeared between the two curves 180° and 150°.

TABLE I. Ratios of the areas of Figs. 2 to 3.

θ	Ratio
180°	0.51%
165°	0.48%
150°	0.43%
130°	0.31%
110°	0.12%

The energy distributions of all of the photons, for the case in which the magnetizations of the scatterers are in opposite direction to each other, are shown in Fig. 3. The ratios of the areas of Figs. 2 and 3 are given in Table I. It is seen that the effect for scattering of $180°\geq\theta_1,\theta_2\geq150°$ is ~0.5% and decreases rapidly for smaller angles. It is to be noted that present Compton-scattering experiments can achieve such an accuracy.

The primary difficulty in the experiment described in the present paper is the loss of intensity due to double scattering. Since however, θ_1 and θ_2 can be smeared between 180° and 165°, for example, or 180° and 150°, much larger solid angles can be used in this experiment as compared to a single-scattering experiment.

The detected intensity of the experiment described in the present paper can be compared with the detected intensity of a typical single-scattering Compton experiment performed today.[1] In a single scattering Compton experiment, the polar angles between the source and target typically are defined to an accuracy $\Delta\theta_s\simeq2°, \Delta\phi_s\simeq2°$. The polar angles between the target and the detector are also defined to an accuracy $\Delta\theta_d\simeq2°$, $\Delta\phi_d\simeq2°$, and the thickness of the target is typically 0.1 mean free path. In the experiment described in the present paper, if we smear θ_1 and θ_2 between 180° and 165° we have $\Delta\theta_1\simeq15°$ and $\Delta\theta_2\simeq15°$. Defining the solid angle between the first scatterer and the source to a comparable accuracy we have $\Delta\theta_s\simeq15°$ and $\Delta\phi_s\simeq15°$. We assume also that the thickness of the first and second scatterers are 0.1 mean free path and $\Delta\phi_1\simeq15°$ and $\Delta\phi_2\simeq15°$. We then have for R, the ratio of the detected intensity of the experiment described in the present paper to the detected intensity of a single-scattering Compton experiment

$$R\sim\frac{(0.1)^2(\frac{15}{180})^3(\frac{15}{360})^3}{(0.1)(\frac{2}{180})^2(\frac{2}{360})^2}=1.1. \qquad (14)$$

The above indicates that the intensity detected in the experiment described in the present paper can be comparable to the de ected intensities of Compton experiments performed today. It is expected to be particularly applicable for experiments when high resolution is not absolutely necessary, for example, the temperature dependence of the number and average momenta of the magnetic electrons in ferromagnetic materials.

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