Theory of $1/f$ noise in metal films and whiskers

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The equilibrium thermal fluctuation in metal films is studied with careful consideration of the finite size of the specimen and the boundary conditions on the metal-vacuum and metal-substrate interfaces. For a freely suspended film, the noise spectrum due to bulk fluctuation has two parts, a frequency-independent part below a characteristic frequency and an $f^{-3/2}$ part above that frequency. The characteristic frequency is the inverse of the diffusion time across the largest dimension of the specimen. The $1/f$ spectrum is obtained when two conditions are met : (a) the metal film is deposited on a substrate of very low bulk thermal conductivity; and (b) the thermal fluctuation in the film is two dimensional and is mainly excited by the random flow of heat between the film and the substrate. In freely standing metal whiskers, the $1/f$ spectrum can be obtained when the fluctuation is excited by white-noise sources at the ends.

I. INTRODUCTION

Recent experiments strongly suggest that $1/f$ noise in metal films at room temperature,[1,2] in superconducting films at the resistive transition,[3] and in Josephson tunnel junctions[4] arises from equilibrium temperature fluctuations. The analysis of the noise spectrum is based on the diffusion equation for temperature fluctuations. In the Langevin approach,[1,2,5] the driving term added to the diffusion equation has the form $\nabla \cdot \vec{F}(\vec{r}, t)$, where $\vec{F}$ is uncorrelated in space and time. This fluctuating term leads to a power spectrum that varies at $\ln(1/f)$ and f^0 over the appropriate frequency ranges, and that does not exhibit a $1/f$ region. Voss and Clarke[2] then incorporated a suggestion of Lundström *et al.*[6] into a theory in which the driving term had the form $P(\vec{r}, t)$, where P is uncorrelated in space and time. This formulation led to a $1/f$ power spectrum over a substantial range of frequencies with an amplitude that was in good agreement with the measured $1/f$ noise. However, no physical mechanism was suggested to account for the origin of this unusual driving term.

In all previous theoretical discussions, the specimen is assumed to be a finite portion of an infinite solid, but in actual measurements metal films of finite dimensions are invariably employed. There has been no attempt to incorporate the boundary condition into the problem because of a general lack of information on boundary effects. The situation is very different now, however, because of a recent measurement by Clarke and Ketchen[7] in which the metal film is peeled off from the substrate and suspended freely in the apparatus. The noise spectrum is found to be very different from that predicted from the three-dimensional model, but in agreement with that for the one-dimensional model.[2] Furthermore, there is no $1/f$ region. This strongly suggests that the $1/f$ noise comes from fluctuating heat flow through the metal-substrate boundary, and that in the freely suspended film, the finite size of the film renders the three-dimensional model inapplicable.

We present in this paper a detailed analysis of boundary effects on the noise spectrum of metal films. It is shown that by imposing a set of physically reasonable boundary conditions, one can explain both the one-dimensional nature of the temperature fluctuation in the freely suspended film and the $1/f$ spectrum of a metal film attached to an insulating substrate.

We have also extended the same consideration to a freely standing metal whisker. It is shown that the noise spectrum can have a $1/f$ part if the noise is excited by white-noise sources at the ends of the whisker. The implications of this finding on the recent experimental results of Dutta *et al.*[8] will be discussed.

II. FREELY SUSPENDED FILMS

We consider a metal film of dimensions $l_1 \gg l_2 \gg l_3$. The heat-diffusion equation in the film is written as

$$\frac{\partial T}{\partial t} - \Lambda \nabla^2 T = -\nabla \cdot \vec{F}(\vec{r}, t) , \tag{1}$$

where $T(\vec{r}, t)$ is the fluctuating part of the temperature, Λ is the thermal-diffusion constant defined by $\Lambda = K/C$, C is the specific heat, and K is the thermal conductivity of the metal. In the driving term, the quantity $\vec{F}(\vec{r}, t)$ is uncorrelated in space and time.

A. Ideal boundary condition

There is negligible heat flow between the metal film and the surrounding space, so the boundary condition on the free surfaces are

$$K\hat{n}\cdot\nabla T=0\,, \tag{2}$$

where $\hat{n}$ is the unit normal of the surface. This boundary condition applies to the surfaces $y=0$, $y=l_2$, $z=0$, and $z=l_3$. The ends $x=0$ and $x=l_1$ are connected to the current source and other measuring apparatus, so we treat the x dimension of the film as a part of a large system.

Before the boundary conditions are imposed, the solution of Eq. (1) is

$$T(\vec{r},t)=\sum_{\vec{q}}\int\frac{d\omega}{2\pi}\,\frac{i\vec{q}\cdot\vec{F}(\vec{q},\omega)}{i\omega-\Lambda q^2}\,e^{i\vec{q}\cdot\vec{r}-i\omega t}\,, \tag{3}$$

where $\vec{F}(\vec{q},\omega)$ is the Fourier transform of $\vec{F}(\vec{r},t)$. The boundary conditions imply that the y,z dependence of the solution must be of the form $\cos(q_y y)\cos(q_z z)$, and that

$$q_y=n_y\pi/l_2,\quad q_z=n_z\pi/l_3\,, \tag{4}$$

where n_y, $n_z=0,1,2,3,\ldots$. Furthermore, only the part of the driving force which is odd in q_y and q_z can excite a response in the system. If we denote this part by $\vec{F}'(\vec{q},\omega)$, then we can write the solution as

$$T(\vec{r},t)=\sum_{\vec{q}}\int\frac{d\omega}{2\pi}\,\frac{i\vec{q}\cdot\vec{F}'(\vec{q},\omega)}{i\omega-\Lambda q^2}\times\cos(q_y y)\cos(q_z z)e^{iq_x x-i\omega t}\,. \tag{5}$$

In the actual measurement, the average temperature fluctuation over the film is measured. When we average the result in Eq. (5) over the film only the terms with $q_y=q_z=0$ will remain. This gives

$$\langle T(\vec{r},t)\rangle=\sum_{q_x}\int\frac{d\omega}{2\pi}\,\frac{iq_xF_x'(q_x,\omega)}{i\omega-\Lambda q_x^2}\times\frac{1-e^{iq_xl_1}}{iq_xl_1}\,e^{-i\omega t}\,. \tag{6}$$

This is just the result of the one-dimensional fluctuation problem considered by Voss and Clarke.[2] The spectrum of the temperature fluctuation was found by these authors as

$$\begin{aligned}S_T(\omega)&=\frac{F_0^2l_1}{2\pi}\int_{-\infty}^{\infty}\frac{q_x^2dq_x}{\omega^2+\Lambda^2q_x^4}\left(\frac{\sin\frac{1}{2}q_xl_1}{\frac{1}{2}q_xl_1}\right)^2,\\&=\frac{F_0^2}{\sqrt{2}\,l_1\pi\Lambda^{1/2}\omega^{3/2}}\left[1-e^{-\theta}(\cos\theta+\sin\theta)\right],\end{aligned} \tag{7}$$

where F_0^2 is the square magnitude of $\vec{F}'(\vec{q},\omega)$, $\theta=(\omega/\omega_1)^{1/2}$, and $\omega_1=2\Lambda/l_1^2$. For $\omega\gg\omega_1$ the spectrum

$$S_T(\omega)\cong F_0^2/\sqrt{2}\,\pi l_1\Lambda^{1/2}\omega^{3/2}\,,$$

and for $\omega\ll\omega_1$

$$S_T(\omega)\cong F_0^2l_1/2\sqrt{2}\,\Lambda^{3/2}\omega^{1/2}\,.$$

The only characteristic frequency of the film is ω_1, which is associated with the largest dimension of the film.

When the frequency is much less than ω_1, another boundary effect may become important. If the measuring apparatus has a much larger heat capacity and consequently much less temperature fluctuation than the film, we must impose the condition $T=0$ at $x=0$ and $x=l_1$ for the film. This produces standing-wave patterns in the x direction and

$$T(\vec{r},t)=\sum_{q_x}\int\frac{d\omega}{2\pi}\,\frac{iq_xF_x'(q_x,\omega)}{i\omega-\Lambda q_x^2}\times\sin(q_xx)e^{-i\omega t}\,, \tag{8}$$

where $q_x=n\pi/l_1$. Averaging over the length l_1 of the film and working out the spectrum, we obtain

$$S_T(\omega)=\sum_{n=\text{odd}}\frac{4F_0^2}{\omega^2+\Lambda^2q_x^4}\,\frac{1}{l_1^2}\,. \tag{9}$$

Since $\Lambda q_x^2=\frac{1}{2}n^2\pi^2\omega_1\gg\omega$, it follows that in the frequency range under consideration the spectrum is frequency independent.

In summary, we find that under ideal boundary conditions the noise spectrum is $f^{-3/2}$ for $\omega\gg\omega_1$ and $f^{-1/2}$ or f^0 for $\omega\ll\omega_1$, depending on the heat capacity of the measuring apparatus.

B. Relaxed boundary condition

Actual metal films are never made with perfect geometric surfaces, so we must find ways to deal with surface imperfections. When the wavelength of the diffusion pattern is small compared with the linear dimension of the specimen but comparable to the roughness of the edges, the standing-wave pattern is so severely disturbed that the spatial quantization condition, Eq. (4), is meaningless. The discreteness of the allowed wave vectors is smeared out, so that it is more appropriate to treat the wave vector as a continuous variable. The net effect of this is equivalent to treating that dimension of the film as a part of a large system.

We first encounter the relaxed boundary condition when the frequency becomes comparable or higher than the second highest natural frequency $\omega_2=2\Lambda/l_2^2$. There is still standing wave in the z direction, but the standing-wave condition in the y direction is relaxed. The average temperature fluctuation is

$$\langle T(\vec{r},t)\rangle=\sum_{\vec{q}}\int\frac{d\omega}{2\pi}\,\frac{i\vec{q}\cdot\vec{F}'(\vec{q},\omega)}{i\omega-\Lambda q^2}\times\frac{1-e^{iq_xl_1}}{iq_xl_1}\,\frac{1-e^{iq_yl_2}}{iq_yl_2}\,e^{-i\omega t}\,, \tag{10}$$

where $\vec{q}=(q_x, q_y, 0)$. The spectrum of the fluctuation is

$$S_T(\omega)=F_0^2\,\frac{l_1 l_2}{(2\pi)^2}\int \frac{q^2\,dq_x\,dq_y}{\omega^2+\Lambda^2 q^4}\times\left(\frac{\sin\frac{1}{2}q_x l_1}{\frac{1}{2}q_x l_1}\right)^2\left(\frac{\sin\frac{1}{2}q_y l_2}{\frac{1}{2}q_y l_2}\right)^2. \tag{11}$$

In the above integration the range of q_x is much smaller than that of q_y because $l_1 \gg l_2$. This allows us to approximate $q_x=0$ in the frequency-dependent factor and carry out the rest of the integration over q_x. The result is

$$S_T(\omega)\cong F_0^2\,\frac{l_2}{2\pi}\int_{-\infty}^{\infty}\frac{q_y^2\,dq_y}{\omega^2+\Lambda^2 q_y^4}\left(\frac{\sin\frac{1}{2}q_y l_2}{\frac{1}{2}q_y l_2}\right)^2. \tag{12}$$

The problem becomes a one-dimensional fluctuation in the y direction, so

$$S_T(\omega)\cong F_0^2/\sqrt{2}\,\pi l_2\Lambda^{1/2}\omega^{3/2} \tag{13}$$

for $\omega \gg \omega_2$. This gives a higher level of noise than that given by Eq. (7) by a factor l_2/l_1. In the actual experiment, one should observe a leveling of the spectrum around $\omega \sim \omega_2$, followed by a $f^{-3/2}$ portion when $\omega \gg \omega_2$.

In a similar manner, we will experience another relaxation of boundary condition in the z direction with another zigzag in the noise spectrum when $\omega \simeq \omega_3$, where $\omega_3 = 2\Lambda/l_3^2$.

The results of the above discussion may be summarized in Fig. 1. The observation of Clarke and Ketchen has confirmed the spectrum around ω_1. Further work at higher frequencies is needed to establish the applicability of the relaxed boundary condition. But for metal films used in the experiments ω_2 is so high that the thermal fluctuation noise around this frequency becomes totally negligible compared with the Johnson noise. So one must seek to verify the relaxed boundary condition in films of large dimensions or low thermal conductivity. Unfortunately, both requirements tend to reduce the noise level.

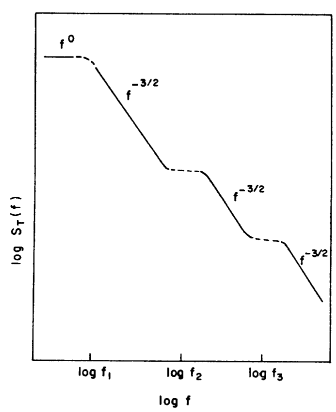

FIG. 1. Predicted spectrum of the equilibrium thermal fluctuation in a freely suspended metal film.

III. FILM ON SUBSTRATE

In this section, we consider a film deposited on a substrate which is made of a material of very low thermal conductivity, denoted by K_s. An example of this combination is the system of gold film on glass substrate used in the experiment of Voss and Clarke.[2] The ratio K/K_s is approximately 300.

On the interface between the metal and the substrate we can write the boundary conditions

$$T(z=0^+)=T(z=0^-), \tag{14}$$

$$J_z(z=0^+)=J_z(z=0^-), \tag{15}$$

where J_z is the z component of the heat flux. The heat flux can be related to the temperature by means of the transport equation. So Eq. (15) may be rewritten as

$$K\left.\frac{\partial T}{\partial z}\right|_{z=0^+} - K_s\left.\frac{\partial T}{\partial z}\right|_{z=0^-} = G(x,y,t), \tag{16}$$

where the fluctuation term $G(x,y,t)$ represents the instantaneous deviation of the heat flux from that given by the equilibrium transport equation.

Ideally, one should solve the coupled heat-diffusion problem for the combined film-substrate system with the above-mentioned boundary conditions. This is not practical, however, because the geometry of the system, that the film covers a small part of one surface of the substrate, is difficult to analyze. Therefore, we simplify the problem by considering two extreme cases of very small and very large thermal-diffusion lengths.

In the small-diffusion-length limit, the film is poorly coupled with the bulk of the substrate. The temperature at the interface is free to fluctuate from the average temperature of the substrate, so the boundary condition in Eq. (14) is unimportant. In the flux equation, the relative importance of the two terms on the left-hand side may be estimated as follows. The length scale of temperature variation in the substrate is the thermal-diffusion length $l_s=(2\Lambda_s/\omega)^{1/2}$, where l_s is the thermal-diffusion constant in glass. Thus, we estimate that the ratio of the second term to the first is of the order $(K_s/K)^{1/2}$, and so we will neglect the second term from now on. As in the bulk fluctuation problem, the driving term $G(x,y,t)$ is taken as totally uncorrelated in space and time.

The boundary conditions on the other surfaces of the film must be reexamined. The free surface boundary condition, Eq. (2), continues to apply to the surface $z=l_3$. However, because of the constant flow of heat across the interface, the temperature distribution in the y direction can no longer maintain sinusoidal standing-wave patterns. The situation is analogous to a rectangular waveguide with one side open. The field distribution inside the waveguide cannot be described by a superposition of the normal modes of the closed waveguide. Since the width of the film is very large compared with its thickness, the effects of the $y=0$ and $y=l_2$ boundaries are limited to a small region near the edges. To a good approximation it is appropriate to treat the y dimension of the film as a part of a large solid, as was done in Ref. 2.

The noise spectrum of the film is the sum of a bulk fluctuation part and a part excited by the surface term $G(x,y,t)$. In the limit of short diffusion length in the substrate, the bulk fluctuation part of the spectrum is the same as that of the freely suspended film, which does not display $1/f$ behavior. In analyzing the effect of the surface excitation we drop the source term in Eq. (1), and write the solution of the diffusion equation as

$$T(\vec{r},t)=\sum_{q_xq_y}\int\frac{d\omega}{2\pi}\,T(q_x,q_y,\omega)\times\cos[q_z(z-l_3)]e^{iq_xx+iq_yy-i\omega t}, \tag{17}$$

where the boundary condition on the $z=l_3$ surface has been imposed. The wave vector q_z is not arbitrary but satisfies

$$q_z^2=i(\omega/\Lambda)-(q_x^2+q_y^2). \tag{18}$$

The boundary condition in Eq. (16) is satisfied by

$$T(q_x,q_y,\omega)=-G(q_x,q_y,\omega)/Kq_z\sin q_zl_3, \tag{19}$$

where $G(q_x,q_y,\omega)$ is the Fourier transform of $G(x,y,t)$. The average temperature over the film is found to be

$$\langle T(\vec{r},t)\rangle=\sum_{q_xq_y}\int\frac{d\omega}{2\pi}\,\frac{-G(q_x,q_y,\omega)}{Kl_3q_z^2}\times\frac{1-e^{iq_xl_1}}{iq_xl_1}\,\frac{1-e^{iq_yl_2}}{iq_yl_2}\,e^{-i\omega t}, \tag{20}$$

and the spectrum of the average temperature is

$$S_T(\omega)=\frac{G_0^2}{C^2l_3^2}\,\frac{l_1l_2}{(2\pi)^2}\int_{-\infty}^{\infty}dq_x\int_{-\infty}^{\infty}dq_y\,\frac{1}{\omega^2+\Lambda^2(q_x^2+q_y^2)^2}\times\left(\frac{\sin\frac{1}{2}q_xl_1}{\frac{1}{2}q_xl_1}\right)^2\left(\frac{\sin\frac{1}{2}q_yl_2}{\frac{1}{2}q_yl_2}\right)^2. \tag{21}$$

In the above equation the quantity G_0^2 is the square amplitude of $G(q_x,q_y,\omega)$.

There is close resemblance between this result and that derived by Voss and Clarke[2] by using a source term $P(\vec{r},t)$ in the diffusion equation, except that in our result there are only two independent wave vectors. The spectrum in various frequency regions can be calculated immediately by the method of Ref. 2. We obtain

$$S_T(\omega)\cong\begin{cases}G_0^2/C^2l_3^2\omega^2 & \text{for } \omega\gg\omega_2\\ G_0^2l_2/C^2\sqrt{2}\Lambda l_3^2\omega^{3/2} & \text{for } \omega_2\gg\omega\gg\omega_1\\ G_0^2l_1l_2/2C^2\Lambda l_3^2\omega & \text{for } \omega\ll\omega_1.\end{cases} \tag{22}$$

The spectrum has the $1/f$ frequency dependence when the frequency is lower than the lowest diffusion frequency $f_1=\omega_1/2\pi$. For the gold film studied in Ref. 2, we estimate $\Lambda=1.3$ cm^2/sec, $l_1=6.25\times10^{-2}$ cm, and consequently $f_1=106$ Hz. The $1/f$ spectrum was observed in the frequency range 1–300 Hz.[2] It is difficult to tell whether there exists a kink in the spectrum in the narrow frequency range above f_1.

We now consider the limit where the thermal-diffusion length in the substrate is comparable to the length of the film. This happens when the frequency is below $\omega_0=2\Lambda_s/l_1^2\cong0.3$ Hz. Under this condition, the temperature fluctuation of the film is strongly coupled to that of the substrate, and the fluctuation spectrum will be the same as that of a part of the substrate of volume $l_1\times l_2\times l_s$, where we take the effective thickness of the substrate as the thermal-diffusion length l_s. In Ref. 2 it was shown that under these conditions the noise spectrum of the substrate is independent of the frequency, so we infer that the $1/f$ spectrum is replaced by f^0 spectrum when $\omega<\omega_0$.

Finally, we normalize the spectrum according to the relation that at room temperature we must have

$$\int_0^\infty S_T(\omega)\,d\omega=\langle(\Delta T)^2\rangle=T^2/3N, \tag{23}$$

where N is the number of atoms in the film. The integral is cut off at ω_1 as a close approximation. This gives

$$S_T(\omega)=\frac{T^2}{3N\omega[1+\ln(\omega_1/\omega_0)]}. \tag{24}$$

The noise spectrum of the voltage fluctuation is obtained by applying the scale factor $(\overline{V}\beta)^2$, where $\overline{V}$ is the average voltage across the length of the film and β is the temperature coefficient of the electrical resistivity of the film. For the gold film on glass substrate studied in Ref. 2, we put in $\omega_1/\omega_0\cong K/K_s\cong300$, $T=293$ K, $\overline{V}=0.81$ V, $\beta=0.0012$ K^{-1}, and the dimensions of the film 250 Å$\times$8 μm

$\times 625$ μm. Then Eq. (24) predicts a voltage noise of 0.6×10^{-6} V^2/Hz at 10 Hz. This is in excellent agreement with the data.

The above analysis points to the source of $1/f$ noise as due to the random flow of heat across the metal-surface interface. It also shows that if the thermal coupling between the metal and the substrate is improved, the noise level should reduce as was observed by Clarke and Hsiang.[3] The $1/f$ noise can be suppressed very effectively if there is good thermal contact between the film and the substrate and the substrate has high thermal conductivity.

IV. $1/f$ NOISE IN METAL WHISKERS

Recently Dutta *et al.*[8] reported the observation of $1/f$ noise in freely standing copper crystal whiskers of various lengths. There are a number of outstanding features in the data that are difficult to explain. Among these are: (a) there is no apparent change in spectrum at the frequency ω_1 characteristic of the length of the whisker and (b) the noise level is two to three orders of magnitude higher than that observed in copper films of the same volume. We discuss here a possible mechanism of $1/f$ noise in thin metal whiskers. While it does not give a satisfactory explanation of the data in Ref. 8, it may shed light on the source of the noise.

In the experiment four gold wires, which are attached to the whisker with silver paint, are used as current and voltage leads. This should be contrasted with the film experiments in Refs. 2 and 3 in which the leads are contiguous parts of the film. As was discussed in the previous section, any junction between two substances may be a source of noise. So in the present model, we consider a one-dimensional heat-flow problem with noise sources at both ends. The one-dimensional model is justified in a thin whisker by the consideration in Sec. II A.

The temperature distribution in the whisker is

$$T(x,t)=\int\frac{d\omega}{2\pi}\left[T(q,\omega)e^{iqx}+T(-q,\omega)e^{-iqx}\right]e^{-i\omega t}\,, \tag{25}$$

where $\Lambda q^2+i\omega=0$. The boundary conditions are

$$T(0,t)=G_1(t),\quad T(l,t)=G_2(t)\,, \tag{26}$$

where both $G_1(t)$ and $G_2(t)$ are white-noise sources. Matching of the boundary conditions gives simple results for $T(\pm q,\omega)$. Then we find

$$T(x,t)=\int\frac{d\omega}{2\pi}\,\frac{e^{-i\omega t}}{\sin ql}\left[G_1(\omega)\sin q(l-x)+G_2(\omega)\sin qx\right]. \tag{27}$$

The average temperature of the wire is

$$\langle T(x,t)\rangle=\int\frac{d\omega}{2\pi}\,\frac{1-\cos ql}{ql\sin ql}\left[G_1(\omega)+G_2(\omega)\right]e^{-i\omega t}\,, \tag{28}$$

and the spectrum of the average temperature is

$$S_T(\omega)=(|G_1|^2+|G_2|^2)|(\tan\tfrac{1}{2}ql)/ql|^2\,, \tag{29}$$

where $|G_1|^2$ and $|G_2|^2$ are both constants. The quantity

$$|ql|=(\omega l^2/\Lambda)^{1/2}=(2\omega/\omega_1)^{1/2}\,,$$

where $\omega_1=2\Lambda/l^2$ is the characteristic frequency. For $\omega\gg\omega_1$ one can find that $|\tan\frac{1}{2}ql|\cong 1$, so

$$S_T(\omega)\propto\omega^{-1}. \tag{30}$$

For $\omega\ll\omega_1$ the spectrum becomes frequency independent. At the high-frequency end the $1/f$ spectrum is cut off at $\omega_2=2\Lambda/d^2$, where d is the diameter of the whisker.

The spectrum is easy to normalize in an approximate manner, with the result

$$S_T(\omega)=\begin{cases}A/\omega_1, & 0<\omega<\omega_1\\ A/\omega, & \omega_1<\omega<\omega_2\end{cases} \tag{31}$$

where $A=kT^2/C[1+\ln(\omega_2/\omega_1)]$. Putting in the typical sizes $l=1$ cm, $d=3$ μm, and $C=3.5\times 10^7$ erg/cm^3 – K for copper at $T=300$ K, we find that $A=1.7\times 10^3/N$, where N is the number of atoms in the whisker. To convert this to the voltage fluctuation spectrum, we multiply A by β^2, where $\beta=0.0038$ K^{-1} is the temperature coefficient of electrical resistivity for copper. This gives

$$S_V(f)/\overline{V}^2=7.7\times 10^{-3}/Nf. \tag{32}$$

This is comparable to the noise level in metal films.[2] The noise level observed in Ref. 8 is two to three orders of magnitude higher.

The implication of this calculation is that the observed noise cannot be due to equilibrium temperature fluctuation in the whisker. The high noise level suggests that a small volume is involved. This seems to indicate that the temperature fluctuation in the junctions is responsible for the noise. Further experiments are necessary to test this speculation.

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